Quantum computation: quantum process and characterization

Xiao Xue

QuTech, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, Netherlands

I	I	I	1	/	/	/	1	1	1	/	1	-	-	-	-	-	-						
1	/	/	/	/	/	1	1	1	1	1	1	1	-	-	-	-	-						
/	1	1	1	1	1	1	1	1	1	1	1	/	1	/	/	/	-						
1	1	1	/	/	/	/	/	/	/	/	/	/	1	-	/	/	/						
/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	1	-	/						
-	/	-	-	-	/	/	/	/	/	/	/	/	/	/	/	/	/						
_	-	-	-	-	-	-	-	-	/	/	/	/	/	/	/	/							
_	_	-	-	-	-	_	-	-	-	-	-	/	/	/	/	/	1						
				_	-	-	-	-	-	-	-	-	/	/	1	1	1						





Quantum computers look like...

AMO





Solid-state From google





Quantum bit

Spin

Oscillator





Pure state and mixed state





Pure state and mixed state





Relaxation and dephasing

Figure: Nielsen & Chuang

$$\rho_{\Psi} = |\Psi\rangle \langle \Psi| = \begin{pmatrix} a^2 & ab^* \\ a^*b & b^2 \end{pmatrix}$$



Relaxation:

$$\rho_{\Psi'} = \begin{pmatrix} 1 - e^{-\frac{t}{T_1}} (1 - a^2) & e^{-\frac{t}{2T_1}} a b^* \\ e^{-\frac{t}{2T_1}} a^* b & e^{-\frac{t}{T_1}} b^2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



Dephasing:

 $\rho_{\Psi'} = \begin{pmatrix} a^2 & e^{-\left(\frac{t}{T_2}\right)^{(2)}} ab^* \\ e^{-\left(\frac{t}{T_2}\right)^{(2)}} a^* b & h^2 \end{pmatrix} \rightarrow \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}$

Depolarizing process

$$\rho_{\Psi} = |\Psi\rangle\langle\Psi| = \begin{pmatrix} a^2 & ab^* \\ a^*b & b^2 \end{pmatrix}$$

Figure: Nielsen & Chuang





State Fidelity

 $F = Tr(\rho_{mea}\rho_{ref})$

Consider $\rho_{ref} = |\alpha\rangle\langle\alpha|as a pure state$

 $F = Tr(\rho_{mea}\rho_{ref}) = \langle \alpha | \rho_{mea} | \alpha \rangle$

Recap:

For a pure state ρ : $Tr(\rho^2) = 1$ For a mixed state ρ : $Tr(\rho^2) < 1$



Measurement fidelity

Let's think about it a bit further...

$$F = Tr(\rho_{mea}\rho_{ref}) = \langle \alpha | \rho_{mea} | \alpha \rangle$$

Consider ρ_{mea} as known, but $\rho_{ref} = |\alpha\rangle\langle\alpha|$ is unknown... F gives "measurement fidelity".

Faulty measurement: $\rho_{ref} = p_{\alpha} |\alpha\rangle \langle \alpha | + p_{\beta} |\beta\rangle \langle \beta |$

$$F = Tr(\rho_{mea}\rho_{ref}) = p_{\alpha}\langle \alpha | \rho_{mea} | \alpha \rangle + p_{\beta}\langle \beta | \rho_{mea} | \beta \rangle$$



Positive operator-valued measurement

For an ideal POVM, $E_m = |\psi_m\rangle \langle \psi_m|$, with $|\psi_m\rangle$ often chosen to be the eigenstates.

Outcome: $P_m = Tr(|\psi\rangle\langle\psi|E_m)$ Single qubit: $\{E_0 = |0\rangle\langle0|, E_1 = |1\rangle\langle1|\}$

Measurement error:

$$\{E_0^e = (1 - \epsilon_0) |0\rangle \langle 0| + \epsilon_1 |1\rangle \langle 1|, E_1^e = \epsilon_0 |0\rangle \langle 0| + (1 - \epsilon_1) |1\rangle \langle 1|\}$$

 ϵ_0 and ϵ_1 are the readout errors of $|0\rangle$ and $|1\rangle$ states

Measurement fidelity:

$$F_{0} = Tr(E_{0}^{e}E_{0}) = Tr(E_{0}^{e}|0\rangle\langle0|) = 1 - \epsilon_{0}$$

$$F_{1} = Tr(E_{1}^{e}E_{1}) = Tr(E_{1}^{e}|1\rangle\langle1|) = 1 - \epsilon_{1}$$



Quantum operation/channel

Consider a state

$$\rho = |\alpha\rangle\langle\alpha| \qquad \qquad \rho = p_{\alpha}|\alpha\rangle\langle\alpha| + p_{\beta}|\beta\rangle\langle\beta|$$

Define a quantum operation/channel

$$\Lambda(\rho) = K|\alpha\rangle\langle\alpha|K^{\dagger} \qquad \qquad \Lambda(\rho) = p_{\alpha}K|\alpha\rangle\langle\alpha|K^{\dagger} + p_{\beta}K|\beta\rangle\langle\beta|K^{\dagger}$$

Sometimes the channel is a statistical mixture of multiple operators

$$\Lambda(\rho) = K_1 \rho K_1^{\dagger} + K_2 \rho K_2^{\dagger} + \dots \qquad \qquad K_1^{\dagger} K_1 + K_2^{\dagger} K_2 + \dots = I$$

defined as Kraus operators



Dephasing channel

If ρ completely loses its phase information:

$$\rho = \begin{pmatrix} a & c^* \\ c & b \end{pmatrix}$$

$$\mathcal{K}_{dephase_{max}}(\rho) = \frac{\rho + Z\rho Z}{2} = a|0\rangle\langle 0| + b|1\rangle\langle 1|$$

The Kraus operators are $I/\sqrt{2}$ and $Z/\sqrt{2}$.

