The Effect of the Rabi Frequency on The Electric Susceptibility for Λ −tyep Three Level Atom When It Interacts With Laser Beam

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Abstract:

For Λ −tyep three level atom interacts with a laser beam, the affect of changing Rabi frequency has been studied. Furthermore a clear distortion has been seen in both the real and imaginary parts of the electric susceptibility. With increasing the Rabi frequency of the laser pulse that in resonance with the lower transition the distortion will produce a new peak in the electric susceptibility parts, both the real and imaginary ones.

Keywords: Electric susceptibility, Rabi Frequency, Three level atom.

Introduction:

The interaction of light with matter has been studied widely, both with quantum treatment, such as in [1], or in semiclassical treatment as the study of Fleischhauer [2], or the study of Scully and Zubairy [3] as examples. In the semiclassical treatment, the atoms of the mater are treated as fully quantum objects, the all fields are treated as classical vector fields. The light usually is taken, as any source of laser because of it is properties, while the matter has different kinds. There are several kinds of atoms that interact with laser field .The simplest kind one them is the two levels system, which is poorly suited to applications in non-linear optics at the single photon level. On the other hand, the three levels atom with the addition of a their level and a second optical field gives rise to a range of coherent phenomena, such as those that suppresses the resonant absorption. Furthermore, at the resonant absorption gives a very large dispersive optical non-linearity, which can be used to control the propagation of light through the medium. In the simple case of a three-level atom coherently coupled to two laser fields, interference between the two excitation pathways has profound consequences for the atomic state evolution as well as for the propagation of the applied optical fields [4]. There are several studies on the three levels systems and their applications [5,6]. Furthermore, even in the three level atom there are three types of ways to study these systems such like the cascade system, V −system, and the Λ −system. In this work we will focus on the Λ −system.
Consider a single three-level atom composed of a ground state $|1\rangle$, a rapidly decaying intermediate state $|2\rangle$ and a highly excited state $|3\rangle$ as it is shown in figure (1), , the decay of the state $|3\rangle$ is negligible slow. Two laser fields, a probe laser field, and a coupling laser field drive the atom. The Rabi frequency of the first pulse Ω_c in the resonance of the lower transition, while the Rabi frequency Ω_p of the second pulse is

in resonance with the upper transition. Usually the probe laser beam operated in resonance with the lower transition and the scanning laser beam operated in resonance with the upper transition. Due to the spatial distribution of the probe Rabi frequency, Ω_p denotes only the peak value of a Gaussian intensity distribution. This leads to a high resonance amplitude in case of large Rabi frequency Ω_p.

Theoretical Treatment:

The interaction of light with matter under circumstances such that the linear superposition principle is violated, resulting the non-linear optics. The non-linear response of a material system to an applied optical wave can be described by expressing the material polarization as a power series expansion in the electric field. The Hamiltonian H for the system can be written as

$$H = H_0 + H_1 \qquad (1)$$

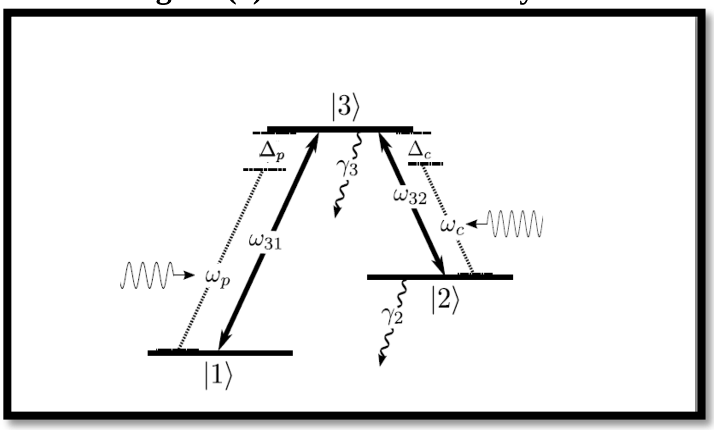
where H_0 describes the Hamiltonian for atom without presenting the electric field of the light, which is the laser. H_1 is the Hamiltonian of the interaction with the electric field of the laser. Now for three principle cases $|n>$ with the condition

