# Quantum computation: quantum process and characterization 

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Quantum computers look like...

AMO


Solid-state


Quantum bit

## Spin

## Oscillator

(4) 1 \&

Magnetic field

$$
\frac{10>}{\frac{1}{g<0}}
$$

(a)


(c)

(d)


Pure state and mixed state


QuTech

Pure state and mixed state


## QuTech

## Relaxation and dephasing

## Figure: Nielsen \& Chuang

$$
\rho_{\Psi}=|\Psi\rangle\langle\Psi|=\left(\begin{array}{cc}
a^{2} & a b^{*} \\
a^{*} b & b^{2}
\end{array}\right)
$$



## Relaxation:

$$
\rho_{\Psi^{\prime}}=\left(\begin{array}{cc}
1-e^{-\frac{t}{T_{1}}} & \left.1-a^{2}\right) \\
e^{-\frac{t}{2 T_{1}}} a b^{*} \\
e^{-\frac{t}{2 T_{1}}} a^{*} b & e^{-\frac{t}{T_{1}}} b^{2}
\end{array}\right) \rightarrow\left(\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right)
$$



Dephasing:

$$
\rho_{\Psi^{\prime}}=\left(\begin{array}{cc}
a^{2} & e^{-\left(\frac{t}{T_{2}}\right)^{(2)}} a b^{*} \\
e^{-\left(\frac{t}{T_{2}}\right)^{(2)}} a^{*} b & b^{2}
\end{array}\right) \rightarrow\left(\begin{array}{cc}
a^{2} & 0 \\
0 & b^{2}
\end{array}\right)
$$

## Depolarizing process

$$
\rho_{\Psi}=|\Psi\rangle\langle\Psi|=\left(\begin{array}{cc}
a^{2} & a b^{*} \\
a^{*} b & b^{2}
\end{array}\right)
$$

Figure: Nielsen \& Chuang


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## State Fidelity

$$
F=\operatorname{Tr}\left(\rho_{\text {mea }} \rho_{\text {ref }}\right)
$$

Consider $\rho_{\text {ref }}=|\alpha\rangle\langle\alpha|$ as a pure state

$$
F=\operatorname{Tr}\left(\rho_{\text {mea }} \rho_{\text {ref }}\right)=\langle\alpha| \rho_{\text {mea }}|\alpha\rangle
$$

Recap:

For a pure state $\rho$ : $\operatorname{Tr}\left(\rho^{2}\right)=1$

For a mixed state $\rho: \quad \operatorname{Tr}\left(\rho^{2}\right)<1$

## Measurement fidelity

Let's think about it a bit further...

$$
F=\operatorname{Tr}\left(\rho_{\text {mea }} \rho_{\text {ref }}\right)=\langle\alpha| \rho_{\text {mea }}|\alpha\rangle
$$

Consider $\rho_{\text {mea }}$ as known, but $\rho_{\text {ref }}=|\alpha\rangle\langle\alpha|$ is unknown... F gives "measurement fidelity".

Faulty measurement: $\rho_{\text {ref }}=p_{\alpha}|\alpha\rangle\langle\alpha|+p_{\beta}|\beta\rangle\langle\beta|$

$$
F=\operatorname{Tr}\left(\rho_{\text {mea }} \rho_{\text {ref }}\right)=p_{\alpha}\langle\alpha| \rho_{\text {mea }}|\alpha\rangle+p_{\beta}\langle\beta| \rho_{\text {mea }}|\beta\rangle
$$

## Positive operator-valued measurement

For an ideal POVM, $E_{m}=\left|\psi_{m}\right\rangle\left\langle\psi_{m}\right|$, with $\left|\psi_{m}\right\rangle$ often chosen to be the eigenstates.
Outcome: $\quad P_{m}=\operatorname{Tr}\left(|\psi\rangle\langle\psi| E_{m}\right) \quad$ Single qubit: $\left\{E_{0}=|0\rangle\langle 0|, E_{1}=|1\rangle\langle 1|\right\}$

Measurement error:

$$
\left\{E_{0}^{e}=\left(1-\epsilon_{0}\right)|0\rangle\langle 0|+\epsilon_{1}|1\rangle\langle 1|, E_{1}^{e}=\epsilon_{0}|0\rangle\langle 0|+\left(1-\epsilon_{1}\right)|1\rangle\langle 1|\right\}
$$

$\epsilon_{0}$ and $\epsilon_{1}$ are the readout errors of $|0\rangle$ and $|1\rangle$ states

Measurement fidelity:

$$
\begin{aligned}
& F_{0}=\operatorname{Tr}\left(E_{0}^{e} E_{0}\right)=\operatorname{Tr}\left(E_{0}^{e}|0\rangle\langle 0|\right)=1-\epsilon_{0} \\
& F_{1}=\operatorname{Tr}\left(E_{1}^{e} E_{1}\right)=\operatorname{Tr}\left(E_{1}^{e}|1\rangle\langle 1|\right)=1-\epsilon_{1}
\end{aligned}
$$

## Quantum operation/channel

Consider a state

$$
\rho=|\alpha\rangle\langle\alpha|
$$

$$
\rho=p_{\alpha}|\alpha\rangle\langle\alpha|+p_{\beta}|\beta\rangle\langle\beta|
$$

Define a quantum operation/channel

$$
\Lambda(\rho)=K|\alpha\rangle\langle\alpha| K^{\dagger} \quad \Lambda(\rho)=p_{\alpha} K|\alpha\rangle\langle\alpha| K^{\dagger}+p_{\beta} K|\beta\rangle\langle\beta| K^{\dagger}
$$

Sometimes the channel is a statistical mixture of multiple operators

$$
\Lambda(\rho)=K_{1} \rho K_{1}^{\dagger}+K_{2} \rho K_{2}^{\dagger}+\cdots
$$

$$
K_{1}^{\dagger} K_{1}+K_{2}^{\dagger} K_{2}+\cdots=I
$$

defined as Kraus operators

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Dephasing channel
If $\rho$ completely loses its phase information:

$$
\rho=\left(\begin{array}{ll}
a & c^{*} \\
c & b
\end{array}\right)
$$

$$
\mathcal{K}_{\text {dephase }_{\text {max }}}(\rho)=\frac{\rho+Z \rho Z}{2}=a|0\rangle\langle 0|+b|1\rangle\langle 1|
$$

The Kraus operators are $I / \sqrt{2}$ and $Z / \sqrt{2}$.

Often $\rho$ loses its phase with probability (error rate) $1-p$

$$
\mathcal{K}_{\text {dephase }}(\rho)=p \rho+\frac{1-p}{2}(\rho+Z \rho Z)=\frac{1+p}{2} \rho+\frac{1-p}{2} Z \rho Z
$$

with the Kraus operators $\sqrt{(1+p) / 2} I$ and $\sqrt{(1-p) / 2} Z$.

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Depolarizing channel
If $\rho$ completely loses its information in all directions:

$$
\rho=\left(\begin{array}{ll}
a & c^{*} \\
c & b
\end{array}\right)
$$

$$
\mathcal{K}_{d e p_{\max }}(\rho)=\frac{\rho+X \rho X+Y \rho Y+Z \rho Z}{4}=I
$$

The Kraus operators of the maximum-depolarizing channel $\mathcal{K}_{\text {dep }_{\max }}$ are $I / 2, X / 2, Y / 2$, and $Z / 2$.

Often $\rho$ loses its information with probability (error rate) $1-p$ :

$$
\begin{aligned}
\mathcal{K}_{\text {dep }}(\rho) & =p \rho+(1-p) I / 2 \\
& =\frac{1+3 p}{4} \rho+\frac{1-p}{4}(X \rho X+Y \rho Y+Z \rho Z)
\end{aligned}
$$

Here, the Kraus operators are $\sqrt{1+3 p} I / 2, \sqrt{1-p} X / 2, \sqrt{1-p} Y / 2$, and $\sqrt{1-p} Z / 2$.

