Quantization of electromagnetic energy and the Heisenberg's uncertainty principle

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Abstract

The Heisenberg's uncertainty principle is a corner stone of quantum physics. Its derivation from the uncertainty theorem of Fourier transform theory demonstrates that the uncertainty principle arises from the dependency of momentum on wave number that exists at the quantum level. It also establishes that this principle is not a quantum mechanical measurement principle per se. In this article we provide detailed evidence that the quantization of electromagnetic energy in turn arises from the Heisenberg's uncertainty principle and the Nyquist-Shannon Sampling theorem and that the uncertainty principle must be regarded as the major trigger for quantization in quantum physics.

In the year 1900 Max Planck introduced the concept of energy quanta in order to be able to use the means of classical statistical physics to calculate the spectrum of the heat radiation[1]. In the further years this general principle of quantization could be scientifically proven. In this context, the quantum mechanism (since 1925) modifies the Hamiltonian mechanics to the effect that the position and momentum coordinates no longer correspond to numbers, but operators ("q number" for "quantum number")[1]. While the quantization in quantum mechanics is an inevitable principle, the exact underlying cause of quantization is still not known. Why is electromagnetic energy emitted in smallest energy quanta? At first sight one cannot discover any obvious reason, as there are other forms of energy that are not emitted in quanta. If one thinks about quantum physics, the first thing that comes to mind is the Heisenberg uncertainty principle and the Schrödinger equation. In the Schrödinger equation the quantization of energy is required in the form that a precisely fitting value of the energy is needed to fulfill this equation, but this states only that energy is or has to be quantized and not why it is quantized. The Schrödinger equation describes the time evolution of the quantum mechanical state of a non-relativistic system in the form of a partial differential equation[1].

The first-mentioned Heisenberg's uncertainty principle[2] (HUP) is a fundamental principle of quantum physics and states, according to the current definition, that the product of momentum and position of a particle must be at least $\hbar/2$ ($\hbar$ is the reduced Planck's constant). In this article we provide detailed evidence that the quantization in quantum physics depends from the HUP, defining a fundamental concept in quantum physics.

1. The Heisenberg uncertainty relation

The original formulation of the uncertainty principle by the physicist Werner Heisenberg[2] was the place multiplied with the impulse is¿ h. In the past years it could be shown that the uncertainty principle is not an exclusive quantum mechanical measuring principle in the sense of a "measurement error", as originally formulated by Heisenberg.

The Planck constant *h* was discovered in 1900 by Max Planck as the last natural constant required for defining the Planck units. The Planck units as defined by Max Planck differ from today's usual Planck units by a factor of 2π, because he did not set $\hbar$ but the Planck's constant *h* to 1. Although Planck devoted a chapter of his book "Theory of Heat Radiation" written 1906 to this system of units and later took up this topic again, it could not be established within physics. However, with the first work on the unification of quantum theory and gravitation in the late 1930s the later field of application of the Planck units emerged. In 1955, when John Archibald Wheeler and Oskar Klein published a value as the limit to the applicability of the general relativity theory, which was smaller by a factor of 2π

than the Planck length, the smallest conceivable distance proposed by Planck, this Planck length was correspondingly corrected by the factor *2π*. Planck's suggestion was taken up again with the difference that the Planck units henceforth contained the variable $\hbar$ instead of *h*. In this course, the term in the Heisenberg uncertainty relation was also changed from *h/2* to $\hbar/2$ to keep the Planck units consistent (since the modified Planck length, which is smaller by a factor of *2π*, multiplied with the Planck momentum, yields the value $\hbar$ and not as originally *h*). However, this does not imply that this corrected value can express the exact uncertainty principle. As known, the Schrödinger equation is also in contradiction to the general theory of relativity.

In a recent paper[3], it was shown by a rigorous mathematical derivation that a consistent application of Fourier transform theory to the derivation of the HUP requires that the determining term of the Heisenberg inequality is *h/2*, not *ħ/2*. This was confirmed when extending the analysis to the Brillouin zones formulation of Solid State Physics[3].

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2} \tag{1}$$

The derivation of the HUP from the uncertainty theorem of Fourier transform theory demonstrates that the HUP arises from the dependency of momentum on wave number that exists at the quantum level[3]. It also establishes that the HUP is purely a relationship between the effective widths of Fourier transform pairs of variables (i.e. conjugate variables), whilst the HUP is not a quantum mechanical measurement principle per se[3].

The derivation of the HUP by Pierre Millette[3] is quit complicated and is based on the Nyquist-Shannon Sampling theorem[4]. A much simpler derivation of the exact formulation of the HUP results from the fact that the motion of a particle can be described by a wave of frequency $f = 1/t = v/\lambda$.

