1.

$x = x_0 \sin(\omega t + \varphi_0)$, $\varphi_0 = 0$ јер је почетни положај равнотежни

$x = x_0 \sin \omega t$

$v = \dot{x} = \omega x_0 \cos \omega t$

$a = \ddot{x} = -\omega^2 x_0 \sin \omega t$

$E_k = \frac{1}{2} m v^2$, $E_p = \frac{1}{2} k x^2$, $E_k + E_p = const$

$a_{max} = \omega^2 x_0$ $\qquad \omega^2 = \frac{0{,}4}{0{,}09} = \frac{40}{9}$

$x_0 = 9\,cm = 0{,}09\,m$

$\boxed{\omega = \frac{2}{3}\sqrt{10}}$

$v(0{,}5\,s) = \frac{2}{3}\sqrt{10} \cdot 0{,}09 \cos\left(\frac{2}{3}\sqrt{10} \cdot 0{,}5\right) = 0{,}094$ m/s

$E_k = \frac{1}{2} \cdot 0{,}01 \cdot 0{,}094^2 = 0{,}000044\,J = 0{,}044\,mJ$

2. $x = x_0 \sin\omega t$

$v = \dot{x} = \omega x_0 \cos\omega t$

$E_k = \frac{1}{2} m v^2 = \frac{m}{2}\omega^2 x_0^2 \cos^2\omega t$

$E_{kmax} = \frac{m}{2}\omega^2 x_0^2$

Да би Ek било $E_k = \frac{1}{2} E_{kmax}$ мора

бити $\cos^2\omega t = \frac{1}{2}$

$\cos 2\pi f t = \frac{1}{\sqrt{2}}$

$2\cancel{\pi} f t = \frac{\cancel{\pi}}{4}$

$t = \frac{1}{8f} = \frac{1}{40} s = 0{,}025 s$

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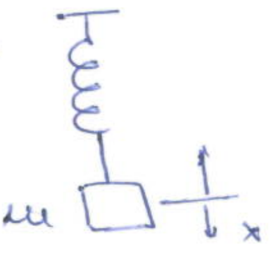
3. 

$x = x_0 \sin\omega t$

a) $E_p = \frac{1}{2} k x^2$

$E_p = \frac{1}{2} k x_0 \sin\omega t$

$E_{uk} = \frac{1}{2} m\omega^2 x_0^2$

код је тело у x_0, амплитудном положају

$E_{uk} = E_p$, одакле налазимо ω

$\frac{\cancel{1}}{2} k \cancel{x_0} = \frac{\cancel{1}}{2} m\omega^2 x_0^{\cancel{2}}$

$\boxed{\omega^2 = \frac{k}{m x_0}}$

$E_p = \frac{1}{2} k x_0 \sin\left(\sqrt{\frac{k}{m x_0}} \cdot t\right)$

само треба заменити вредности

б) $E_{k} = E_{p\,max} = \frac{1}{2} k x_0$

4. $x = x_0 \sin \omega t$

$$E_p = \frac{1}{2} k x^2 = \frac{1}{2} k x_0^2 \sin^2 \omega t$$

$$v = \dot{x} = \omega x_0 \cos \omega t$$

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$$

$$\frac{1}{2} m \omega^2 \cancel{x_0^2} \cos^2 \omega t = k \cancel{x_0^2} \sin^2 \omega t$$

$$\sin^2 \omega t = \frac{m\omega^2}{2k} \cos^2 \omega t$$

$$\sin^2 \omega t = \frac{m\omega^2}{2k} - \frac{m\omega^2}{2k} \sin^2 \omega t$$

$$\left(1 + \frac{m\omega^2}{2k}\right) \sin^2 \omega t = \frac{m\omega^2}{2k}$$

$$\sin^2 \omega t = \frac{\frac{m\omega^2}{2k}}{1 + \frac{m\omega^2}{2k}} = \frac{\frac{m\omega^2}{\cancel{2k}}}{\frac{2k + m\omega^2}{\cancel{2k}}}$$

$$\sin^2 \omega t = \frac{m\omega^2}{2k + m\omega^2} = \frac{1}{1 + \frac{2k}{m\omega^2}}, \quad \omega^2 = \frac{k}{m x_0}$$

$$\sin^2 \omega t = \frac{1}{1 + \frac{2\cancel{k}}{\cancel{m}} \frac{m x_0}{k}} = \frac{1}{1 + 2x_0}$$

$$\sin \omega t = \frac{1}{\sqrt{1 + 2x_0}}$$

$$\boxed{x = \frac{x_0}{\sqrt{1 + 2x_0}}}$$