Recurring $\mathcal{K}_{\text {dep }}$ will finally fully depolarize the state:

$$
\lim _{N \rightarrow \infty} \mathcal{K}_{\text {dep }}{ }^{N}(\rho)=\mathcal{K}_{\text {dep }_{\text {max }}}(\rho)=I .
$$

## Kraus operator in Pauli basis: $\chi$-matrix

$$
\begin{array}{lll} 
& P_{0}=I \\
P_{1}=\sum^{2} & P_{j}=X \\
P_{2} & a_{i j} P_{j} & P_{1}=Y \\
P_{3}=Z
\end{array}
$$

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Single qubit case:

$$
K_{i}=a_{i 0} I+a_{i 1} X+a_{i 2} Y+a_{i 3} Z
$$

$$
\chi_{j k}=\sum_{i} a_{i j} a_{i k}^{*}
$$

$\chi$-matrix is a $d^{2} \times d^{2}$ dimensional complex matrix

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## Examples of $\chi$-matrices

$$
\begin{aligned}
& \mathcal{K}_{\text {dephase }}(\rho)=p \rho+\frac{1-p}{2}(\rho+Z \rho Z) \\
& =\frac{1+p}{2} \rho+\frac{1-p}{2} Z \rho Z \\
& \chi_{\text {dephasing }}=\left(\begin{array}{cccc}
I & X & Y & Z \\
\frac{1+p}{2} 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1-p}{2}
\end{array}\right) \quad \begin{array}{c} 
\\
X \\
Y \\
Z
\end{array} \\
& \mathcal{K}_{\text {dep }}(\rho)=p \rho+(1-p) I \\
& =\frac{1+3 p}{4} \rho+\frac{1-p}{4}(X \rho X+Y \rho Y+Z \rho Z) \\
& \chi_{\text {dep }}=\left(\begin{array}{cccc}
\frac{1+3 p}{4} & 0 & 0 & 0 \\
0 & \frac{1-p}{4} & 0 & 0 \\
0 & 0 & \frac{1-p}{4} & 0 \\
0 & 0 & 0 & \frac{1-p}{4}
\end{array}\right)
\end{aligned}
$$

- $\chi$-matrix is a matrix of coefficients.
- It's always symmetric.


## Examples of $\chi$-matrices

It's difficult to use $\chi$-matrix for multiple operations


$$
\Lambda(\rho)=\sum_{j, k=1}^{d^{2}} \chi_{j k} P_{j} \rho P_{k}
$$

$$
\Lambda_{2}\left(\Lambda_{1}(\rho)\right)=\sum_{p, q=1}^{d^{2}} \chi_{p q}^{2} P_{p}\left(\sum_{j, k=1}^{d^{2}} \chi_{j k}^{1} P_{j} \rho P_{k}\right) P_{q}
$$

## QuTech

## Superoperators

## Revision of state fidelity

Operators can be decomposed into Paulis

Same for density matrices (density operators)

$$
K_{i}=a_{i 0} I+a_{i 1} X+a_{i 2} Y+a_{i 3} Z
$$

$$
\rho=\frac{1}{2}\left(I+r_{x} X+r_{y} Y+r_{z} Z\right)
$$

Now let's imagine performing POVM:

On the $x$-axis: $\quad F=\operatorname{Tr}(\rho X)=r_{x}$

On the $y$-axis: $\quad F=\operatorname{Tr}(\rho Y)=r_{y}$

$$
\begin{aligned}
& \operatorname{Tr}(I X)=0 \\
& \operatorname{Tr}(Y X)=0 \\
& \operatorname{Tr}(Z X)=0 \\
& \operatorname{Tr}(X X)=1
\end{aligned}
$$

On the $z$-axis: $\quad F=\operatorname{Tr}(\rho Z)=r_{z}$

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## Quantum state tomography

How to perform POVM along the $x$-axis?

$$
F=\operatorname{Tr}\left(\rho \rho_{r e f}\right)
$$

- $\quad \rho_{\text {ref }}$ must be a physical state (not necessarily a

$$
\begin{aligned}
X & =\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)=\frac{1}{2}\left(\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right)-\frac{1}{2}\left(\begin{array}{cc}
1 & -1 \\
-1 & 1
\end{array}\right) \\
& =\frac{1}{2}(|0\rangle+|1\rangle)(\langle 0|+\langle 1|)-\frac{1}{2}(|0\rangle-|1\rangle)(\langle 0|-\langle 1|) \\
r_{x} & =\operatorname{Tr}(\rho X)=\operatorname{Tr}\left(\rho \rho_{0+1}\right)-\operatorname{Tr}\left(\rho \rho_{0-1}\right) \\
r_{y} & =\operatorname{Tr}(\rho Y)=\operatorname{Tr}\left(\rho \rho_{0+i 1}\right)-\operatorname{Tr}\left(\rho \rho_{0-i 1}\right) \\
r_{z} & =\operatorname{Tr}(\rho Z)=\operatorname{Tr}\left(\rho \rho_{0}\right)-\operatorname{Tr}\left(\rho \rho_{1}\right)
\end{aligned}
$$

pure state), with $\operatorname{Tr}\left(\rho_{r e f}\right)=1$.

- Pauli matrices cannot be prepared as they are not density matrices of any physical state.



## Super-operator and Pauli transfer matrix

$$
\rho=\frac{1}{2}\left(I+r_{x} X+r_{y} Y+r_{z} Z\right) \stackrel{\text { def }}{=}\left(\begin{array}{c}
\operatorname{Tr}(\rho I) \\
\operatorname{Tr}(\rho X) \\
\operatorname{Tr}(\rho Y) \\
\operatorname{Tr}(\rho Z)
\end{array}\right)=\left(\begin{array}{c}
1 \\
r_{x} \\
r_{y} \\
r_{z}
\end{array}\right)
$$

- A $d \times d$ dimensional density matrix can be represented now as a $d^{2}$ dimensional vector.
- The first entry is always 1 : Trace preserving (TP).
- The other entries correspond to the projection onto $x / y / z$ axes in Bloch sphere.

Pauli transfer matrix (quantum channel)

$$
\Lambda_{i j}^{\mathcal{K}}=\frac{1}{d} \operatorname{Tr}\left[P_{i} \mathcal{K}\left(P_{j}\right)\right] \quad \rho=\frac{1}{2}\left(I+r_{x} X+r_{y} Y+r_{z} Z\right) \stackrel{\text { def }}{=}\left(\begin{array}{c}
1 \\
r_{x} \\
r_{y} \\
r_{z}
\end{array}\right)
$$

PTM is expressed in the Pauli operator basis, meaning it can be directly applied to a state vector in the super-operator format

$$
\begin{array}{lc}
|\rho\rangle\rangle=\frac{1}{\sqrt{2}}\left(\begin{array}{c}
\operatorname{Tr}(\rho I) \\
\operatorname{Tr}(\rho X) \\
\operatorname{Tr}(\rho Y) \\
\operatorname{Tr}(\rho Z)
\end{array}\right)=\frac{1}{\sqrt{2}}\left(\begin{array}{c}
1 \\
r_{x} \\
r_{y} \\
r_{z}
\end{array}\right) & \{I / \sqrt{d}, X / \sqrt{d}, Y / \sqrt{d}, Z / \sqrt{d}\} \\
\left\langle\langle E|=\frac{1}{\sqrt{2}}(1, \operatorname{Tr}(E X), \operatorname{Tr}(E Y), \operatorname{Tr}(E Z)) \quad\right. \text { State/measurement fidelity: } \\
\operatorname{Tr}[E \rho]=\langle\langle E \mid \rho\rangle\rangle
\end{array}
$$

## Examples of PTMs

$$
\begin{aligned}
\mathcal{K}_{\text {dephase }}(\rho) & =p \rho+\frac{1-p}{2}(\rho+Z \rho Z) \\
& =\frac{1+p}{2} \rho+\frac{1-p}{2} Z \rho Z
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K}_{d e p}(\rho) & =p \rho+(1-p) I \\
& =\frac{1+3 p}{4} \rho+\frac{1-p}{4}(X \rho X+Y \rho Y+Z \rho Z)
\end{aligned}
$$

Output density operator

$$
\begin{gathered}
\Lambda_{\text {dephasing }}=\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & p & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & 1
\end{array}\right) \quad \begin{array}{l}
I \\
X \\
Y \\
Z
\end{array} \quad \Lambda_{\text {dep }}=\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & p & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & p
\end{array}\right) \\
I \\
X
\end{gathered} \quad Y \quad Z \quad \begin{array}{ll}
\text { Input density operator }
\end{array}
$$

- $\chi$-matrix is a matrix of input-output.
- It's not always symmetric.