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m v \lambda f \tag{2}$$

For the sake of simplicity, the following substitution is carried out:

$$a = m v \lambda \tag{3}$$

Then

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m v \lambda f = \frac{1}{2} a f = \frac{a}{2t} \rightarrow Et = \frac{a}{2} \tag{4}$$

If, for example, an (arbitrary) measuring time $\Delta t < t = 1/f$ is chosen or an energy exchange takes place in a time $\Delta t < t$, then an exchange process with a linear time dependence would have

$$\frac{\Delta E}{E}=\frac{\Delta t}{t}\rightarrow E\Delta t=\frac{1}{2}af\,\Delta t=\frac{1}{2}af't=\Delta Et=\frac{\Delta E}{f}=\frac{1}{2}mv\lambda'=\frac{1}{2}p\lambda'<\frac{a}{2} \quad (5)$$

In this case $E\Delta t$is ¿$a/2$ (ΔE is the measurable or exchanged energy after time $\Delta t<t$). Based on this transformation, it can be seen that the wavelength of the wave would decrease at the time Δt due to a measurement or the energy exchange, which would be inconsistent, since $\Delta E<E$ implies that $f'<f$. However, when the frequency and at the same time the wavelength decreases, this is no longer consistent with $v=\lambda f$, since the wavelength and the frequency are inversely proportional to each other. In the case of an energy exchange of the energy E, f'would be the frequency of the energy exchanged after the time$\Delta t<t$, that means that only a partial exchange would occur and not an exchange of the total energy. From this it can be concluded that

$$E\Delta t\geq\frac{a}{2};\Delta E\geq E \quad (6)$$

In addition, under such circumstances

$$P=\frac{\Delta E}{\Delta t}=\frac{E}{t}<\frac{E}{\Delta t}=P'\rightarrow P'>P \quad (7)$$

(*P* is the power in the context of a measurement or an energy exchange, P' is the power necessary for the conversion of the whole energy E), which would be inconsistent for $\Delta t<t$ for instance in the context of an energy exchange or other conversion process (e.g., scattering processes, particle transformation processes) and would result in energy loss. Since, however, energy cannot be lost due to the law of conservation of energy, it follows that

$$\Delta E\geq E \quad (8)$$

from equation (6) and (8) results:

$$\Delta E\,\Delta t\geq E\,\Delta t\geq\frac{a}{2} \quad (9)$$

$$\rightarrow\Delta E\Delta t\geq\frac{a}{2} \quad (10)$$

This relationship applies to all moving particles and masses for any given velocity. In this case, the wave property of the particle motion is not relevant for this derivation, because instead of λ, the distance $x=vt=a/mv$ that traverses the particle in the time *t* can also be used. As I will show, *a* has the same size and is constant for all particles and velocities.

Using the Lorentz factor, we can show the dependence of the moving mass m_v from f at a velocity close to *c* is:

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{f^2\lambda^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{f^2\lambda^2 m_v{}^2 v^2}{m^2 c^2 v^2}}} = \frac{m_0}{\sqrt{1-\frac{f^2 a^2}{m^2 c^2 v^2}}} \tag{11}$$

$$m_v{}^2 = \frac{m_0{}^2 m_v{}^2 c^2 v^2}{m_v{}^2 c^2 v^2 - f^2 a^2}; m_v{}^4 c^2 v^2 - m_v{}^2 a^2 f^2 = m_0{}^2 m_v{}^2 c^2 v^2 \tag{12}$$

For $m' \gg m_0$ or $m_0 = 0$ we obtain

$$m_v = \frac{af}{vc} \tag{13}$$

In addition:

$$\lambda v = \frac{a}{m} = \frac{a}{\frac{m_v}{\gamma}} = \frac{a\gamma}{\frac{af}{vc}} = \frac{vc\gamma}{f} = \gamma\lambda'' c \rightarrow mv\lambda = m_v c\lambda'' \tag{14}$$

(γ is the Lorentz factor). From this one obtains:

$$\begin{gathered} mv\lambda = m_v c\lambda''; m_{Ph} c\lambda_{Ph} = m_v c\lambda'' \rightarrow m_{Ph}\lambda_{Ph} = m_v\lambda'' = \frac{h}{c} \rightarrow m_v c\lambda'' = h = mv\lambda \\ \rightarrow a = h \end{gathered} \tag{15}$$

(λ'' is the minimum wavelength of a particle at the velocity (near) *c*, the index *Ph* refers to variables of the photon). From $a = h$ follows:

$$\Delta E \cdot \Delta t \geq \frac{h}{2} \tag{16}$$

The same applies to the product of momentum and position, which can be derived from the product energy and time. However, this is less an uncertainty in the sense of an indeterminacy, as the name of the priciple says, but rather the inability of a particle to implement or realize the condition $\Delta E \cdot \Delta t < h/2$, which is, however, only relevant in the context of a measurement or because of another energetic relevance.