$$\sum_n |n><n| = 1 \qquad <n|m> = \delta_{nm} \qquad (2)$$

We can rewrite the Hamiltonian H_0 as

$$H_0 = (\sum_n |n><n|)H(\sum_n |n><n|) = \begin{bmatrix} h\omega_1 & 0 & 0 \\ 0 & h\omega_2 & 0 \\ 0 & 0 & h\omega_3 \end{bmatrix} \qquad (3)$$

Figure (1): Three Level Λ −system



The electric field of the laser can be written as

$$E = \xi_p \cos(\omega_p t) + \xi_c \cos(\omega_c t) \quad (4)$$

where ξ_c the amplitude of the coupling laser field that has a frequency ω_c , while ξ_p the amplitude of the probe laser field that has a frequency ω_p . The interaction Hamiltonian is given as

$$H_1 = \frac{-1}{2}\begin{bmatrix} 0 & 0 & \xi_p \varrho_{13} e^{i\omega_p t} \\ 0 & 0 & \xi_p \varrho_{23} e^{i\omega_c t} \\ \xi_p \varrho_{31} e^{-i\omega_p t} & \xi_c \varrho_{32} e^{-i\omega_c t} & 0 \end{bmatrix} \quad (5)$$

with the elements ϱ_{nm} of the dipole operator $\varrho = q\hat{r}$ that satisfy the condition $\varrho_{nm} = \varrho^*_{mn} = \langle n|\varrho|m\rangle$. The Rabi frequencies defined as $\Omega_p = \frac{\xi_p|\varrho_{13}|}{h}$, $\Omega_c = \frac{\xi_c|\varrho_{23}|}{h}$, this gives the interaction Hamiltonian with the Rabi frequencies as

$$H_1 = \frac{-h}{2}\begin{bmatrix} 0 & 0 & \Omega_p e^{i\varphi_p} e^{i\omega_p t} \\ 0 & 0 & \Omega_c e^{i\varphi_c} e^{i\omega_c t} \\ \Omega_p e^{-i\varphi_p} e^{-i\omega_p t} & \Omega_c e^{-i\varphi_c} e^{-i\omega_c t} & 0 \end{bmatrix} \quad (6)$$

Using eq. (3) and (6) , we can rewrite eq.(1) as

$$H = \frac{h}{2}\begin{bmatrix} 2\omega_1 & 0 & -\Omega_p e^{i\varphi_p} e^{i\omega_p t} \\ 0 & 2\omega_2 & -\Omega_c e^{i\varphi_c} e^{i\omega_c t} \\ -\Omega_p e^{-i\varphi_p} e^{-i\omega_p t} & -\Omega_c e^{-i\varphi_c} e^{-i\omega_c t} & 2\omega_3 \end{bmatrix} \quad (7)$$

With time independent elements the Hamiltonian is

$$\tilde{H} = \frac{h}{2}\begin{bmatrix} 0 & 0 & -\Omega_p \\ 0 & -2(\omega_2 + \omega_c - \omega_1 - \omega_p) & -\Omega_c \\ -\Omega_p & -\Omega_c & 2(\omega_3 - \omega_1 - \omega_p) \end{bmatrix} \quad (8)$$

Defined $\Delta p = \omega_p - \omega_3 + \omega_1$ and $\Delta c = \omega_c - \omega_3 + \omega_2$ gives

$$\tilde{H} = \frac{h}{2}\begin{bmatrix} 0 & 0 & \Omega_p \\ 0 & -2(\Delta p - \Delta c) & \Omega_c \\ \Omega_p & \Omega_c & 2\Delta p \end{bmatrix} \quad (9)$$