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$$
\begin{aligned}
\mathcal{K}_{\text {dephase }}(\rho) & =p \rho+\frac{1-p}{2}(\rho+Z \rho Z) & & \rho=\frac{1}{2}\left(I+r_{x} X+r_{y} Y+r_{z} Z\right) \\
& =\frac{1+p}{2} \rho+\frac{1-p}{2} Z \rho Z & &
\end{aligned}
$$

Output density operator
$\Lambda_{\text {dephasing }}=\left(\begin{array}{llll}1 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & 1\end{array}\right) \quad \begin{gathered}I \\ X \\ I \\ X\end{gathered}$
Input density operator

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## Pauli transfer matrix


$\Lambda_{2}\left(\Lambda_{1}(\rho)\right)=\sum_{p, q=1}^{d^{2}} \chi_{p q}^{2} P_{p}\left(\sum_{j, k=1}^{d^{2}} \chi_{j k}^{1} P_{j} \rho P_{k}\right) P_{q}$

A complete circuit:
$\left.p_{E}=\left\langle\langle E| \mathcal{G}_{N} \ldots \mathcal{G}_{2} \mathcal{G}_{1} \mid \rho\right\rangle\right\rangle$
$\Lambda_{2}\left(\Lambda_{1}(\rho)\right) \Rightarrow G_{2} G_{1} \mid \rho \gg$
$G_{i}$ is the PTM of $\Lambda_{i}$
where $\mathcal{G}_{i}$ is the PTM of the $i$-th gate.

Average gate fidelity:

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$$
F_{g}=\frac{\operatorname{Tr}\left[\mathcal{G}^{-1} \mathcal{G}^{i d e a l}\right]+d}{d(d+1)}
$$

Example: controlled-Z gate
Red: +1
Red: +1/4
Blue: -1


# Characterize a quantum process 

## Quantum process tomography

Measure the complete input-output correlation.

Intuition: prepare different states in experiment, and apply the operator on them, followed by measurement of the outcome states.


This can be described in either $\chi$-matrix or PTM, but PTM is easier.

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## Quantum process tomography

Entry of a PTM

$$
\mathcal{G}_{i j}=\left\langle\left\langle P_{i}\right| \mathcal{G} \mid P_{j}\right\rangle
$$

$$
\begin{aligned}
\mathcal{G}_{13} & =\frac{1}{2}\langle\langle X| \mathcal{G} \mid Z\rangle \\
& \left.=\frac{1}{2}\left\langle\left\langle\rho_{\hat{x}}-\rho_{-\hat{x}}\right| \mathcal{G} \mid \rho_{0}-\rho_{1}\right\rangle\right\rangle
\end{aligned}
$$

$$
\rho_{ \pm \hat{x}} \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle \pm|1\rangle)
$$

$$
\rho_{0 / 1} \longrightarrow \quad|0\rangle /|1\rangle
$$

Therefore, a PTM $\mathcal{G}$ can be fully reconstructed by preparing and measuring the state in all the basis states $\left\{\rho_{0}, \rho_{1}, \rho_{\hat{x}}, \rho_{-\hat{x}}, \rho_{\hat{y}}, \rho_{-\hat{y}}\right\}$ before and after the process respectively.

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## Quantum process tomography

Entry of a PTM

$$
\mathcal{G}_{i j}=\left\langle\left\langle P_{i}\right| \mathcal{G} \mid P_{j}\right\rangle
$$

$$
\begin{aligned}
\mathcal{G}_{13} & \left.=\frac{1}{2}\langle\langle X| \mathcal{G} \mid Z\rangle\right\rangle \\
& \left.=\frac{1}{2}\left\langle\left\langle\rho_{\hat{x}}-\rho_{-\hat{x}}\right| \mathcal{G} \mid \rho_{0}-\rho_{1}\right\rangle\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
& \rho_{ \pm \hat{x}} \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle \pm|1\rangle) \\
& \rho_{0 / 1} \longrightarrow \quad|0\rangle /|1\rangle
\end{aligned}
$$

Limitation:
State Preparation and Measurement (SPAM) error
Solution:

- Gate set tomography
- Randomized benchmarking
- Others...


# Randomized benchmarking 

Depolarizing error

$$
F_{g}=\frac{\operatorname{Tr}\left[\mathcal{G}^{-1} \mathcal{G}^{\text {ideal }}\right]+d}{d(d+1)} \quad \mathcal{G}=\Lambda \mathcal{G}^{\text {ideal }} \quad \quad \text { Error: } \Lambda
$$

$$
\operatorname{Tr}\left[\mathcal{G}^{-1} \mathcal{G}^{\text {ideal }}\right]=\operatorname{Tr}\left[\Lambda^{-1} I\right]
$$

Coherent error: rotation angle, rotation axis... Incoherent error: dephasing, depolarizing, relaxation...

Assumptions:

1. The error of a gate does not depend on the previous gates (Markovian).
2. The error of different gates are the same (gate-independent).
(Imagine the error is decoherence and all gates are equally long)

## Depolarizing error

"Twirling" a small error with unitary operators.

$$
\Lambda_{d e p} \approx \sum_{i} \mathcal{U}_{i}^{\dagger} \Lambda \mathcal{U}_{i}
$$

In reality, we can use Clifford group to efficiently approximate the twirling process

$$
\Lambda_{\text {dep }}(\rho)=\frac{1}{K} \sum_{k=1}^{K} \mathcal{C}_{k}^{\dagger} \Lambda \mathcal{C}_{k}|\rho\rangle \quad \quad \Lambda_{\text {dep }}(\rho)=p \rho+(1-p) I / d
$$

Fidelity of error $\Lambda$ :

$$
F_{\Lambda}=F_{\Lambda_{d e p}} \quad F_{\Lambda}=\operatorname{Tr}\left[\Lambda^{-1} I\right]=p+\frac{1-p}{d}
$$

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Clifford gates
Clifford stabilizes Paulis:

$$
C_{k} P_{i} C_{k}^{\dagger}=P_{j}
$$



| Single qubit Cliffords |  |
| :---: | :---: |
| Paulis | I |
|  | X |
|  | Y |
|  | Y, X |
| $2 \pi / 3$ rotations | $\mathrm{X} / 2, \mathrm{Y} / 2$ |
|  | $\mathrm{X} / 2,-\mathrm{Y} / 2$ |
|  | -X/2, Y/2 |
|  | -X/2, -Y/2 |
|  | $\mathrm{Y} / 2, \mathrm{X} / 2$ |
|  | $\mathrm{Y} / 2,-\mathrm{X} / 2$ |
|  | -Y/2, X/2 |
|  | -Y/2, -X/2 |
| $\pi / 2$ rotations | X/2 |
|  | -X/2 |
|  | Y/2 |
|  | -Y/2 |
|  | -X/2, Y/2, X/2 |
|  | -X/2, -Y/2, X/2 |
| Hadamard-like | $\mathrm{X}, \mathrm{Y} / 2$ |
|  | $\mathrm{X}, \quad-\mathrm{Y} / 2$ |
|  | $\mathrm{Y}, \mathrm{X} / 2$ |
|  | $\mathrm{Y}, \quad-\mathrm{X} / 2$ |
|  | $\mathrm{X} / 2, \mathrm{Y} / 2, \mathrm{X} / 2$ |
|  | -X/2, Y/2, -X/2 |