The following examples can demonstrate and validate this relation:

1) De Broglie generalized the fundamental relations for photons $\lambda = h/p$ to matter particles whose motion he described as a matter wave[5] and in which the kinetic particle energy forms energy packets ΔE in the time $\Delta t <\geq t = 1/f$. This can also be derived directly from the Heisenberg uncertainty principle:

$$E\Delta t=\frac{1}{2}mv^2\Delta t<\frac{h}{2}\rightarrow\Delta E\Delta t:=\frac{h}{2}\rightarrow\Delta E=\frac{h}{2\Delta t}=\frac{1}{2}hf';mv\lambda f'=hf';mv\lambda=h;$$

$$\lambda=\frac{h}{p} \tag{17}$$

$$E\Delta t=\frac{1}{2}mv^2\Delta t\geq\frac{h}{2}\rightarrow\frac{1}{2}mv^2=\frac{1}{2}mv\lambda f=\frac{h}{2\Delta t}\frac{\Delta t}{t}=\frac{1}{2}hf;mv\lambda=h;\lambda=\frac{h}{p} \tag{18}$$

Hence, the De-Broglie relationship is $mv^2=hf$.

2) In the case of radioactive α-decay, the energy of the lost mass Δm is equal to the kinetic energy of the emitted α-particles, which can also be described as matter waves. This follows directly from the Heisenberg uncertainty principle, as:

$$\Delta E\Delta t=\Delta mc^2\Delta t<\frac{h}{2}\rightarrow\Delta mc^2\Delta t:=\frac{h}{2}\rightarrow\Delta E=\Delta mc^2=\frac{1}{2}hf=\frac{1}{2}mv^2=E_{kin,\alpha} \tag{19}$$

3) The minimum width of a tunnel barrier σ_x can be determined by the uncertainty principle

$$\Delta L\Delta\varphi=mv\sigma_x\Delta\varphi\geq\frac{h}{2};m\sigma_x v=\frac{\hbar}{2} \tag{20}$$

($\Delta\varphi_{max}=2\pi;mv\sigma_x\Delta\varphi<h/2$¿. If one considers an electron with the moving mass $m_e$and the electric charge e, which tunnels through a potential difference U, then the minimum width of the tunnel barrier results as:

$$\sigma_x=\frac{\hbar}{2m_e v}=\frac{\hbar}{2m_e\sqrt{\frac{eU}{\frac{1}{2}m_e}}}=\frac{\hbar}{\sqrt{8m_e eU}} \tag{21}$$

With a potential difference of 100 mV, as occurs for example in the scanning tunneling microscopy, a smallest tunnel barrier of about 0.3 nm results from this relationship,, which coincides well with experimental observations.

4) The electron of the hydrogen atom moves on an orbital path with the most probable radius $r_B=0{,}529\cdot10^{-10}$ m. The velocity of the electron on this orbit is $2.2\cdot10^6$ m / s and can be determined from the uncertainty principle:

$$\Delta L\Delta\varphi=J\bar{\omega}\cdot2\pi=2\pi m_e r_B^{\,2}\bar{\omega}=2\pi r_B m_e\bar{v}=\pi r_B m_e v<\frac{h}{2}\rightarrow\pi r_B m_e v=\frac{h}{2};v=\frac{h}{2\pi r_B m_e} \tag{22}$$

or

$$\Delta L=\frac{\frac{1}{2}m_e\bar{v}^2}{\frac{1}{2}\bar{\omega}}=m_e\bar{v}r_B=\frac{h}{2\Delta\varphi}=\frac{\hbar}{2};\ 4\pi r_B m_e\bar{v}=h\rightarrow v=\frac{h}{2\pi r_B m_e}=2{,}2\cdot 10^6 m/s \qquad (23)$$

(the maximum angle $\Delta\varphi=2\pi$). If the uncertainty relation were ≥ ħ / 2, the velocity of the electron would be only 3.5$\cdot 10^5$ m/s and the electron would fall on the nucleus due to Coulomb attraction. The examples validating the uncertainty relation $\Delta x\cdot\Delta p_x\geq h/2$ can be continued indefinitely.