Often ρ loses its phase with probability (error rate) 1 - p

$$\mathcal{K}_{dephase}(\rho) = p\rho + \frac{1-p}{2}(\rho + Z\rho Z) = \frac{1+p}{2}\rho + \frac{1-p}{2}Z\rho Z$$

with the Kraus operators $\sqrt{(1+p)/2I}$ and $\sqrt{(1-p)/2Z}$.



Depolarizing channel

If ρ completely loses its information in all directions:

$$\rho = \begin{pmatrix} a & c^* \\ c & b \end{pmatrix}$$

$$\mathcal{K}_{dep_{max}}(\rho) = \frac{\rho + X\rho X + Y\rho Y + Z\rho Z}{4} = I$$

The Kraus operators of the maximum-depolarizing channel $\mathcal{K}_{dep_{max}}$ are *I*/2, *X*/2, *Y*/2, and *Z*/2.

Often ρ loses its information with probability (error rate) 1 - p: $\mathcal{K}_{dep}(\rho) = p\rho + (1 - p)I/2$ $= \frac{1 + 3p}{4}\rho + \frac{1 - p}{4}(X\rho X + Y\rho Y + Z\rho Z)$

Here, the Kraus operators are $\sqrt{1+3p}I/2$, $\sqrt{1-p}X/2$, $\sqrt{1-p}Y/2$, and $\sqrt{1-p}Z/2$.

Recurring \mathcal{K}_{dep} will finally fully depolarize the state:

$$\lim_{N \to \infty} \mathcal{K}_{dep}{}^{N}(\rho) = \mathcal{K}_{dep_{max}}(\rho) = I.$$



Kraus operator in Pauli basis: χ -matrix

Single qubit case:

 $K_i = a_{i0}I + a_{i1}X + a_{i2}Y + a_{i3}Z$



 χ -matrix is a $d^2 \times d^2$ dimensional complex matrix



Examples of \chi-matrices



- χ -matrix is a matrix of coefficients.
- lt's always symmetric.



Examples of \chi-matrices

It's difficult to use χ -matrix for multiple operations



$$\Lambda(\rho) = \sum_{j,k=1}^{d^2} \chi_{jk} P_j \rho P_k$$

$$\Lambda_2(\Lambda_1(\rho)) = \sum_{p,q=1}^{d^2} \chi_{pq}^2 P_p(\sum_{j,k=1}^{d^2} \chi_{jk}^1 P_j \rho P_k) P_q$$





Superoperators

Revision of state fidelity

Operators can be decomposed into Paulis

$$K_i = a_{i0}I + a_{i1}X + a_{i2}Y + a_{i3}Z$$

Same for density matrices (density operators)

$$\rho = \frac{1}{2}(I + r_x X + r_y Y + r_z Z)$$

Now let's imagine performing POVM:

On the *x*-axis: $F = Tr(\rho X) = r_x$

On the y-axis: $F = Tr(\rho Y) = r_y$

$$Tr(IX) = 0$$
$$Tr(YX) = 0$$
$$Tr(ZX) = 0$$
$$Tr(XX) = 1$$

On the *z*-axis: $F = Tr(\rho Z) = r_z$



Quantum state tomography

How to perform POVM along the *x*-axis?

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$
$$= \frac{1}{2} (|0\rangle + |1\rangle) (\langle 0| + \langle 1|) - \frac{1}{2} (|0\rangle - |1\rangle) (\langle 0| - \langle 1|)$$

$$r_x = Tr(\rho X) = Tr(\rho \rho_{0+1}) - Tr(\rho \rho_{0-1})$$

 $r_y = Tr(\rho Y) = Tr(\rho \rho_{0+i1}) - Tr(\rho \rho_{0-i1})$ $r_z = Tr(\rho Z) = Tr(\rho \rho_0) - Tr(\rho \rho_1)$



- ρ_{ref} must be a physical state (not necessarily a pure state), with $Tr(\rho_{ref}) = 1$.
- Pauli matrices cannot be prepared as they are not density matrices of any physical state.





Super-operator and Pauli transfer matrix

$$\rho = \frac{1}{2} \left(I + r_{\chi} X + r_{y} Y + r_{z} Z \right) \stackrel{\text{def}}{=} \begin{pmatrix} Tr(\rho I) \\ Tr(\rho X) \\ Tr(\rho Y) \\ Tr(\rho Z) \end{pmatrix} = \begin{pmatrix} 1 \\ r_{\chi} \\ r_{y} \\ r_{z} \end{pmatrix}$$

- A $d \times d$ dimensional density matrix can be represented now as a d^2 dimensional vector.
- The first entry is always 1 : Trace preserving (TP).
- The other entries correspond to the projection onto x/y/z axes in Bloch sphere.



Pauli transfer matrix (quantum channel)

$$\Lambda_{ij}^{\mathcal{K}} = \frac{1}{d} Tr[P_i \mathcal{K}(P_j)] \qquad \qquad \rho = \frac{1}{2} \left(I + r_x X + r_y Y + r_z Z \right) \stackrel{\text{def}}{=} \begin{pmatrix} 1 \\ r_x \\ r_y \\ r_z \end{pmatrix}$$

PTM is expressed in the Pauli operator basis, meaning it can be directly applied to a state vector in the super-operator format

$$|\rho\rangle\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} Tr(\rho I) \\ Tr(\rho X) \\ Tr(\rho Y) \\ Tr(\rho Z) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\{I/\sqrt{d}, X/\sqrt{d}, Y/\sqrt{d}, Z/\sqrt{d}\}$$

$$\langle\!\langle E| = \frac{1}{\sqrt{2}}(1, Tr(EX), Tr(EY), Tr(EZ))\rangle$$

State/measurement fidelity: $Tr[E\rho] = \langle\!\langle E | \rho \rangle\!\rangle$

. . .



