

# Properties of the Multidimensional Fourier Transform

$f(\mathbf{x}) = \int_{\mathbb{R}^D} F(\mathbf{u}) \exp(j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{u}$	$F(\mathbf{u}) = \int_{\mathbb{R}^D} f(\mathbf{x}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{x}$
$\text{rect}(x, y)$	$\frac{\sin \pi u}{\pi u} \frac{\sin \pi v}{\pi v}$
$\text{circ}(x, y)$	$\frac{1}{\sqrt{u^2+v^2}} J_1(2\pi\sqrt{u^2+v^2})$
$\exp(-(x^2 + y^2)/2r^2)$	$2\pi r^2 \exp(-2\pi^2(u^2 + v^2)r^2)$
$\cos(\pi(x^2 + y^2)/r^2)$	$r^2 \sin(\pi(u^2 + v^2)r^2)$
$\exp(j\pi(x^2 + y^2)/r^2)$	$jr^2 \exp(-j\pi(u^2 + v^2)r^2)$
$\delta(\mathbf{x})$	1

# Two-dimensional Fourier Transform of Selected Functions

	$f(\mathbf{x}) = \int_{\mathbb{R}^D} F(\mathbf{u}) \exp(j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{u}$	$F(\mathbf{u}) = \int_{\mathbb{R}^D} f(\mathbf{x}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}) d\mathbf{x}$
(i)	$af_1(\mathbf{x}) + bf_2(\mathbf{x})$	$aF_1(\mathbf{u}) + bF_2(\mathbf{u})$
(ii)	$f(\mathbf{x} - \mathbf{x}_0)$	$F(\mathbf{u}) \exp(-j2\pi\mathbf{u} \cdot \mathbf{x}_0)$
(iii)	$f(\mathbf{x}) \exp(j2\pi\mathbf{u}_0 \cdot \mathbf{x})$	$F(\mathbf{u} - \mathbf{u}_0)$
(iv)	$f(\mathbf{A}\mathbf{x})$	$\frac{1}{ \det \mathbf{A} } F(\mathbf{A}^{-T}\mathbf{u})$
(v)	$f_1(\mathbf{x}) * f_2(\mathbf{x})$	$F_1(\mathbf{u})F_2(\mathbf{u})$
(vi)	$f_1(\mathbf{x})f_2(\mathbf{x})$	$F_1(\mathbf{u}) * F_2(\mathbf{u})$
(vii)	$\nabla_{\mathbf{x}} f(\mathbf{x})$	$j2\pi\mathbf{u}F(\mathbf{u})$
(viii)	$\mathbf{x}f(\mathbf{x})$	$\frac{j}{2\pi} \nabla_{\mathbf{u}} F(\mathbf{u})$
(ix)	$F(\mathbf{x})$	$f(-\mathbf{u})$
(x)	$f^*(\mathbf{x})$	$F^*(-\mathbf{u})$
(xi)	$f_1(x)f_2(y)$	$F_1(u)F_2(v)$
(xii)	$\int_{\mathbb{R}^D}  f(\mathbf{x}) ^2 d\mathbf{x} = \int_{\mathbb{R}^D}  F(\mathbf{u}) ^2 d\mathbf{u}$	