R. Barends et al., Nature 2014

## Clifford gates



## QuTech

## Clifford gates



| Single qubit Cliffords |  |
| :---: | :---: |
|  |  |
| Paulis | X |
|  | Y |
|  | Y, X |
| $2 \pi / 3$ rotations | $\mathrm{X} / 2, \mathrm{Y} / 2$ |
|  | $\mathrm{X} / 2,-\mathrm{Y} / 2$ |
|  | -X/2, Y/2 |
|  | -X/2, -Y/2 |
|  | $\mathrm{Y} / 2, \mathrm{X} / 2$ |
|  | $\mathrm{Y} / 2,-\mathrm{X} / 2$ |
|  | -Y/2, X/2 |
|  | -Y/2, -X/2 |
|  | $\mathrm{X} / 2$ |
| $\pi / 2$ rotations | -X/2 |
|  | Y/2 |
|  | -Y/2 |
|  | -X/2, Y/2, X/2 |
|  | -X/2, -Y/2, X/2 |
| Hadamard-like | $\mathrm{X}, \mathrm{Y} / 2$ |
|  | $\mathrm{X}, \quad-\mathrm{Y} / 2$ |
|  | $\mathrm{Y}, \mathrm{X} / 2$ |
|  | $\mathrm{Y}, \quad-\mathrm{X} / 2$ |
|  | $\mathrm{X} / 2, \mathrm{Y} / 2, \mathrm{X} / 2$ |
|  | -X/2, Y/2, -X/2 |

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## Clifford gates



| Single qubit Cliffords |  |  |
| :---: | :---: | :---: |
| Paulis | I |  |
|  | X |  |
|  | Y |  |
|  | Y, | X |
| $2 \pi / 3$ rotations | X/2, | Y/2 |
|  | X/2, | -Y/2 |
|  | -X/2, | Y/2 |
|  | -X/2, | -Y/2 |
|  | Y/2, | $\mathrm{X} / 2$ |
|  | Y/2, | -X/2 |
|  | -Y/2, | $\mathrm{X} / 2$ |
|  | -Y/2, | -X/2 |
| $\pi / 2$ rotations | X/2 |  |
|  | -X/2 |  |
|  | Y/2 |  |
|  | -Y/2 |  |
|  | -X/2, | $\mathrm{Y} / 2, \mathrm{X} / 2$ |
|  | -X/2, | -Y/2, X/2 |
| Hadamard-like | X, |  |
|  | X, | -Y/2 |
|  | Y, | X/2 |
|  |  | -X/2 |
|  | X/2, | Y/2, X/2 |
|  | -X/2, | $\mathrm{Y} / 2,-\mathrm{X} / 2$ |

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## Clifford gates



| Single qubit Cliffords |  |  |
| :---: | :---: | :---: |
|  |  |  |
| Paulis | X |  |
|  | Y |  |
|  | Y, | X |
| $2 \pi / 3$ rotations | $\mathrm{X} / 2$, |  |
|  |  | -Y/2 |
|  | -X/2, | Y/2 |
|  | -X/2, | -Y/2 |
|  | Y/2, | $\mathrm{X} / 2$ |
|  | Y/2, | -X/2 |
|  | -Y/2, | $\mathrm{X} / 2$ |
|  | -Y/2, | -X/2 |
| $\pi / 2$ rotations | X/2 |  |
|  | -X/2 |  |
|  | Y/2 |  |
|  | -Y/2 |  |
|  | -X/2, | $\mathrm{Y} / 2, \mathrm{X} / 2$ |
|  | -X/2, | -Y/2, X/2 |
| Hadamard-like | X, |  |
|  |  | -Y/2 |
|  |  | $\mathrm{X} / 2$ |
|  | Y, | -X/2 |
|  | $\mathrm{X} / 2$, | Y/2, X/2 |
|  | -X/2, | Y/2, -X/2 |

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## Concatenated depolarizing channel

1-step depolarizing channel:

$$
\left.\Lambda_{\text {dep }}(\rho)=\frac{1}{K} \sum_{k=1}^{K} \mathcal{C}_{k}^{\dagger} \Lambda \mathcal{C}_{k}|\rho\rangle\right\rangle
$$

Outcome state:

$$
\Lambda_{d e p}(\rho)=p \rho+(1-p) I / d
$$

m-step depolarizing channel :
$\Lambda_{d e p}^{m}(\rho)=\int_{k_{m}, \ldots, k_{2}, k_{1}} \frac{1}{K^{m}} \mathcal{C}_{k_{m}}^{\dagger} \Lambda \mathcal{C}_{k_{m}} \ldots \mathcal{C}_{k_{2}}^{\dagger} \Lambda \mathcal{C}_{k_{2}} \mathcal{C}_{k_{1}}^{\dagger} \Lambda \mathcal{C}_{k_{1}}|\rho\rangle$

Outcome state:

$$
\begin{aligned}
\Lambda_{d e p}^{m}(\rho) & \left.=\Lambda_{\operatorname{dep}_{m}} \ldots \Lambda_{\operatorname{dep}_{2}} \Lambda_{\operatorname{dep}_{1}}|\rho\rangle\right\rangle \\
& =p^{m} \rho+\frac{1-p^{m}}{d} I
\end{aligned}
$$

Sequence fidelity:

$$
\begin{aligned}
F_{\text {seq }}\left(\rho_{\psi}\right) & =\operatorname{Tr}\left[E_{\psi} \Lambda^{m}\left(\rho_{\psi}\right)\right] \\
& =\operatorname{Tr}\left[E_{\psi}\left(\rho_{\psi}-\frac{I}{d}\right)\right] p^{m}+\operatorname{Tr}\left[E_{\psi} \frac{I}{d}\right] \\
& =A p^{m}+B .
\end{aligned}
$$

Fidelity of error $\Lambda$ :

$$
F_{\Lambda}=\operatorname{Tr}\left[\Lambda^{-1} I\right]=p+\frac{1-p}{d}
$$

Randomized benchmarking

$$
\Lambda_{d e p}^{m}(\rho)=\int_{k_{m}, \ldots, k_{2}, k_{1}} \frac{1}{K^{m}} \mathcal{C}_{k_{m}}^{\dagger} \Lambda \mathcal{C}_{\left.k_{m} \ldots \mathcal{C}_{k_{2}}^{\dagger} \Lambda \mathcal{C}_{k_{2}} \mathcal{C}_{k_{1}}^{\dagger} \Lambda \mathcal{C}_{k_{1}}|\rho\rangle\right\rangle \quad \begin{aligned}
m & \Lambda_{d e p}^{m}(\rho)
\end{aligned}=\Lambda_{\left.d e p_{m} \ldots \Lambda_{d e p_{2}} \Lambda_{d e p_{1}}|\rho\rangle\right\rangle}=p^{m} \rho+\frac{1-p^{m}}{d} I}
$$

Now let $\mathcal{C}_{l_{1}}=\mathcal{C}_{k_{1}}, \mathcal{C}_{l_{2}}=\mathcal{C}_{k_{2}} \mathcal{C}_{k_{1}}^{\dagger}, \ldots$, and $\mathcal{C}_{l_{m}}=\mathcal{C}_{k_{m}} \mathcal{C}_{k_{m-1}}^{\dagger}$, and let $\mathcal{C}_{l_{m+1}}=\mathcal{C}_{k_{m}}^{\dagger}=\left(\mathcal{C}_{l_{m}} \ldots \mathcal{C}_{l_{2}}\right.$ $\left.\mathcal{C}_{l_{1}}\right)^{\dagger}$, the sequence can be written as:

$$
\left.\Lambda_{d e p}^{m}(\rho)=\int_{l_{m}, \ldots, l_{2}, l_{1}} \frac{1}{K^{m}} \mathcal{C}_{l_{m+1}} \Lambda \mathcal{C}_{l_{m}} \ldots \Lambda \mathcal{C}_{l_{2}} \Lambda \mathcal{C}_{l_{1}}|\rho\rangle\right\rangle .
$$

Recall: $\mathcal{G}=\Lambda \mathcal{G}^{\text {ideal }}$
RB measures Clifford gate fidelity.