Examples of PTMs

$$\mathcal{K}_{dephase}(\rho) = p\rho + \frac{1-p}{2}(\rho + Z\rho Z)$$
$$= \frac{1+p}{2}\rho + \frac{1-p}{2}Z\rho Z$$

$$\mathcal{K}_{dep}(\rho) = p\rho + (1-p)I = \frac{1+3p}{4}\rho + \frac{1-p}{4}(X\rho X + Y\rho Y + Z\rho Z)$$



$$\Lambda_{dephasing} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{array}{c} I \\ X \\ Y \\ Z \\ I \\ X \\ Y \\ Z \end{array}$$

 $\Lambda_{dep} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$

- Input density operator
 - χ -matrix is a matrix of input-output.
 - It's not always symmetric.



PTM Application

Density operator







Pauli transfer matrix



$$\Lambda_2(\Lambda_1(\rho)) = \sum_{p,q=1}^{d^2} \chi_{pq}^2 P_p(\sum_{j,k=1}^{d^2} \chi_{jk}^1 P_j \rho P_k) P_q$$

A complete circuit: $p_E = \langle\!\langle E | \mathcal{G}_N ... \mathcal{G}_2 \mathcal{G}_1 | \rho \rangle\!\rangle$ where \mathcal{G}_i is the PTM of the *i*-th gate.

Average gate fidelity:

QuTech

$$F_g = \frac{Tr[\mathcal{G}^{-1}\mathcal{G}^{ideal}] + d}{d(d+1)}$$

 $\Lambda_2(\Lambda_1(\rho)) \Rightarrow G_2G_1|\rho \gg$ G_i is the PTM of Λ_i

Example: controlled-Z gate



Red: +1/4 Blue: -1/4



Characterize a quantum process



Quantum process tomography

Measure the complete input-output correlation.

Intuition: prepare different states in experiment, and apply the operator on them, followed by measurement of the outcome states.

This can be described in either χ -matrix or PTM, but PTM is easier.





Quantum process tomography

Entry of a PTM

 $\mathcal{G}_{ij} = \langle\!\langle P_i | \mathcal{G} | P_j \rangle\!\rangle$

$$\mathcal{G}_{13} = \frac{1}{2} \langle\!\langle X | \mathcal{G} | Z \rangle\!\rangle$$
$$= \frac{1}{2} \langle\!\langle \rho_{\hat{x}} - \rho_{-\hat{x}} | \mathcal{G} | \rho_0 - \rho_1 \rangle\!\rangle$$

$$\rho_{\pm \hat{x}} \longrightarrow \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$$

 $\rho_{0/1} \longrightarrow$

 $|0\rangle / |1\rangle$

Therefore, a PTM \mathcal{G} can be fully reconstructed by preparing and measuring the state in all the basis states $\{\rho_0, \rho_1, \rho_{\hat{x}}, \rho_{-\hat{x}}, \rho_{\hat{y}}, \rho_{-\hat{y}}\}$ before and after the process respectively.



Quantum process tomography

Entry of a PTM

 $\mathcal{G}_{ij} = \langle\!\langle P_i | \mathcal{G} | P_j \rangle\!\rangle$

$$\mathcal{G}_{13} = \frac{1}{2} \langle\!\langle X | \mathcal{G} | Z \rangle\!\rangle$$
$$= \frac{1}{2} \langle\!\langle \rho_{\hat{x}} - \rho_{-\hat{x}} | \mathcal{G} | \rho_0 - \rho_1 \rangle\!\rangle$$

$$\rho_{\pm \hat{x}} \longrightarrow \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$$

 $\rho_{0/1} \longrightarrow |0\rangle / |1\rangle$

Limitation:

State Preparation and Measurement (SPAM) error

Solution:

- Gate set tomography
- Randomized benchmarking
- Others...
- QuTech

Randomized benchmarking



Depolarizing error

$$F_{g} = \frac{Tr[\mathcal{G}^{-1}\mathcal{G}^{ideal}] + d}{d(d+1)} \qquad \qquad \mathcal{G} = \Lambda \mathcal{G}^{ideal} \qquad \qquad \text{Error: } \Lambda$$

$$Tr[\mathcal{G}^{-1}\mathcal{G}^{ideal}] = Tr[\Lambda^{-1}I]$$

Coherent error:rotation angle, rotation axis...Incoherent error:dephasing, depolarizing, relaxation...

Assumptions:

- 1. The error of a gate does not depend on the previous gates (Markovian).
- 2. The error of different gates are the same (gate-independent).

(Imagine the error is decoherence and all gates are equally long)



Depolarizing error

"Twirling" a small error with unitary operators. $\Lambda_{dep} \approx \sum_{i} \mathcal{U}_{i}^{\dagger} \Lambda \mathcal{U}_{i}$

In reality, we can use Clifford group to efficiently approximate the twirling process

$$\Lambda_{dep}(\rho) = \frac{1}{K} \sum_{k=1}^{K} C_k^{\dagger} \Lambda C_k |\rho\rangle \qquad \qquad \Lambda_{dep}(\rho) = p\rho + (1-p)I/d$$

Fidelity of error Λ :

$$F_{\Lambda} = F_{\Lambda_{dep}} \qquad \qquad F_{\Lambda} = Tr[\Lambda^{-1}I] = p + \frac{1-p}{d}$$



Clifford stabilizes Paulis: $C_k P_i C_k^{\dagger} = P_j$



Single qubit Cliffords	
	Ι
Doulis	Х
Paulis	Y
	Y, X
	X/2, Y/2
	X/2, -Y/2
	-X/2, Y/2
0 - 12 rotations	-X/2, -Y/2
$2\pi/3$ rotations	Y/2, X/2
	Y/2, -X/2
	-Y/2, X/2
	-Y/2, -X/2
	X/2
	-X/2
10 metertiana	Y/2
$\pi/2$ rotations	-Y/2
	-X/2, Y/2, X/2
	-X/2, -Y/2, X/2
	X. Y/2
	X, -Y/2
TT 1 141	Y, X/2
Hadamard-like	Y, -X/2
	X/2, Y/2, X/2
	-X/2, Y/2, -X/2