On the other hand, the density matrix can be defined as

$$\rho = \sum_n P_n |n>< n| \quad (10)$$

where P_n is the probability of the occupation of the system in case n . The Von Neumann equation $\dot{\rho} = \frac{-i}{h}[H,\rho]$, for system under the study , using eq. (9) and eq. (10) , the Von Neumann equation is

$$\dot{\rho}_{ij} = \frac{-i}{h}\sum_n \left[\left(H_{ik}\rho_{kj} - \rho_{ik}H_{kj}\right) + \frac{1}{2}\left(\Gamma_{ik}\rho_{kj} + \rho_{ik}\Gamma_{kj}\right)\right] \quad (11)$$

For N atoms, the polarization using the density matrix is

$$\mathrm{P} = \mathrm{N}\langle\varrho\rangle = \mathrm{N\,Tr}(\rho\varrho) = N\,Tr\left(\begin{bmatrix}\rho_{11} & \rho_{12} & \rho_{13}\\ \rho_{21} & \rho_{22} & \rho_{23}\\ \rho_{31} & \rho_{32} & \rho_{33}\end{bmatrix}\begin{bmatrix}0 & 0 & \varrho_{13}\\ 0 & 0 & \varrho_{23}\\ \varrho_{31} & \varrho_{32} & 0\end{bmatrix}\right) \quad (12)$$

and polarization will be simply rewritten as

$$P = N\left(\varrho_{31}\tilde{\rho}_{13}e^{i\omega_p t}e^{i\varphi_p} + \varrho_{32}\tilde{\rho}_{23}e^{i\omega_c t}e^{i\varphi_c} + \varrho_{13}\tilde{\rho}_{31}e^{-i\omega_p t}e^{-i\varphi_p} + \varrho_{23}\tilde{\rho}_{32}e^{-i\omega_c t}e^{-i\varphi_c}\right) \quad (13)$$

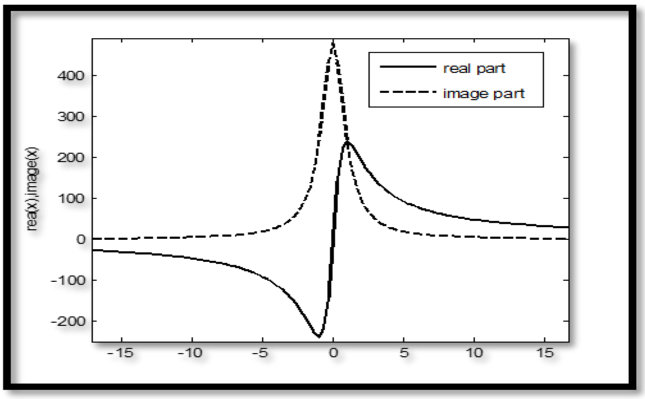
From the polarization P we can calculate the electric susceptibility for the system

$$\chi = \frac{2N|\varrho_{13}|}{\varepsilon_0\xi_p}\frac{-2(\gamma_{12}-i(\Delta_c-\Delta_p)\Omega_p}{4(\gamma_{13}+i\Delta_p)(-i\gamma_{12}-\Delta_c+\Delta_p)-i\Omega_c^2} \quad (14)$$

Results and Discussion:

Using the equations that gotten in the well known electric susceptibility , and with the help of matlab software, we have calculate the electric susceptibility for Λ −tyep three level atom interacts with a laser beam for different values of Ω_c and Ω_p . Figure(2) represents the real and imaginary parts of the electric susceptibility with $\Omega_c = 0\ MHz$ and $\Omega_p = 3MHz$. It is clear from the figure that both parts of the electric susceptibility are in well-known shape as it is calculated in several references [7, 8]. This has been done as a comparison with considering values of Rabi frequency, the figure is in agreement with the work that done in Ref [9]. With increasing the value of Rabi frequency Ω_c at the same value of Ω_p a distortion has been appeared on the real and imaginary parts of the susceptibility. Figure (3) shows that at $\Omega_c = 0.5\ MHz$ at the same parameters. Moreover , with higher increase the distortion becomes clearer as it seen in figure (4), which drown at $\Omega_c = 1.5\ MHz$.