## QuTech

Randomized benchmarking

$$
\Lambda_{d e p}^{m}(\rho)=\int_{l_{m}, \ldots, l_{2}, l_{1}} \frac{1}{K^{m}} \mathcal{C}_{l_{m+1}} \Lambda \mathcal{C}_{l_{m}} \ldots \Lambda \mathcal{C}_{l_{2}} \Lambda \mathcal{C}_{l_{1}}|\rho\rangle .
$$



A real Clifford gate with error

$$
\begin{aligned}
\Lambda_{d e p}^{m}(\rho) & =\Lambda_{\text {dep } \left._{m} \ldots \Lambda_{d e p_{2}} \Lambda_{d e p_{1}}|\rho\rangle\right\rangle} \\
& =p^{m} \rho+\frac{1-p^{m}}{d} I
\end{aligned}
$$

$$
F_{\Lambda}=\operatorname{Tr}\left[\Lambda^{-1} I\right]=p+\frac{1-p}{d}
$$

## QuTech

## Randomized benchmarking

$$
\Lambda_{d e p}^{m}(\rho)=\int_{k_{m}, \ldots, k_{2}, k_{1}} \frac{1}{K^{m}} \mathcal{C}_{k_{m}}^{\dagger} \Lambda \mathcal{C}_{\left.k_{m} \ldots \mathcal{C}_{k_{2}}^{\dagger} \Lambda \mathcal{C}_{k_{2}} \mathcal{C}_{k_{1}}^{\dagger} \Lambda \mathcal{C}_{k_{1}}|\rho\rangle\right\rangle \longrightarrow \begin{array}{l}
\Lambda_{d e p}^{m}(\rho)
\end{array}=\Lambda_{\left.\operatorname{dep}_{m} \ldots \Lambda_{d e p_{2}} \Lambda_{d e p_{1}}|\rho\rangle\right\rangle}=p^{m} \rho+\frac{1-p^{m}}{d} I}
$$



## Single-qubit RB

Single exponential decay:

$$
F_{\sigma_{z}}=A p^{m}+B
$$



## Two-qubit RB

Single exponential decay:

$$
F_{\sigma_{z} \otimes \sigma_{z}}=A p^{m}+B
$$



In total
11520 elements

QuTech


Simultaneous RB
Three-fold exponential decay:

$$
F_{\sigma_{z} \otimes \sigma_{z}}=\mathrm{A}_{1} p_{1}^{m}+\mathrm{A}_{2} p_{2}^{m}+\mathrm{A}_{12} p_{12}^{m}+B
$$



Simultaneous RB Three-fold exponential decay:

$$
\begin{array}{ll} 
& \\
\text { With correlated errors: } & p_{12} \neq p_{1} \cdot p_{2} \quad \square \\
\text { No correlated errors: } & p_{12}=p_{1} \cdot p_{2} \otimes \sigma_{z}=\mathrm{A}_{1} p_{1}^{m}+\mathrm{A}_{2} p_{2}^{m}+\mathrm{A}_{12} p_{12}^{m}+B \\
p_{e f f}=\frac{3}{15}\left(p_{1}+p_{2}\right)+\frac{9}{15} p_{12} \\
\text { General case } & \square \quad \Lambda_{d e p}^{s i m}(\rho)=\left(p_{1} \rho^{1}+\frac{1-p_{1}}{2} I\right) \otimes\left(p_{2} \rho^{2}+\frac{1-p_{2}}{2} I\right)
\end{array}
$$

Sequence randomly sampled for each qubit


## Separating the three channels

Two-qubit space
$\mathbf{I}_{1}$ subspace $\left(\begin{array}{ccccc}1 & -\underline{0} & 0 & 0 \\ 0 & p_{1} \mathbf{I}_{1} & 0 & 0 \\ 0 & \neg & 0 & p_{2} \mathbf{I}_{2} & 0 \\ 0 & 1 & 0 & 0 & p_{12} \mathbf{I}_{12}\end{array}\right) \begin{gathered}\sigma_{0} \otimes \sigma_{0} \\ \sigma_{i} \otimes \sigma_{0} \\ \sigma_{0} \otimes \sigma_{j} \\ \sigma_{i} \otimes \sigma_{j} \\ \\ \\ 1\end{gathered}$
Isolate the $\sigma_{z} \otimes \sigma_{0}$ term (SPAM bases)

Use additional two-qubit Pauli operators to flip the initial state $\rho \quad \Lambda\left(\sigma_{i} \otimes \sigma_{j} \rho \sigma_{i} \otimes \sigma_{j}\right)$

$$
\sum_{i, j} \chi_{\sigma_{z} \otimes \sigma_{0}}\left(\sigma_{i} \otimes \sigma_{j}\right) \cdot \Lambda\left(\sigma_{i} \otimes \sigma_{j} \rho \sigma_{i} \otimes \sigma_{j}\right)
$$

$$
\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & p_{1}
\end{array}\right) \quad(\rho)
$$

$p_{2} \mathbf{I}_{2}$ and $p_{12} \mathbf{I}_{12}$ become all 0

Character function: $\chi_{\sigma_{z} \otimes \sigma_{0}}\left(\sigma_{i} \otimes \sigma_{j}\right)$
TABLE I. Values for the character function $\chi_{P}(\sigma)$ for $P \in\left\{\left(\sigma_{z} \otimes I\right),\left(I \otimes \sigma_{z}\right),\left(\sigma_{z} \otimes \sigma_{z}\right)\right\}$.
X.Xue., et al. PRX 2019

QuTech

| $P \backslash \sigma$ | $I I$ | $\sigma_{z} I$ | $I \sigma_{z}$ | $\sigma_{z} \sigma_{z}$ | $\sigma_{x} I$ | $I \sigma_{x}$ | $\sigma_{x} \sigma_{x}$ | $\sigma_{y} I$ | $I \sigma_{y}$ | $\sigma_{y} \sigma_{y}$ | $\sigma_{z} \sigma_{x}$ | $\sigma_{x} \sigma_{z}$ | $\sigma_{z} \sigma_{y}$ | $\sigma_{y} \sigma_{z}$ | $\sigma_{x} \sigma_{y}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\sigma_{z} I$ | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 |
| $I \sigma_{z} \sigma_{x}$ | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 |
| $\sigma_{z} \sigma_{z}$ | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 |

## Add Pauli operators at the beginning



## Character randomized benchmarking



## QuTech

# Quantum computation with spin qubits in semiconductor 

Xiao Xue

QuTech, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, Netherlands

"Quantum phenomena do not occur in a Hilbert space. They occur in a laboratory."
-- Asher Peres

## Future quantum computer

Linke, et al, PNAS 2017


Semiconducter (quantum dot, donor...)

* Scaling, high density;
* Coherence;
* "Hot" (cryo-electronics, easy wiring).


Loss, DiVincenzo, PRA 1998

Kane, Nature 1998


## 1 billion qubits

Trapped ions
$100 \times 100 \mathrm{~m}^{2}$


Much smaller for surface trap


Spin qubits
$5 \times 5 \mathrm{~mm}^{2}$


## QuTech

## Transistor v.s. quantum dot

Transistor: 1 gate / 1 device


Loss and DiVincenzo, PRA 1998

## All-electrical operation

- Tunable energy of electrons
- Tunable tunnel barriers
- Electrical contacts

QDots: $2 N+3$ gates / $N$ devices

A good starting point for scalability
QuTechlaurand et al, Nature Nano 2016


## Intel-QuTech collaboration



10 years, 50 M \$
Silicon spin qubits Transmon qubits

Architecture, Cryo-CMOS, interconnects

Leo DiCarlo
PI of SC qubit group

Lieven Vandersypen
Director of QuTech PI of spin qubit group

Jim Clark
PI Intel Quantum group

Mike Mayberry
Vice president Intel

## Qubits made at Intel

L. M. K. Vandersypen, M. A. Eriksson. Physics Today, 2019


Spin qubits made in Intel Fin-FET


For comparison:

## a A A A A B E

R. Pillarisetty, et al., IEDM 2019

## Semiconductor heterostructure

GaAs
AlGaAs with Si dopant (p-type)
AlGaAs
---------------------

## GaAs



## SiO2

Si

Argument: bury the electrons deeply
For Ge , the carriers are holes instead.


## Semiconductor heterostructure



Argument: bury the electrons deeply
For Ge , the carriers are holes instead.


## Semiconductor heterostructure



## QuTech

## Artificial atom

Single electron energy diagram
-> orbital energy


Multiple electron energy diagram -> orbital energy + charging energy


- Ideally, the dot is a quantum harmonic oscillator.
- In reality, there's always some deviation.
- For simplicity, we often plot it as a finite square potential well.