2. The uncertainty principle and the quantization of electromagnetic energy

An energy wave packet hf is cut off after energy emission after a distance $\Delta x=\lambda$ (after the time $\Delta t=\Delta x/c=1/f$). This follows directly from the Heisenberg uncertainty principle, which I will show below. Since the two principal force vectors are perpendicular to each other, the two major components of the energy of the photon (expressed here as a function of the force F) must be considered separately. The uncertainty equation $\Delta E\cdot\Delta t\geq\frac{h}{2}$ can be derived from equation (1) and determines that

$$\Delta E_w\Delta t=\frac{1}{2}m_v c^2\Delta t=\frac{1}{2}m_v c\Delta x=\frac{1}{2}m_v c\lambda\geq\frac{h}{2};\rightarrow m_v c\lambda\geq h \qquad (24)$$

(E_w is the kinetic energy of the forward motion of the electromagnetic wave particle, $m_v$is the moving mass).The condition $mc\lambda\geq h$ can be distributed onto two condition, namely 1) $mc\lambda=h$ and 2) $mc\lambda>h$.

1) Considering the first case $mc\lambda=h$

$$m_v c\lambda=h\rightarrow m_v c^2=\overbrace{m_v c\lambda}^{h} f=hf=m_v c\underbrace{\lambda f}_{c} \qquad (25)$$

(for $m'>0$). Often, the "moving or relativistic mass" is seen in direct connection with the Lorentz transformation, being a Lorentz invariant. Since photons have the rest mass $m_0=0$ and the velocity $v=c$, the moving mass is often considered as not applicable for photons due to the Lorentz factor (equation 11), which has an undefined value in the case of $v=c$. However, the speed of light has not been transformed in the derivation of the Lorentz transformations. In fact, the special relativity theory results from this observation: hypothesizing a constant light velocity in all reference systems, the equations for the Lorentz transformations result. Moreover, for $v=c$ the equivalent equation for the moving mass of a particle (equation 13) cannot be transformed back into the original equation

(11), since the term in the equation (12) becomes undefined. Hence, the Lorentz factor cannot be applied for calculation of the moving mass of photons, but also above all because the mass and the velocity of a photon do not "increase". The physicists Rebka and Pound succeeded in 1960 in confirming the Einsteinian prediction of a photon mass in an impressively experiment[6]. In the gravitational field, quanta are not accelerated but are more energetic, which manifests itself in a frequency increase. The experiment showed that photons which had been emitted at the top had a higher frequency upon reaching the bottom than the photons which were emitted at the bottom. Photons which were emitted at the bottom had a lower frequency upon reaching the top than the photons emitted at the top. The value measured by Pound and Rebka for the frequency shift was very much in agreement with the theoretically derived frequency shift using the equation $m_v c^2 = hf$. Thus, this experiment confirms that photons can be assigned a real moving mass.

2) Considering the second case $m_v c\lambda > h$:

The determining term in the HUP is dependent on the wave number at quantum level[2].

$$h := \frac{2\pi \vec{p}}{\vec{k}}; \vec{k} := \frac{2\pi}{\lambda}; h < m_v c\lambda = p\lambda = \frac{2\pi p}{\vec{k}} = h; \tag{26}$$

which is a contradiction per se. Moreover, the value Δx can only resolve values of p up to its Nyquist value p_N given by the Nyquist-Shannon Sampling theorem[3], in apparent contradiction to the HUP[2], namely in the form of

$$\Delta x \Delta p \leq \frac{h}{2} \tag{27}$$

Hence

$$m_v c\lambda \equiv h \rightarrow E_w = \frac{1}{2} m_v c^2 = \frac{1}{2} m_v c\lambda f = \frac{1}{2} hf \tag{28}$$

From equation (28) and the principle of equivalence of mass and energy we receive:

$$E = m_v c^2 = hf \tag{29}$$

for $m_0 \ll m'$ or $m_0 = 0$. Furthermore for any value of λ:

$$m_v = \frac{hf}{c^2} = \frac{\overbrace{m_v c\lambda}^{\not h} f}{c^2} = \frac{m_v c\lambda c}{c^2 \lambda} = \frac{m_v c^2}{c^2} = m_v \tag{30}$$

These equations provide evidence that the HUP is able to precisely specify the energy of a wave particle as well as its mass. The velocity of matter waves particle and electromagnetic waves is thereby limited to c, since for $v > c$

$$\lambda = \frac{h}{m_v v} = \frac{m_v c \lambda}{m_v v} < \frac{h}{m_v c} = \lambda \rightarrow v \leq c \tag{31}$$

(32)

The model of the matter waves states that a wave of a certain wavelength can be also assigned to each small particle with a specific momentum. With the matter wave, quantum mechanical effects of matter, for example the interference, can be described, which cannot be explained by the particle model of classical mechanics. The square magnitude of the wave describes the probability density for the position of a particle. Louis de Broglie generalized the equation $\lambda = h/p$ to arbitrary particles[5]. This De-Broglie low[5] could be repeatedly demonstrated experimentally and is for instance the basis of the Bohr's atom model. The reason for this is that the distance covered by small particles in a given time Δt (i.e. the time in which an electron orbits the nucleus or the measuring time) and their momentum are also subjected to the HUP, while wave packets of $hf = hv/\lambda$ (λ is the De-Broglie-wavelength) are created.