R. Barends et al., Nature 2014



	Single qubit Cliffords	
Paulis		I X Y Y. X
$2\pi/3$ rotations		X/2, Y/2 X/2, -Y/2 -X/2, Y/2 -X/2, -Y/2 Y/2, X/2 Y/2, -X/2 -Y/2, X/2 -Y/2, -X/2
$\pi/2$ rotations		X/2 -X/2 Y/2 -Y/2 -X/2, Y/2, X/2 -X/2, -Y/2, X/2
Hadamard-like		X, Y/2 X, -Y/2 Y, X/2 Y, -X/2 X/2, Y/2, X/2 -X/2, Y/2, -X/2





Single qubit Clifford	S
Paulis	I X Y
	Y, X X/2 V/2
	X/2, -Y/2
Hard Salary Month	-X/2, Y/2 -X/2, -Y/2
$2\pi/3$ rotations	Y/2, X/2
	Y/2, -X/2 -Y/2, X/2
	-Y/2, -X/2
	X/2 X/2
- /9 rotations	-X/2 Y/2
$\pi/2$ rotations	-Y/2
	-X/2, Y/2, X/2 -X/2, -Y/2, X/2
	X, Y/2
	X, -Y/2
Hadamard like	Y, X/2
Hadamard-fike	Y, -X/2
	X/2, Y/2, X/2
	-X/2, Y/2, -X/2





Single qubit Cliffords	
	Ι
Paulis	Х
1 dulls	Y
	Y, X
	X/2, Y/2
	X/2, -Y/2
	-X/2, Y/2
2 /2	-X/2, -Y/2
$2\pi/3$ rotations	Y/2, X/2
	Y/2, -X/2
	-Y/2, X/2
	-Y/2, -X/2
	X/2
	-X/2
10	Y/2
$\pi/2$ rotations	-Y/2
	-X/2, Y/2, X/2
	-X/2, -Y/2, X/2
	X. Y/2
	XY/2
	Y. X/2
Hadamard-like	YX/2
	X/2 Y/2 X/2
	-X/2 Y/2 -X/2
	-102, 112, -102


Clifford gates



Single qubit Cliffords	
D. I'	I X
Paulis	Y V X
S	$\frac{1, \Lambda}{Y/2 - Y/2}$
	X/2, 1/2 X/2 X/2
	X/2, -1/2 X/2, X/2
	-X/2, 1/2 X/2 X/2
$2\pi/3$ rotations	-X/2, -1/2
	1/2, X/2 X/2 X/2
	1/2, -A/2 N/2 N/2
	-Y/2, X/2
	-Y/2, -X/2
	X/2
	-X/2
$\pi/2$ rotations	Y/2
x/2 rotations	-Y/2
	-X/2, Y/2, X/2
	-X/2, -Y/2, X/2
	X, Y/2
	X, -Y/2
II. J	Y, X/2
Hadamard-like	Y, -X/2
	X/2, Y/2, X/2
	-X/2, Y/2, -X/2



Concatenated depolarizing channel

1-step depolarizing channel:

$$\Lambda_{dep}(\rho) = \frac{1}{K} \sum_{k=1}^{K} \mathcal{C}_{k}^{\dagger} \Lambda \mathcal{C}_{k} |\rho\rangle$$

Outcome state:

$$\Lambda_{dep}(\rho) = p\rho + (1-p)I/d$$

m-step depolarizing channel :

 $\Lambda_{dep}^{m}(\rho) = \int_{k_{m}, k_{2}, k_{1}} \frac{1}{K^{m}} \mathcal{C}_{k_{m}}^{\dagger} \Lambda \mathcal{C}_{k_{m}} \dots \mathcal{C}_{k_{2}}^{\dagger} \Lambda \mathcal{C}_{k_{2}} \mathcal{C}_{k_{1}}^{\dagger} \Lambda \mathcal{C}_{k_{1}} |\rho\rangle\rangle$

Outcome state:

$$\begin{split} \Lambda^m_{dep}(\rho) = &\Lambda_{dep_m} ... \Lambda_{dep_2} \Lambda_{dep_1} |\rho\rangle \\ = &p^m \rho + \frac{1 - p^m}{d} I, \end{split}$$



Sequence fidelity:

Fidelity of error Λ :

$$\begin{split} F_{seq}(\rho_{\psi}) = &Tr[E_{\psi}\Lambda^{m}(\rho_{\psi})] \\ = &Tr[E_{\psi}(\rho_{\psi} - \frac{I}{d})]p^{m} + Tr[E_{\psi}\frac{I}{d}] \\ = &Ap^{m} + B. \end{split}$$

$$F_{\Lambda} = Tr[\Lambda^{-1}I] = p + \frac{1-p}{d}$$



Now let $C_{l_1} = C_{k_1}$, $C_{l_2} = C_{k_2}C_{k_1}^{\dagger}$,..., and $C_{l_m} = C_{k_m}C_{k_{m-1}}^{\dagger}$, and let $C_{l_{m+1}} = C_{k_m}^{\dagger} = (C_{l_m}...C_{l_2} - C_{l_1})^{\dagger}$, the sequence can be written as:

$$\Lambda^{m}_{dep}(\rho) = \int_{l_m,\dots,l_2,l_1} \frac{1}{K^m} \mathcal{C}_{l_{m+1}} \Lambda \mathcal{C}_{l_m} \dots \Lambda \mathcal{C}_{l_2} \Lambda \mathcal{C}_{l_1} |\rho\rangle .$$

Recall:
$$\mathcal{G} = \Lambda \mathcal{G}^{ideal}$$
 RB measures Clifford gate fidelity.

$$\Lambda^m_{dep}(\rho) = \int_{l_m,\dots,l_2,l_1} \frac{1}{K^m} \mathcal{C}_{l_{m+1}} \Lambda \mathcal{C}_{l_m} \dots \Lambda \mathcal{C}_{l_2} \Lambda \mathcal{C}_{l_1} |\rho\rangle .$$