## QuTech

Transport

"Single electron transistor"

The energy levels are controlled
$+$ via a metal gate on top

Coulomb peaks:



## Single electron spin state

Apply a magnetic field
Last electron


Orbital N


QuTech


## Spin-to-charge conversion



- Spin-up ---> 0 electron
- Spin-down ---> 1 electron


## Two electrons in one dot



Orbital $\mathrm{N}=0$


$$
(|\downarrow \uparrow\rangle-|\uparrow \downarrow\rangle) / \sqrt{2}
$$


$|\uparrow \uparrow\rangle$

$|\downarrow \downarrow\rangle$

Spin-singlet, with total spin 0 . They are not distinguishable.


Spin-triplet Total spin1

## Charge sensor (SET)

A QD with one electron
Spin qubit

A QD with many electrons
Degenerate states-> spin doesn't matter


Charge sensor (SET)
Monitor the current through the sensor


## Initialization-readout cycle



Double quantum dot (DQD)
Fermi-Hubbard model


"Charge stability diagram"

Pauli spin blockade


## "Detuning"



Open barrier


## QuTech

## Two-spin energy diagram



Zeeman energy difference:

- Non-uniform g-factor
- Different local B field

$|\uparrow \uparrow\rangle$

$|\uparrow \uparrow\rangle$
${ }^{|L \uparrow\rangle}$ Adiabatic transfer $(|\downarrow \uparrow\rangle+|\uparrow \nu\rangle) / \sqrt{2}$
|ヶЬ) $(|\downarrow \uparrow\rangle-|\uparrow \downarrow\rangle) / \sqrt{2}$
$|\downarrow \downarrow\rangle$


## Single-shot readout - spin-charge conversion

Spin-selective tunneling:


Pauli spin blockade:




## Recap of readout

## Elzerman readout:



Fermi energy can be thermal-broadened
Must be operated at high field
Need an electron reservoir

Pauli spin blockade:


No thermal-broadening
Can be operated at low field
No need for electron reservoir

Charge stability diagram


## QuTech

## Common methods are not scalable



Charge stability diagram quadruple dot


Charge stability diagram hextuple dot

Controlled filling becomes challenging due to cross-capacitances and latching effects. QuTech

## Cross capacitance



Qubit control

| LP | B | RP |
| :---: | :---: | :---: |
| + | ${ }^{\mathrm{R}}$ | ${ }^{-}$ |



Vector source generator (IQ modulation, 10-20 GHz)

Arbitrary waveform generator
(M)


Readout signal

## Current-meter



Barrier control
(exchange interaction, two-qubit gate)

## QuTech

## Entering therotating frame



Larmor precession: electron "spins" aroud the $B_{z}$.

Q1: how to make the spin rotate around the x -axis?

A: Apply a field $B_{x}$.


A: Simply applying a $B_{\chi}$ field does not work.

## QuTech

## Enter the rotating frame

Step 1: ignore global phase

$$
\begin{aligned}
&|0(t)\rangle=e^{-i E_{0} t}|0(t=0)\rangle \\
&|1(t)\rangle=e^{-i E_{1} t}|1(t=0)\rangle \\
&=e^{-i E_{0} t} e^{-i\left(E_{1}-E_{0}\right) t}|1(t=0)\rangle \\
&=e^{-i E_{0} t} e^{-i g \mu_{B} B_{z} t}|1(t=0)\rangle \\
& E_{1}=g \mu_{B} B_{Z} \\
& E_{0}
\end{aligned}
$$

$$
|0(t)\rangle \stackrel{\text { def }}{=}|0(t=0)\rangle
$$

$$
|1(t)\rangle \stackrel{\text { def }}{=} e^{-i E_{z} t}|1(t=0)\rangle
$$

$$
|0(t)\rangle+|1(t)\rangle \stackrel{\text { def }}{=}|0(t=0)\rangle+e^{-i E_{z} t}|1(t=0)\rangle
$$

Q3: why is the qubit vector static in Bloch sphere?

## QuTech



## Enter the rotating frame



Simply applying a $B_{\chi}$ field does not work.
For the electron, $B_{x}$ is osillating.

Q2: Why do we use microwave to rotate the spin?

Hint: It's an oscillating electro-magnetic field.


## Enter the rotating frame

Step 2: Decompose oscillating field into two rotating fields.


One will rotate in same direction as spins.


QuTech


Static field in the rotating frame


Oscillating twice as fast -> ignored

## Enter the rotating frame



Q3: Why is the qubit vector static in Bloch sphere?
A: Bloch sphere is plotted in the rotating frame.


Precession


Rotating frame

Nutation


Laboratory frame

## Z gate and dephasing

$$
\begin{aligned}
& |0(t)\rangle=|0(t=0)\rangle \xrightarrow{\text { rotating frame }}|0\rangle \\
& |1(t)\rangle=e^{-i E_{z} t}|1(t=0)\rangle \xrightarrow{\text { rotating frame }}|1\rangle
\end{aligned}
$$

A rotating frame is determined by the energy splitting (frequency) of the qubit.
$|0(t)\rangle+|1(t)\rangle=|0(t=0)\rangle+e^{-i E_{z} t}|1(t=0)\rangle \xrightarrow{\text { rotating frame }}|0\rangle+|1\rangle$

Q4: What if we change the qubit energy intentionally?
A: Z gate.

$$
|0(t)\rangle+|1(t)\rangle=|0(t=0)\rangle+e^{-i\left(E_{z}+\Delta E\right) t}|1(t=0)\rangle \xrightarrow{\text { rotating frame }}|0\rangle+e^{-i \Delta E t}|1\rangle
$$

Q5: What if the qubit energy fluctuates under environmental noise?
A: Dephasing.

$$
|0(t)\rangle+|1(t)\rangle=|0(t=0)\rangle+e^{-i\left(E_{z}+\delta E(t)\right) t}|1(t=0)\rangle \xrightarrow{\text { rotating frame }}|0\rangle+e^{-i \delta E(t) t}|1\rangle
$$

## QuTech

## Single-qubit gate: ESR and EDSR

Electron spin resonance


Oscillating B field

Electric dipole spin resonance


Cobalt micromagnet enabling single-qubit gates


## QuTech

## Frequency selectivity and Crosstalk



QuTech



## Two-qubit gate: exchange interaction

Eigenstates (uncoupled)
$|\uparrow \uparrow\rangle$
$|\downarrow \downarrow\rangle$

Eigenstates (coupled)
$|\uparrow \uparrow\rangle$
$(|\downarrow \uparrow\rangle+|\uparrow \downarrow\rangle) / \sqrt{2}$
$(|\downarrow \uparrow\rangle-|\uparrow \downarrow\rangle) / \sqrt{2}$
$|\downarrow \downarrow\rangle$


$$
H_{e x c}=J \overrightarrow{S_{1}} \cdot \overrightarrow{S_{2}}=J\left(\mathcal{S}_{1 x} \mathcal{S}_{z x}+\mathcal{S}_{1 y} \mathcal{S}_{z y}+S_{1 z} \cdot S_{2 z}\right)
$$

## Two-qubit gates



Conditional rotation:


C-Phase: $\quad U_{J}(t)=\left(\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & e^{i J(\epsilon) t / 2 \hbar} & 0 & 0 \\ 0 & 0 & e^{i J(\epsilon) t / 2 \hbar} & 0 \\ 0 & 0 & 0 & 1\end{array}\right)$
CZ: $Z_{1}\left(-\frac{\pi}{2}\right) Z_{2}\left(-\frac{\pi}{2}\right) U_{J}\left(\frac{\pi \hbar}{J(\epsilon)}\right)=\left(\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1\end{array}\right)$

## Pulse sequence





QuTech

## Error mechanism: Nuclear spins

$$
\mathcal{H}=g \mu_{B} \vec{S} \vec{B}+\underbrace{\vec{S} A_{i} A_{i} \vec{I}_{i}}_{\text {Overhauser field } B_{N}}
$$

| Full polarization <br> $\dagger \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$ | $A=\sum_{i} A_{i}$ | GaAs: A~ <br> Paget, 1977 |
| :--- | :--- | :--- |


|  | Statistical polarization $\downarrow \neg \downarrow \downarrow \downarrow 1 / \backslash \downarrow \nearrow 1$ | $\begin{array}{rc} A / \sqrt{N} & \text { GaAs dot: } N \sim 10^{6} \\ & B_{N}=A / N^{1 / 2} \sim 5 \mathrm{mT} \\ \text { Merkulov, Efros, Rosen, PRB } 2002 \\ \text { Khaetskii, Loss, Glazman, PRL } 2002 \end{array}$ |
| :---: | :---: | :---: |
| QuTech |  |  |