Figure(2): The real and imaginary parts of the electric susceptibility ($\Omega_c = 0\ MHz$)

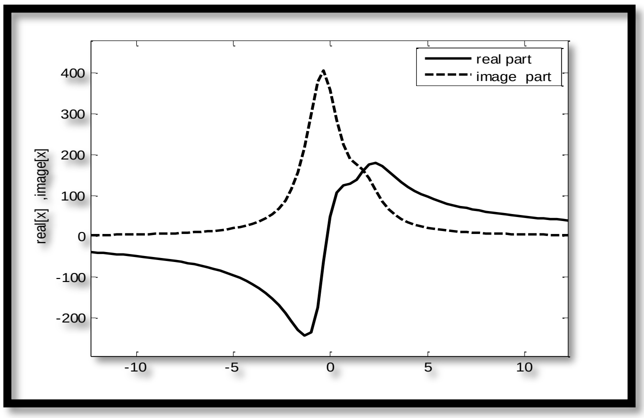


$\longrightarrow \Delta_p/\gamma_{13}$

$\Omega_p = 3.0MHz$ ، $\Delta_c = 0MHz$، $\gamma_{13} = 1.0MHz$ ،$\varrho_{13} = 6.0$ ، $\xi_p = 1.0$ ، $\gamma_{12} = 0.1MHz$

This behavior can be understand as that the increasing of Rabi frequency at higher values an another peak will appear in both cures for the real and the imaginary parts of the electric susceptibility . This beak will be clearer with increasing the Rabi frequency's value.

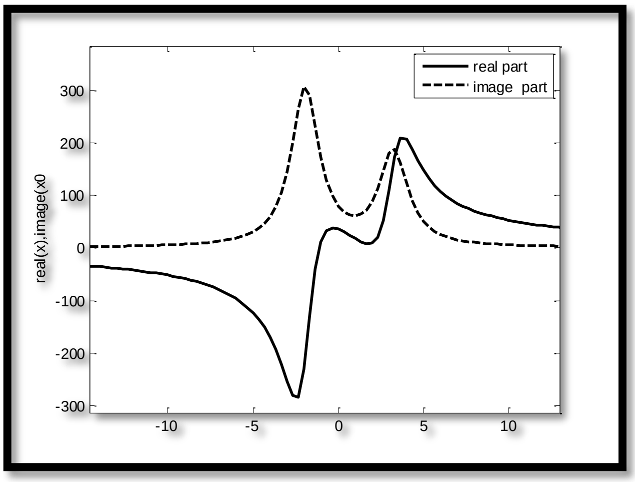
Figure(3): The real and imaginary parts of the electric susceptibility ($\Omega_c = 0.5MHz$)



$\longrightarrow \Delta_p/\gamma_{13}$

$\Omega_p = 3.0MHz$ ، $\Delta_c = 0MHz$، $\gamma_{13} = 1.0MHz$ ،$\varrho_{13} = 6.0$ ، $\xi_p = 1.0$ ، $\gamma_{12} = 0.1MHz$

Figure (4) The real and imaginary parts of the electric susceptibility ($\Omega_c = 1.5\ MHz$)

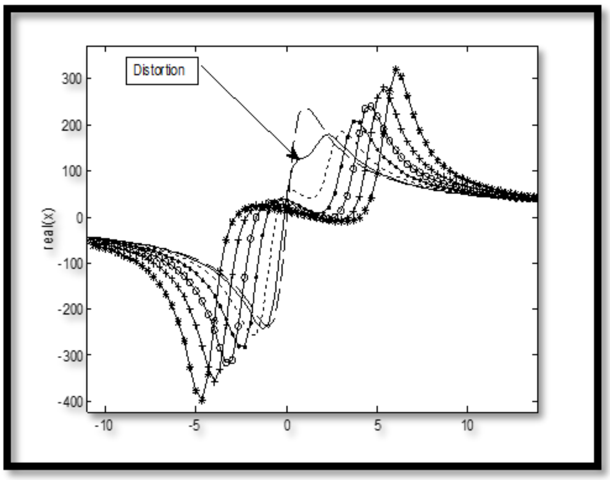


$\longrightarrow \Delta_p/\gamma_{13}$

$\Omega_p = 3.0MHz$ ، $\Delta_c = 0MHz$، $\gamma_{13} = 1.0MHz$ ،$\varrho_{13} = 6.0$ ، $\xi_p = 1.0$ ، $\gamma_{12} = 0.1MHz$