A real Clifford gate with error





Single-qubit RB

Single exponential decay:

$$F_{\sigma_z} = Ap^m + B$$





Two-qubit RB

Single exponential decay:

$$F_{\sigma_z \otimes \sigma_z} = Ap^m + B$$





Simultaneous RB

Three-fold exponential decay:

$$F_{\sigma_{z}\otimes\sigma_{z}} = A_{1}p_{1}^{m} + A_{2}p_{2}^{m} + A_{12}p_{12}^{m} + B$$



Simultaneous RB

Three-fold exponential decay:

$$F_{\sigma_z \otimes \sigma_z} = A_1 p_1^m + A_2 p_2^m + A_{12} p_{12}^m + B$$

$$p_{eff} = \frac{3}{15}(p_1 + p_2) + \frac{9}{15}p_{12}$$

With correlated errors:

No correlated errors:

$$p_{12} = p_1 \cdot p_2 \quad \Longrightarrow \quad \Lambda_a^s$$

 $p_{12} \neq p_1 \cdot p_2$

$$\Lambda_{dep}^{sim}(\rho) = (p_1 \rho^1 + \frac{1 - p_1}{2}I) \otimes (p_2 \rho^2 + \frac{1 - p_2}{2}I)$$

General case



QuTech



J.Helsen, **X.X.**, npj Quantum Info 2019 **X.X.**, et al. PRX 2019 See also: J.M.Gambetta et al, PRL 2012

Separating the three channels Use additional two-qubit Pauli operators to flip

Two-qubit space



TABLE I. Values for the character function $\chi_P(\sigma)$ for $P \in \{(\sigma_z \otimes I), (I \otimes \sigma_z), (\sigma_z \otimes \sigma_z)\}$.

Character function: $\chi_{\sigma_z \otimes \sigma_0}(\sigma_i \otimes \sigma_j)$

X.Xue., et al. PRX 2019

$P \setminus \sigma$	II	$\sigma_z I$	$I\sigma_z$	$\sigma_z \sigma_z$	$\sigma_{x}I$	$I\sigma_x$	$\sigma_x \sigma_x$	$\sigma_y I$	$I\sigma_y$	$\sigma_y \sigma_y$	$\sigma_z \sigma_x$	$\sigma_x \sigma_z$	$\sigma_z \sigma_y$	$\sigma_y \sigma_z$	$\sigma_x \sigma_y$	$\sigma_y \sigma_x$
$\sigma_z I$	1	1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	-1	-1
$I\sigma_z$	1	1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1
$\sigma_z \sigma_z$	1	1	1	1	-1	-1	1	-1	-1	1	-1	-1	-1	-1	1	1



Add Pauli operators at the beginning



Character randomized benchmarking





J.Helsen, **X.X.**, npj Quantum Info 2019 **X.X.**, et al. PRX 2019 J.Helsen

Quantum computation with spin qubits in semiconductor

Xiao Xue

QuTech, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, Netherlands

I	I	I	1	/	/	/	1	1	1	/	1	-	-	-	-	-	-						
1	/	/	/	/	/	1	1	1	1	1	/	/	/	-	-	-	-						
/	1	1	1	1	1	1	1	1	1	1	1	/	/	/	/	/	/						
1	/	/	/	/	/	/	/	/	1	1	/	/	/	-	/	-	/						
/	/	/	/	/	/	/	/	/	/	/	-	/	-	/	-	-	/						
-	-	/	/	/	-	/	/	/	/	/	/	-	/	/	1	/	/						
-	-	-	-	-	-	-	-	-	-	-	-	/	/	/	/	/							
	-	_	-	-	-	-	-	-	-	-	-	/	/	/	1	/	1						
				-	-	-	-	-	-	-	-	-	/	/	1	/	1						





"Quantum phenomena do not occur in a Hilbert space. They occur in a laboratory."

-- Asher Peres



Future quantum computer

Linke, et al, PNAS 2017



Kane, Nature 1998



Semiconducter (quantum dot, donor...)

- Scaling, high density;
- ✤ Coherence;
- ✤ "Hot" (cryo-electronics, easy wiring).







1 billion qubits

Trapped ions 100 X 100 m^2



Much smaller for surface trap



Spin qubits $5 \times 5 mm^2$ 50 nm 💳



Transistor v.s. quantum dot

Transistor: 1 gate / 1 device



Intel-QuTech collaboration



10 years, 50 M\$

Silicon spin qubits **Transmon qubits**

Architecture, Cryo-CMOS, interconnects

Leo DiCarlo PI of SC qubit group

Director of QuTech PI of spin qubit group

> Achievement: quantum dot arrays made @ Intel 300 mm cleanroom QuTech

Qubits made at Intel

L. M. K. Vandersypen, M. A. Eriksson. Physics Today, 2019



Spin qubits made in Intel Fin-FET

Zwerver et al., Nature Electronics 2022



For comparison:







300mm fabrication

R. Pillarisetty, et al., IEDM 2019



Semiconductor heterostructure



Argument: bury the electrons deeply

For Ge, the carriers are holes instead.





Semiconductor heterostructure



Argument: bury the electrons deeply

For Ge, the carriers are holes instead.





Semiconductor heterostructure





Artificial atom

Single electron energy diagram -> orbital energy

Multiple electron energy diagram -> orbital energy + charging energy



QuTech



- Ideally, the dot is a quantum harmonic oscillator.
- In reality, there's always some deviation.
- For simplicity, we often plot it as a finite square potential well.



Transport



"Single electron transistor"



Coulomb peaks:



N+1

Single electron spin state

Apply a magnetic field Zeeman splitting: $E_z = g\mu_B B_z$



Spin-to-charge conversion



- Spin-up ---> 0 electron
- Spin-down ---> 1 electron



Two electrons in one dot





Spin-singlet, with total spin 0. They are not distinguishable.