## Error mechanism: Nuclear spins

## $\mathcal{H}=g \mu_{B} \vec{S} \vec{B}+\underbrace{\vec{S} \sum_{i} A_{i} \vec{I}_{i}}$

## Overhauser field $B_{N}$



## Materials impact on coherence time

GaAs


$\mathrm{T}_{2}{ }^{*} \sim 10 \mathrm{~ns}$
$\mathrm{T}_{2} \mathrm{DD}<0.2 \mathrm{~ms}$
Petta et al, Science 2005

Si


$$
\begin{aligned}
& \mathrm{T}_{2}^{*} \sim 1 \mu \mathrm{~s} \\
& \mathrm{~T}_{2}^{\mathrm{DD}}>0.5 \mathrm{~ms}
\end{aligned}
$$

Kawakami, Scarlino, et al, Nature Nano 2014
${ }^{28} \mathrm{Si}$


$$
\begin{aligned}
& \mathrm{T}_{2}{ }^{*} \sim 100 \mu \mathrm{~s} \\
& \mathrm{~T}_{2}{ }^{\mathrm{DD}} \sim 28 \mathrm{~ms}
\end{aligned}
$$

Veldhorst, et al, Nature Nano 2014

Error mechanism: Charge noise


Arash Sheikholeslam et al., Journal of Material Chemistry C 2016

## Charge trap


$|0\rangle+e^{-i \delta E(x(t)) t}|1\rangle$
$\delta E(x(t))=g \mu_{B} B_{z}(x(t))$


## Valleys in silicon



Orbital splitting: >1 meV
Valley splitting: $0-300 \mu \mathrm{eV}$
Zeeman splitting: 30-80 $\mu \mathrm{eV}$
$1 \mathrm{GHz}=4 \mu \mathrm{eV}$


Strained Si QW

Quantum Dot Spin Qubits


- Two dots
- Control with one gate voltage
- One electric axis
- One magnetic axis
- J. Levy (2002)
- Two dots
- Control with one gate voltage
- Two electric axes
- Z. Shi, et al., (20I2)


## Alternative: Singlet-Triplet qubit



Eigenstates (coupled)
$|\uparrow \uparrow\rangle$


Cphase gate through capacitive coupling
$|\downarrow \downarrow\rangle$

$|\downarrow \downarrow\rangle$

M. Shulman et al, Science 2012

## Alternative: Exchange-only qubit


D. DiVincenzo et al, Nature 2000

$$
\left|0_{L}\right\rangle=|S\rangle|\uparrow\rangle \quad\left|1_{L}\right\rangle=(2 / 3)^{1 / 2}\left|T_{+}\right\rangle|\downarrow\rangle-(1 / 3)^{1 / 2}\left|T_{0}\right\rangle|\uparrow\rangle
$$

J. Medford et al., Nature Nano 2013



QuTech
detuning $\varepsilon$

## Alternative: donors




Kane, Nature 1998

## QuTech

## Alternative: donors

Flip-flop qubit


Electron spin: $\sim 1 \mathrm{~ms}$ Nucler spin: ~1s

## QuTech

# Quantum computation with spin qubits in semiconductor 

Xiao Xue

QuTech, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, Netherlands


## Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control \& readout


## QuTech

## Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control \& readout

High-fidelity operations
L. M. K. Vandersypen et al., npj Quantum Info (2017)

## QuTech

## Materials impact on coherence time

GaAs


$\mathrm{T}_{2}{ }^{*} \sim 10 \mathrm{~ns}$
$\mathrm{T}_{2} \mathrm{DD}<0.2 \mathrm{~ms}$
Petta et al, Science 2005

Si


$$
\begin{aligned}
& \mathrm{T}_{2}^{*} \sim 1 \mu \mathrm{~s} \\
& \mathrm{~T}_{2}^{\mathrm{DD}}>0.5 \mathrm{~ms}
\end{aligned}
$$

Kawakami, Scarlino, et al, Nature Nano 2014
${ }^{28} \mathrm{Si}$


$$
\begin{aligned}
& \mathrm{T}_{2}{ }^{*} \sim 100 \mu \mathrm{~s} \\
& \mathrm{~T}_{2}{ }^{\mathrm{DD}} \sim 28 \mathrm{~ms}
\end{aligned}
$$

Veldhorst, et al, Nature Nano 2014

## Materials impact on fidelities

GaAs



2-spin exchange: Q > 50

Martins et al, PRL 2016

Si


1-Q gate: >99\%
2-Q gate: $92.0 \%$
Reed, et al, PRL 2016
Watson, et al, Nature 2018 X.X., et al, PRX 2019
${ }^{28} \mathrm{Si}$


1-Q gate: >99.9\%
2-Q gate: 98.0\%
Yoneda, et al, Nat Nano 2018 Huang, et al, Nature 2019

## Device

## purified ${ }^{28} \mathrm{Si}$



Dephasing times
$T_{2}{ }^{*}: 20 \mu \mathrm{~s}, 10 \mu \mathrm{~s}(8 \mathrm{~min} \mathrm{avg})$ Valley splitting: >140 ueV

Charge stability diagram



Detuning:
Couples strongly to charge noise

Barrier control at symmetry point:

Improvement of coherence by a factor of 5~6

## Symmetry operation against charge noise

Fix the barrier pulse amplitude
Sweep the detuning



Decoupled CPhase Watson et al, Nature 2018

Symmetry point: Reed et al, PRL 2016, Martins et al, PRL 2016

## QuTech

## Adiabatic CZ gate



## Optimize pulse shape using Gate Set Tomography



Analysis using PyGSTi (Sandia) - http://www.pigsty.info

## QuTech

$\left\{\begin{array}{l}\text { Qubit frequency }(2 x) \\ 1 Q \text { gate duration }(2 x) \\ 1 Q \text { phase shifts }(1 Q)(4 x) \\ C Z \text { amplitude } \\ 1 Q \text { phase shifts }(C Z)(2 x)\end{array}\right.$

$\sim 98 \%$ CZ gate before optimization

## Optimize pulse shape using GST



Analysis using PyGSTi (Sandia) - http://www.pigsty.info

## QuTech




$>99.5 \% \mathrm{CZ}$ gate after optimization

## Two-qubit CZ fidelity of $99.65 \% \pm 0.15 \%$

Measured CZ


See also: ${ }^{31} \mathrm{P}$ donors: Madzik Nature 2022 and silicon (J always-on): Noiri et al., Nature 2022 and silicon (CZ): Mills et al., arxiv preprint

Two-qubit gate set tomography


1685 such sequences in total
R. Blume-Kohout et al., PRX Quantum 2022

## QuTech

## Modular design


L. M. K. Vandersypen et al., npj Quantum Info (2017)

## QuTech

## Building lattices from the bottom-up



Local electrodes allow individual tunability


Towards larger 2D array


## Qubit arrays



Hendrix, et al, Nature 2021


## Gen 3

Si/SiGe

## QuTech

3-qubit GHZ states

## Single-qubit control (EDSR)



## Exchange control and Bell states

| $(\mathrm{MHz})$ | $\mathrm{J}_{12}$ | $\mathrm{~J}_{23}$ | $\mathrm{~J}_{34}$ | $\mathrm{~J}_{45}$ | $\mathrm{~J}_{56}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~J}_{12}$ on | 12.1 | 0.023 | 0.018 | $<0.03$ | 0.040 |
| $\mathrm{~J}_{23}$ on | $<0.05$ | 11.1 | $<0.30$ | $<0.03$ | 0.040 |
| $\mathrm{~J}_{34}$ on | 0.050 | $<0.03$ | 6.6 | $<0.07$ | 0.042 |
| $J_{45}$ on | 0.038 | $<0.03$ | 0.031 | 9.8 | 0.250 |
| $J_{56}$ on | 0.033 | $<0.03$ | $<0.02$ | $<0.03$ | 5.3 |



| Qubits | Fidelity (\%) | Concurence (\%) |
| :---: | :---: | :---: |
| $1-2$ | $89.2 \pm 2.2$ | $86.7 \pm 3.2$ |
| $2-3$ | $90.1 \pm 2.2$ | $83.9 \pm 3.8$ |
| $3-4$ | $88.3 \pm 3.6$ | $87.9 \pm 5.0$ |
| $4-5$ | $95.6 \pm 2.0$ | $94.9 \pm 3.2$ |
| $5-6$ | $94.1 \pm 1.4$ | $90.6 \pm 3.6$ |