For different values of Rabi frequency the real (imaginary) part of the electric susceptibility drown in figure (5) (figure (6)) to prove the conclusion that we have reach.

Figure (5) The real part of the electric susceptibility ($\Omega_c = 0, 0.5 , 1, 1.5 , 2 , 2.5 , 3\ MHz$)



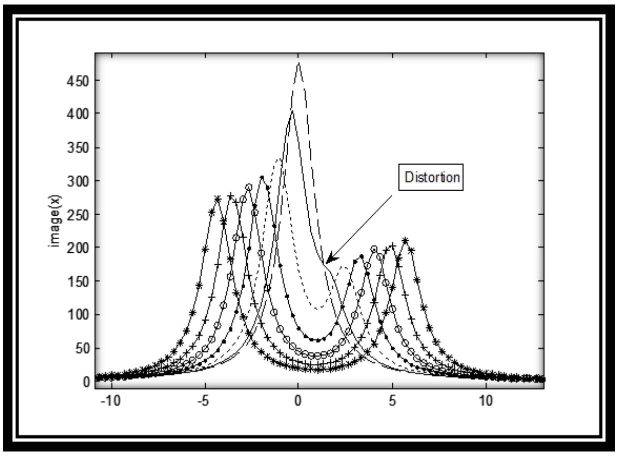
$\longrightarrow \quad \Delta_p/\gamma_{13}$

$\Omega_p = 3.0MHz$ ، $\Delta_c = 0MHz$، $\gamma_{13} = 1.0MHz$ ،$\varrho_{13} = 6.0$ ، $\xi_p = 1.0$ ، $\gamma_{12} = 0.1MHz$

Conclusion:

With the incusing the Rabi of the coupled laser beam that interact with Λ −type three level system a clear distortion will appear in the both the real and imaginary parts of the electric susceptibility . This distortion will be higher with more increasing of the Rabi frequency and will produce a new beak in both parts of the electric susceptibility. Moreover, the separation between the old and new peaks becomes wider with more increase in Rabi frequency of the coupled laser beam that in resonance with the lower transition of the Λ −type three level system.

Figure (6) the imaginary part of the electric susceptibility
($\Omega_c = 0, 0.5, 1, 1.5, 2, 2.5, 3\ MHz$)



Δ_p/γ_{13}

$\Omega_p = 3.0MHz$ ، $\Delta_c = 0MHz$ ، $\gamma_{13} = 1.0MHz$ ، $\varrho_{13} = 6.0$ ، $\xi_p = 1.0$ ، $\gamma_{12} = 0.1MHz$

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الملخص

تم دراسة تغيير تردد رابي للأنظمة الذرية ذات الثلاثة مستويات من نوع Λ عند تفاعلها مع اشعة الليزر. اوضحت الدراسة تشوه واضح في الجزئيين الحقيقي و التخيليلي من المتآثرية الكهربية للمواد تحت الدراسة. هذا التشويه تمثل في ظهور قمة جديدة في منحني الجزئيين الحقيقيى و التخيليى للمتآثرية الكهربية للنظام تحت الدراسة عند زيادة قيمة تردد رابي المقترن الذي يقع في وضع رنيني مع الانتقال السفلي للنظام ذو الثلاثة مستويات المدروس.

كلمات مفتاحية: المتآثرية الكهربية ، تردد ربي ، الذرات ذات الثلاثة مستويات.