Charge sensor (SET)

A QD with one electron Spin qubit

A QD with many electrons Degenerate states-> spin doesn't matter



Charge sensor (SET)

Monitor the current through the sensor



Initialization-readout cycle



Double quantum dot (DQD)

Fermi-Hubbard model charge sensor gradient 13 23 -50 33 12 LP В RP (mV) 22 -10032 01 RР 11 -150 21 31 00 10 -20050 -50 100 0 LP (mV)

"Charge stability diagram"



Pauli spin blockade

QuTech



Two-spin energy diagram



Single-shot readout – spin-charge conversion



Recap of readout

Elzerman readout:



Fermi energy can be thermal-broadened

Must be operated at high field

Need an electron reservoir

Pauli spin blockade:



No thermal-broadening

Can be operated at low field

No need for electron reservoir
Charge stability diagram







Device fabricaton by U. Mukhopadhyay and J.P. Dehollain

Common methods are not scalable



Charge stability diagram quadruple dot

Charge stability diagram hextuple dot

Co QuTech

Controlled filling becomes challenging due to cross-capacitances and latching effects.

Cross capacitance











Larmor precession: electron "spins" aroud the B_z .

Q1: how to make the spin rotate around the x-axis?

A: Apply a field B_{χ} .



A: Simply applying a B_x field does not work.



Step 1: ignore global phase

 $|0(t)\rangle = e^{-iE_0t}|0(t=0)\rangle$

 $|1(t)\rangle = e^{-iE_1t}|1(t=0)\rangle$

$$= e^{-iE_0t} e^{-i(E_1 - E_0)t} |1(t = 0)\rangle$$

$$=e^{-iE_0t}e^{-ig\mu_BB_zt}|1(t=0)\rangle$$

$$|0(t)\rangle \stackrel{\text{\tiny def}}{=} |0(t=0)\rangle$$

$$|1(t)\rangle \stackrel{\text{\tiny def}}{=} e^{-iE_Z t} |1(t=0)\rangle$$

$$|0(t)\rangle + |1(t)\rangle \stackrel{\text{\tiny def}}{=} |0(t=0)\rangle + e^{-iE_Z t}|1(t=0)\rangle$$

V

Q3: why is the qubit vector static in Bloch sphere?







For the electron, B_{χ} is osillating.

Q2: Why do we use microwave to rotate the spin?



Electron

Hint: It's an oscillating electro-magnetic field.





 B_{z}

Step 2: Decompose oscillating field into two rotating fields.





One will rotate in same direction as spins.





b

Q3: Why is the qubit vector static in Bloch sphere?

A: Bloch sphere is plotted in the rotating frame.



Z gate and dephasing

$$|0(t)\rangle = |0(t=0)\rangle \xrightarrow{\text{rotating frame}} |0\rangle$$

 $|1(t)\rangle = e^{-iE_z t} |1(t=0)\rangle \xrightarrow{\text{rotating frame}} |1\rangle$

A rotating frame is determined by the energy splitting (frequency) of the qubit.

$$|0(t)\rangle + |1(t)\rangle = |0(t=0)\rangle + e^{-iE_z t}|1(t=0)\rangle \xrightarrow{\text{rotating frame}} |0\rangle + |1\rangle$$

Q4: What if we change the qubit energy intentionally?

A: Z gate. $|0(t)\rangle + |1(t)\rangle = |0(t=0)\rangle + e^{-i(E_z + \Delta E)t} |1(t=0)\rangle \xrightarrow{\text{rotating frame}} |0\rangle + e^{-i\Delta Et} |1\rangle$

Q5: What if the qubit energy fluctuates under environmental noise?

A: Dephasing. $|0(t)\rangle + |1(t)\rangle = |0(t = 0)\rangle + e^{-i(E_z + \delta E(t))t} |1(t = 0)\rangle \xrightarrow{\text{rotating frame}} |0\rangle + e^{-i\delta E(t)t} |1\rangle$ QuTech

Single-qubit gate: ESR and EDSR

Electron spin resonance



Oscillating B field

Electric dipole spin resonance



Cobalt micromagnet enabling single-qubit gates

-->



Effective oscillating magnetic field





Frequency selectivity and Crosstalk



Two-qubit gate: exchange interaction

QuTech





$$H_{exc} = J\overrightarrow{S_1} \cdot \overrightarrow{S_2} = J(\underbrace{S_{1x}} \cdot \underbrace{S_{2x}} + \underbrace{S_{1y}} \cdot \underbrace{S_{2y}} + S_{1z} \cdot S_{2z})$$

Two-qubit gates



Conditional rotation:



Conditional phase:



Pulse sequence

▲ RP Voltage

Time











Time



Pulse sequence





Error mechanism: Nuclear spins



Error mechanism: Nuclear spins





Materials impact on coherence time







Valleys in silicon

k_{z↑}







Slide courtesy Mark Eriksson

Alternative: Singlet-Triplet qubit



(2,0,1), TAINTA/ADO LING DRAW ON AT ORIGINAL AND THE TOTAL AND TOTAL AN tternattive choree Udenoted M Fland and signal spins. An alternative choice Ovenoteemer National Institute of Standards and Technology, Gai eyice to remain in the f (S_D) nstormations are equivalent to excent to excent the entry of the service of the entry of the service of the ser (ns) begins again. in this d D to poles of an axis I <u>+</u> <u>1</u>/2/ ь con aberald's average between left and middle dots the store of the state of the store 1 configura-⁻¹The Ham measured in a two-stan subsp initial spin state is based on gate voltage noise with a uniform power speetrum. The meory unand ϵ during the oped and it is shown initialization of of uning ins suffices for operation. Requirements for of the multi-40 qubit 2 phirol asing only exchanged and electrostatic interactions are putlinged? inscillates showing / 10ung leedurbit fro pera De agaigned/physicevB.82.075403 /3.21[°].La. 03.67.Ly expected from Fig 1,0,2) where the spins are denoted S_1 , S_2 , S_3 , the magnetic field is along the z-axis, and units are chosen so that Planck's con-(QPC) Gates L and R are quantum point contacts **Adi**ng to dephasing nected to coaxia Mined allowing ranted value (Fig. 4(b)]. **B0**stan**t** is $\hbar = 1$. Electron spins confined in quantum dots are an attractive Janmarks uning the exchange interaction Gantum computing because of their long coher applied. The de Grundforskningstond tic₂fie perature in a d ence times and potential to: scaling.1-3 In the simplest/2 075403-4 Danish National i*≨10*0 mT appli proposal, single spins form the logical basis, with single ergy levels are tuned with an external magance An alternative scheme,/2 A frequency Drefled etry circuit^{9,10} d ind by using gate voltages to with prical theis corrections singlet and triplet states of dent 2 with MHz ban width Fig23 (a). Jarallel reso requires informageneous static magnetic fiel $N_{\rm L}$, $N_{\rm L}$, tank circuits incorporating left and right QPCs were for he nature wand 2013 the each oubit removes the need for an inhomogeneous field; te electron occupancies of angle interactions between adjacent spins suffice for Salv from nearby inductor $1_{\rm R} = 910$ pH and $1_{\rm R} = 750$ nH +1/2 er with the parasitic capacitance \mathbb{C} and \mathbb{C}_{R}^{P} of the where Bias tees compled to each tank circuit allowed the ively (see Appendix A) or perfiniting ubic perfiting in this paper, we expericonductances g_L, g_R of left and right QPCs to be mea lifference between (2,0,1) o amount by a company of the manipulation with an simultaneously with the reflectance of the RF circui its of goto voltage) the quantum dot. This operation constitutes a rotation around each QPC was pinched off, a separate dip developed a

Alternative: donors





Kane, Nature 1998

Alternative: donors



Quantum computation with spin qubits in semiconductor

Xiao Xue

QuTech, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, Netherlands

I	1	I	1	/	/	/	1	1	1	/	1	-	-	-	-	-	-						
1	/	/	/	/	/	1	1	1	1	1	1	1	-	-	-	-	-						
/	1	1	1	1	1	1	1	1	1	1	1	/	1	/	/	/	-						
1	1	1	/	/	/	/	/	/	/	/	/	/	1	-	/	/	/						
/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	1	-	/						
-	-	-	-	/	/	/	/	/	/	/	/	/	/	/	/	/	/						
_	-	-	-	-	-	-	-	-	/	/	/	/	/	/	/	/							
_	_	-	-	-	-	_	-	-	-	-	-	/	/	/	/	/	1						
				_	-	-	-	-	-	-	-	-	/	/	/	1	1						





Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control & readout

L. M. K. Vandersypen et al., npj Quantum Info (2017)



Modular design



• Local operations in each module

- Remote couplers between modules
- Integrated electronics for control & readout

High-fidelity operations

L. M. K. Vandersypen et al., npj Quantum Info (2017)



Materials impact on coherence time





Materials impact on fidelities



Martins et al, PRL 2016

Reed, et al, PRL 2016 Watson, et al, Nature 2018 **X.X.**, et al, PRX 2019

Yoneda, et al, Nat Nano 2018 Huang, et al, Nature 2019



Device





V_{RP} (mV)

Dephasing times

 T_2^* : 20 µs, 10 µs (8 min avg) Valley splitting: >140 ueV

Detuning:

Couples strongly to charge noise

Barrier control at symmetry point:

Improvement of coherence by a factor of 5~6

Symmetry point: Reed et al, PRL 2016, Martins et al, PRL 2016 6

Symmetry operation against charge noise



Symmetry point: Reed et al, PRL 2016, Martins et al, PRL 2016



Adiabatic CZ gate



Optimize pulse shape using Gate Set Tomography



Optimize pulse shape using GST


Two-qubit CZ fidelity of 99.65% ± 0.15%



11

Modular design



• Local operations in each module

- Remote couplers between modules
- Integrated electronics for control & readout

Multi-qubit arrays

L. M. K. Vandersypen et al., npj Quantum Info (2017)



Building lattices from the bottom-up





Local electrodes allow individual tunability









Towards larger 2D array

GaAs





GaAs

Note: not an actual device





SiMOS/ SiGe

Delft Ongoing

Qubit arrays



Hendrix, et al, Nature 2021

Ge/SiGe

3-qubit phase correction code



S. Philips, M. Mądzik, et al, arXiv:2202.09252

Si/SiGe





3-qubit GHZ states

90 nm dot pitch

Single-qubit control (EDSR)



Exchange control and Bell states

(MHz)	J ₁₂	$J_{_{23}}$	$J_{_{34}}$	$J_{_{45}}$	$J_{_{56}}$
J_{12} on	12.1	0.023	0.018	<0.03	0.040
$J_{_{23}}$ on	<0.05	11.1	<0.30	<0.03	0.040
$J_{_{34}}$ on	0.050	<0.03	6.6	<0.07	0.042
$J_{_{45}}$ on	0.038	<0.03	0.031	9.8	0.250
$J_{_{56}}$ on	0.033	<0.03	<0.02	<0.03	5.3



J on/off ratios $\gtrsim 100$

Qubits	Fidelity (%)	Concurence (%)
1-2	89.2 ± 2.2	86.7 ± 3.2
2-3	90.1 ± 2.2	83.9 ± 3.8
3-4	88.3 ± 3.6	87.9 ± 5.0
4-5	95.6 ± 2.0	94.9 ± 3.2
5-6	94.1 ± 1.4	90.6 ± 3.6



Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control & readout

Spin Shuttling

L. M. K. Vandersypen et al., npj Quantum Info (2017)



Virtual gates application: Shuttling



Mills, et al, Nat Comm 2019





Seidler, et al., arxiv 2021



Courtesy of Inga Seidler & Lars Schreiber (RWTH Aachen)