Moreover, most particles have also a second motion component, which is their self-rotation. If the self-rotation of electromagnetic or other self-rotating particles can be attributed to a matter wave, the rotation energy E_{rw} of this motion in case of an electromagnetic particle (in the time $\Delta t = 1/f$ in which the photon and its rotation is generated) is calculated as:

$$E_{rw} \Delta t = \frac{E_{rw}}{f} = \frac{m_v \lambda_{rw} f_{rw} c}{f} = \frac{h}{2}; f_{rw} = f \rightarrow E_{rw} = \frac{1}{2} hf \tag{33}$$

$$E = E_w + E_{rw} := hf \tag{34}$$

(f_{rw} is the frequency of the rotation wave; λ_{rw} is the wavelength of the rotation wave, f is the frequency of the electromagnetic wave). We can see that the rotation energy is "quantized", its value being equal to the "pure kinetic energy" of the forward motion of the particle, and that the rotation wave energy adjusts its frequency to the frequency of the particle wave. The rotation energy is also a kinetic energy, being nothing but the sum of the kinetic energies of the individual mass points of the particle, since kinetic energies have no direction and therefore the amounts are summed up. Hence, the total kinetic energy is higher than the "pure" kinetic energy of the forward motion of wave particles. However, taking the very small value of the rotation velocity in case of electrons or protons into account, this rotation energy, if not quantized because of measurements or other relevance,

does not relevantly alter the value of the kinetic energy ¿ $\frac{1}{2} m_v v^2 = E - m_0 c^2$ of self-rotating particles (for $v \ll c$).

Photons have an angular momentum that can be measured as spin 1, -1 or 0. But how does the photon actually behave? It is generally assumed that the spin is quantized and can only assume certain quantum states and that the particle does not rotate in real. In turn, I tried to imagine how a real rotation of a photon with the energy $\frac{1}{2} J \omega^2 = \frac{1}{2} m_v r^2 \omega^2 = \frac{1}{2} m_v c^2$ and the spin 1 might look. Thereby, I assumed that a particle strives to fulfill at least in part the quantized, oversized value. If the photon would not rotate around itself, but instead describes a circle or ellipse, whose radius corresponds exactly to the wavelength of the rotation wave (which is equal to the wavelength of the photon), the quantized rotational energy value would be fully satisfied ($J = m_v r^2, L = \hbar, v = c$). The resulting movement of the photon would be in such a case a helical path, but also other more complex trajectories are conceivable.

The circular or elliptical polarizations of light (ideal helix line) and other possible trajectories may similarly be described and construed as elements of a 2-dimensional vector space. What is physically a superposition of states corresponds mathematically to a linear combination of vectors. Since amplitude and phase must be taken into account when superimposing, complex numbers serve as scalars: the trajectories of light therefore form a vector space over the body of the complex numbers. As a basis (with which one can represent each vector), the two (left- and right-handed) circular polarizations (ideal helices) can be used, into which the other motion vectors can be transformed.

The motion model of a photon described here is consistent with all experiments with light done so far (e.g. double split experiments, scattering experiments, polarization experiments) and with the fact that unpolarized light is a transverse wave whose trajectory is exactly as described above.

3. Conclusions

In conclusion, the presented equations show that the Heisenberg's uncertainty principle causes the quantization of electromagnetic energy, which can be regarded as the reason why the quantization phenomenon in quantum physics is as fundamental as it is. Moreover, the uncertainty principle is the main trigger for any quantization in quantum physics, as I was able to show in a further publication[2]. The quantization of the emitted energy always occurs when the wave particle is generated in a time $\Delta t = \Delta x / c$ (photons, gluons), which is not given by a defined lifetime ¿ $h / 2\Delta E$ (as it is the case for W- and Z-bosons). In the case of electromagnetic waves, the forward motion, and separately, the

rotation wave is subjected to the uncertainty principle, causing the frequency of the rotation wave to be equal to the frequency of the particle, while the total energy of a photon is hf and the possible motion of the electromagnetic wave is described by a helix as well as other more complex trajectories whose motion vectors can be converted into a helical path.

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