## Modular design


L. M. K. Vandersypen et al., npj Quantum Info (2017)

## QuTech

## Virtual gates application: Shuttling

Material: $\mathrm{Si} / \mathrm{SiGe}$


2 Electron shuttling

## QuTech

Charge shuttling





3 Electron shuttling


## Conveyor mode shuttling



Seidler, et al., arxiv 2021
$10 \mu \mathrm{~m}$ long $\mathrm{Si} / \mathrm{SiGe}, 35 \mathrm{~nm}$ gate pitch, 280 gates connected to 4 sets



## Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control \& readout
L. M. K. Vandersypen et al., npj Quantum Info (2017)


## QuTech

Connecting a double-dot to a resonator


## QuTech

## Charge-photon admixing

Samkharadze, Zheng, et al., Science 2018


Theory: Benito et al., PRB 96 (2017) High-Z resonator: Samkharadze et al., PRApplied 5 (2016)

## Spin-charge admixing

Samkharadze, Zheng, et al., Science 2018


Theory: Benito et al., PRB 96 (2017) High-Z resonator: Samkharadze et al., PRApplied 5 (2016)

## Spin-charge admixing

Samkharadze, Zheng, et al., Science 2018


Si/SiGe growth by A. Sammak and G. Scappucci
$\begin{aligned} & \text { Condition for } \\ & \text { strong coupling }\end{aligned} g_{s} \propto g_{c} \frac{\Delta B_{x}}{2 t_{c} / h-f_{r}}>\kappa, \gamma_{s}$,
Theory: Benito et al., PRB 96 (2017)
High-Z resonator: Samkharadze et al., PRApplied 5 (2016)

## Vacuum Rabi splitting



QuTech

## Remote spin-spin coupling

Spin-spin dispersive coupling




## QuTech

## Modular design


L. M. K. Vandersypen et al., npj Quantum Info (2017)

## QuTech

## 1 billion qubits

Trapped ions
$100 \times 100 \mathrm{~m}^{2}$


Much smaller for surface trap


Spin qubits
$5 \times 5 \mathrm{~mm}^{2}$


## QuTech

## Superconducting qubits

Chip-to-chip GHZ state transfer (fidelity: 65.6\%)

Chip-to-chip entanglement (fidelity: 73\%)
DiCarlo group (Delft)
C. Dickel, et al, PRB 2018



Fridge-to-fridge entanglement (fidelity: 79.5\%) Wallraff group (ETH)
P. Magnard, et al, PRL 2020


## A ‘supreme’ quantum computer

Google Sycamore quantum processor


Delft lab

## QuTech

## IQ modulation right now

Arbitrary waveform generator (AWG, Keysight)

Vector microwave source (Keysight)


QuTech

## Way forward: cryo-electronics

## Integrated electronics

$1 \%$ accuracy in all parameters


## QuTech

E.Charbon, et al., "Cryo-CMOS for Quantum Computing", IEDM 2016.

## Cryo-CMOS approach

Use 3 K stage for qubit control using cryo-CMOS integrated circuits.


## Horse Ridge

## Horse Ridge, Oregon



News Byte
December 9, 2019

INTEL NTROOUCES HORSE RIDEE TOEMABLE COMMERCCALIVVVABLEDUANTUMCOMPUUERS


Stefano Pellerano, principal engineer at Intel Labs, holds Horse Ridge. The new cryogenic control chip will speed development of full-stack quantum computing systems, marking a milestone in the development of a commercially viable quantum computer. (Credit: Walden Kirsch/Intel Corporation)
» Click for full image
with Charbon \& Sebastiano groups at QuTech and with Pellerano et al from Intel ISSCC 2020



## Cuicul

## Horse Ridge micrograph



## Intel 22 nm FFL Technology

- 4 Transmitters (Each with 32 Channels Multiplexed) $=128$ qubits.
- Supports 2-20 GHz Microwave Output (transmons and spin qubits)
- Power Consumption = 330 mW (digital, clock, 1GHz, 5 times lower at 200 MHz ), 54 mW (analog)
- $\quad$ SNR > 44 dB ( 25 MHz bandwidth)
with Charbon \& Sebastiano groups at QuTech and with Pellerano et al from Intel


## Fidelity benchmark




State tomography


RT setup: Tektronix AWG 5014C
+
+

## Cryo-CMOS: "Horse Ridge"


X. Xue, B. Patra, et al., Nature (2021) with Charbon \& Sebastiano et al @QuTech and Intel Quantum

## "Hot" qubits

Urdampilleta, et al., Nat Nano 2019 (Grenoble) Petit, et al., Nature 2020
Yang, et al., Nature 2020
Geyer, et al., Nat Electronics 2022
(Delft)
(UNSW)
(Basel)

High-fidelity readout up to 1 K Universal two-qubit operations above 1 K Single-qubit gates above 1 K $98 \%$ single-qubit gate at 4.2 K

## QuTech

## Modular design


L. M. K. Vandersypen et al., npj Quantum Info (2017)

## QuTech

## T1 and charge noise vs temperature

M. Veldhorst group @ Delft




## Coherence times vs temperature

A. Dzurak group @ UNSW


## Qubits at 1.1 K

## M. Veldhosrt group @ QuTech Delft


L. Petit, et al., Nature 2020

## Coherence times vs temperature

## Coherence

c


- $J=0.5 \mathrm{MHz}$
$\diamond J=2.5 \mathrm{MHz}$


Two-qubit fidelity



Maurand, et al., Nature Electronics 2022

## Qubits at > 4K

Coherence


Single-qubit fidelity


# Quantum simulation 

## A 2X2 array





Mukhopadhyay, Dehollain et. al. APL 2018

See also
Thalineau et al, APL 2013

## Controllable Tunnel Coupling



## Controllable Tunnel Coupling



## Quantum simulation: Nagaoka Ferromagnetism

```
PHYSICAL REVIEW
VOLUME 147. NUMBER1
8 JULY 1966
```


## Ferromagnetism in a Narrow, Almost Half-Filled $s$ Band*

```
Yosuke Nagaoka \(\dagger\)
Department of Physics, University of California, San Diego, La Jolla, California
(Received 17 January 1966)
```

EIGENVALUES AND MAGNETISM OF ELECTRONS ON AN ARTIFICIAL MOLECULE

International Journal of Nanoscience Vol. 2, No. 3 (2003) 165-170
D. C. MATTIS

Department of Physics, University of Utah


Quantum dots
plaquette:
B. Wunsch,
M. Rudner,

LMKV,
E. Demler

Nagaoka Ferromagnetism




## Experimental procedure

Dehollain, Mukhopadhyay, et. al., Nature 2020


QuTech

## Protocol and main observation

Dehollain, Mukhopadhyay, et. al., Nature 2020


## Adiabatic to diabatic transition, and equilibration

Dehollain, Mukhopadhyay, et. al., Nature 2020



QuTech

## Test 1: Change topology

Dehollain, Mukhopadhyay, et. al., Nature 2020

$[19,15,17,19] \mu \mathrm{V}$


$[16,8,20,19] \mu \mathrm{V}$


$[18,0,21,21] \mu \mathrm{eV}$

## lest 2: Introduce Aharonov-Bohm phase ( $B$-field)



Weak B-field destroys magnetization

## QuTech

## Test 3: Offset local potentials

Dehollain, Mukhopadhyay, et. al., Nature 2020


QuTech
Magnetic ground state survives potential offsets exceeding hopping

