Lecture - 12

Friday, 19 August 2016 (16:15- 17:05)

Contention Resolution

In this lecture we read the subsection titled "Contention Resolution" from the chapter "Randomized Algorithms" in the book "Algorithm Design" by Kleinberg and Tardos.

Further, we discussed the following variant to the contention resolution problem: Let say the processor accepts precisely two jobs, unlike the previous scenario where only one job was accepted. Hence, a round is termed a "failure" for the processor if zero, one, three, four, five..., or n (where n is the number of processes) processes try accessing the processor in this particular round, otherwise the round is termed a "success".

Further let p represent the probability with which any process asks for the processor in a given round. Let $E[i,t]$ represent the event: i^{th} process receives the processor's time in the t^{th} round. Our aim will be to find the value of p that maximizes $E[i,t]$.

Lemma 1. $P(E[i,t]) = (n-1)p^2(1-p)^{n-2}$

The above lemma follows directly from the definition of $E[i,t]$. Further let us define a function $f(x) = (n-1)x^2(1-x)^{n-2}$, where x ranges from 0 to 1.

Lemma 2. *The function $f(x)$ reaches its maximum value at $x = 2/n$.*

Proof. First we will find the root of $f'(x)$ and further check if the root(s) represents a maxima or a minima.

$$\begin{aligned}
0 = f'(x) &= (n-1)(2x(1-x)^{n-2} - (n-2)x^2(1-x)^{n-3}) \\
&\implies 0 = 2(1-x) - x(n-2) \\
&\implies 0 = 2 - nx \\
&\implies \text{2/n is the only root of } f'(x)
\end{aligned}$$

Further observe that $f''(2/n) < 0$, hence the function f reaches its maximum value at $2/n$. $\square$

Q) Further try solving the following variant of the contention resolution problem: There are n processes $P_1, P_2, \ldots . P_n$ who wish to acquire some of the processor's time in each round. Let say each process asks for the processor's time with probability p in each round. Further suppose that the processor accepts requests in each round if the number of requests are less than or equal to 2 (i.e. 1 or 2). Let $E[i,t]$ represent the event: i^{th} process receives the processor's time in the t^{th} round. Find the value of p for which the $E[i,t]$ is maximized.