QuTech

Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control & readout

Spin-photon interface

L. M. K. Vandersypen et al., npj Quantum Info (2017)



Connecting a double-dot to a resonator





Charge-photon admixing



Si/SiGe growth by A. Sammak and G. Scappucci

Theory: Benito *et al.*, PRB 96 (2017) High-*Z* resonator: Samkharadze *et al.*, PRApplied 5 (2016)

Spin-charge admixing



Samkharadze, Zheng, et al., Science 2018

↑ B_{ext} RP B B 100nm LP

Spin-charge admixing

Samkharadze, Zheng, et al., Science 2018



High-Z resonator: Samkharadze et al., PRApplied 5 (2016)

Vacuum Rabi splitting







Remote spin-spin coupling

F. Borjans et al., Nature (2020) P. Harvey-Collard et al., PRX 2022





Modular design



- Local operations in each module
- Remote couplers between modules
- Integrated electronics for control & readout

Quantum-classical interface

L. M. K. Vandersypen et al., npj Quantum Info (2017)



1 billion qubits

Trapped ions 100 X 100 m^2



Much smaller for surface trap



Spin qubits $5 \times 5 mm^2$ 50 nm 💳



Superconducting qubits

Chip-to-chip entanglement (fidelity: 73%)

DiCarlo group (Delft) C. Dickel, et al, PRB 2018





Chip-to-chip GHZ state transfer (fidelity: 65.6%)

Cleland group (U Chicago) Y. Zhong, et al, Nature 2021



Fridge-to-fridge entanglement (fidelity: 79.5%)

Wallraff group (ETH) P. Magnard, et al, PRL 2020



A 'supreme' quantum computer

Google Sycamore quantum processor





Control & Readout

Qubits



Bardin, et al., ISSCC (2019) Arute, et al., Nature (2019)







IQ modulation right now

Arbitrary waveform generator (AWG, Keysight)

Vector microwave source (Keysight)





Way forward: cryo-electronics







E.Charbon, et al., "Cryo-CMOS for Quantum Computing", IEDM 2016.

Cryo-CMOS approach

Use 3K stage for qubit control using cryo-CMOS integrated circuits.



State-of-the-art

Qubits and control in an integrated system



with Charbon & Sebastiano groups at QuTech and with Pellerano et al from Intel ISSCC 2020



Horse Ridge

Horse Ridge, Oregon (very cold)



Self heating characterization





(intel)

News Byte



control chip will speed development of full-stack quantum computing systems, marking a milestone in the development of a commercially viable quantum computer. (Credit: Walden Kirsch/Intel Corporation) » Click for full image

with Charbon & Sebastiano groups at QuTech and with Pellerano et al from Intel **ISSCC 2020**





Horse Ridge micrograph



2 mm

Intel 22 nm FFL Technology

- 4 Transmitters (Each with 32 Channels Multiplexed) = 128 qubits.
- Supports 2-20 GHz Microwave Output (transmons and spin qubits)
- Power Consumption = 330 mW (digital, clock, 1GHz, <u>5 times</u> <u>lower at 200MHz</u>), 54 mW (analog)
- SNR > 44 dB (25 MHz bandwidth)

with Charbon & Sebastiano groups at QuTech and with Pellerano et al from Intel ISSCC 2020



Fidelity benchmark

AIIXY



Cryo-CMOS: "Horse Ridge"





X. Xue, B. Patra, et al., Nature (2021) with Charbon & Sebastiano et al @QuTech and Intel Quantum



"Hot" qubits

Urdampilleta, et al., Nat Nano 2019(Grenoble)Petit, et al., Nature 2020(Delft)Yang, et al., Nature 2020(UNSW)Geyer, et al., Nat Electronics 2022(Basel)

High-fidelity readout up to 1K Universal two-qubit operations above 1K Single-qubit gates above 1K 98% single-qubit gate at 4.2K

Modular design



L. M. K. Vandersypen et al., npj Quantum Info (2017)



T1 and charge noise vs temperature



Coherence times vs temperature



SiMOS





C.H.Yang, et al., Nature 2020

Qubits at 1.1K







L. Petit, et al., Nature 2020

SiMOS

Coherence times vs temperature



QuTech

L. Petit, et al., Nature 2020



......

Maurand, et al., Nature Electronics 2022
Qubits at > 4K





Camenzind, et al., Nature Electronics 2022

Quantum simulation









-569





Mukhopadhyay, Dehollain et. al. APL 2018

See also Thalineau et al, APL 2013







 $5 \text{ GHz} \approx 20 \,\mu\text{eV}$





 $10 \text{ GHz} \approx 40 \,\mu\text{eV}$

Quantum simulation: Nagaoka Ferromagnetism



VOLUME 147, NUMBER 1

8 JULY 1966

Ferromagnetism in a Narrow, Almost Half-Filled s Band*

YOSUKE NAGAOKA[†] Department of Physics, University of California, San Diego, La Jolla, California (Received 17 January 1966)

EIGENVALUES AND MAGNETISM OF ELECTRONS ON AN ARTIFICIAL MOLECULE

International Journal of Nanoscience Vol. 2, No. 3 (2003) 165–170

D. C. MATTIS

Department of Physics, University of Utah



Nagaoka Ferromagnetism



Experimental procedure

Dehollain, Mukhopadhyay, et. al., Nature 2020





Protocol and main observation

Dehollain, Mukhopadhyay, et. al., Nature 2020



Adiabatic to diabatic transition, and equilibration

Dehollain, Mukhopadhyay, et. al., Nature 2020



Wait time at N τ_{wait} [µs]



Test 1: Change topology





Q Magnetic GS disappears for a linear chain (consistent with Lieb-Mattis)

Test 2: Introduce Aharonov-Bohm phase (*B*-field)



Dehollain, Mukhopadhyay, et. al., Nature 2020

Weak B-field destroys magnetization



Test 3: Offset local potentials

QuTech

Dehollain, Mukhopadhyay, et. al., Nature 2020



