SIAM J. COMPUT.
Vol. 9, No. 3, August 1980

0097-5397/80/0903-0004 $01.00/0

A CORRECT PREPROCESSING ALGORITHM FOR BOYER–MOORE STRING-SEARCHING*

WOJCIECH RYTTER†

Abstract. We present the correction to Knuth's algorithm [2] for computing the table of pattern shifts later used in the Boyer–Moore algorithm for pattern matching.

Key words. algorithm, pattern-matching, string, overlap

The key to the Boyer–Moore algorithm for the fast pattern matching is the application of the table of pattern shifts which is denoted in [1] by Δ_2 and in [2] by dd'. Let us denote this table by D.

Assume that the pattern is given by the array pattern $[1:n]$, so D is given as an array $D\ [1:n]$. For every $1 \leqq j \leqq n$, $D[j]$ gives the minimum shift $d > 0$ such that the pattern with the right end placed at the position $k+d$ of the processing string is compatible with the part of string scanned before, where k is the last scanned position in the string and j is the last scanned position in the pattern.

The formal definition of D given in [2] is:

$$D[j] = \text{MIN}\ \{s+n-j \mid s \geqq 1 \text{ and } (s \geqq j \text{ or pattern } [j-s] \neq \text{pattern } [j]) \text{ and } ((s \geqq i \text{ or pattern } [i-s] = \text{pattern } [i]) \text{ for } j < i \leqq n)\}.$$

Algorithm A given by Knuth is:

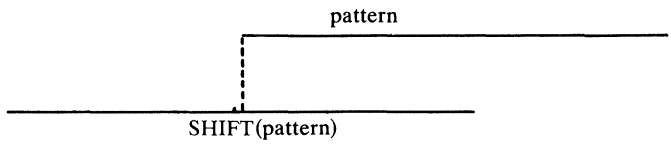
```
A1. for k := 1 step 1 until n do D[k] := 2*n - k;
A2. j := n; t := n + 1;
    while j > 0 do
    begin
    f[j] := t;
    while t ≦ n and pattern [j] ≠ pattern [t] do
    begin
                          D[t] := MIN (D[t], n - j);
                              t := f[t];
    end
    t := t - 1; j:-j - 1;
    end;
A3. for k := 1 step 1 until t do

                  D[k] := MIN (D[k], n + t - k);
```

Algorithm A computes also the auxiliary table $f[0:n]$, for $j<n$ defined as follows: $f[j] = \min\{i \mid j < i \leqq n$ and pattern $[i+1] \cdots$ pattern $[n] =$ pattern $[j+1] \cdots$ pattern $[n+j-i]\}$; the final value of t corresponds to $f[0]$. $f[0]$ is the minimum non-zero shift of pattern on itself; let us denote this value by SHIFT (pattern).

* Received by the editor January 18, 1979, and in revised form May 25, 1979.

† Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México Apartado Postal 20-726, México 20, D.F. On leave of absence from Institute of Informatics, Department of Mathematics, Warsaw University, Warsaw, Poland.



Take as inputs to Algorithm A the following two strings: pattern 1 = aaaaaaaaaa and pattern 2 = abaabaabaa. Denoting by def$D$ and D' respectively the value of D according to the definition and computed by Algorithm A we obtain the following results:

j	=	1	2	3	4	5	6	7	8	9	10
pattern 1[j]	=	a	a	a	a	a	a	a	a	a	a
Def$D$[j]	=	10	10	10	10	10	10	10	10	10	10
D'[j]	=	10	18	17	16	15	14	13	12	11	10

SHIFT(pattern 1) = 1.

pattern 2[j]	=	a	b	a	a	b	a	a	b	a	a
Def$D$[j]	=	12	11	10	12	11	10	12	11	2	2
D'[j]	=	12	11	10	16	15	14	13	12	2	2

SHIFT(pattern 2) = 3.

The disagreement between Def$D$ and D' demonstrates explicitly that Knuth's algorithm is incorrect.

There are three cases which are considered in the design of Algorithm A for computing the value of $D[j]$:

Case (1). $D[j] = 2*n - j$. This is the most simple case computed in the part A1 of Algorithm A.

Case (2). $D[j] < n$ and pattern $[l] \neq$ pattern $[j]$, where $l = n - D[j]$. In this case $D[j]$ is computed in the part A2.

Case (3). $n \leqq D[j] < 2*n - j$ and $j \leqq$ SHIFT(pattern) $= f[0] = t$. In this case $D[j]$ is computed in the part A3 of Algorithm A.

However, another case occurs which is not covered by Cases (1), (2) and (3):

Case (4). $n < D[j] < 2*n - j$ and $j >$ SHIFT(pattern). For example it occurs for pattern = pattern 2 and $j = 5$. To correct Algorithm A, we have to consider not only the minimal nonzero shift of the string on itself but all shifts, namely all i such that $0 < i \leqq n$ and pattern $[i+1] \cdots$ pattern $[n]$ = pattern $[1] \cdots$ pattern $[n-i]$. Let us denote the set of all such i by ALLSHIFTS(pattern). Using the method of computing the failure function in the pattern-matching algorithm of Knuth, Morris and Pratt [2], we give below a correct version of the algorithm, where A1, A2 denote the corresponding parts of Algorithm A.

ALGORITHM B.

```
A1; A2;
q := t; t := n+1-q; q1 := 1;
    B1. j1 := 1; t1 := 0;
        while j1 ≦ t do
        begin
        f1[j1] := t1;
        while t1 ≧ 1 and pattern [j1] ≠ pattern [t1] do t1 := f1[t1];
            t1 := t1+1; j1 := j1+1;
        end;
```

```
B2. while q < n do
    begin
    for k := q1 step 1 until q do D[k] := Min (D[k], n + q - k);
    q1 := q + 1; q := q + t - f1[t];
        t := f1[t]; end;
```

The part B1 computes the auxiliary table $f1[1:t']$ where $t' = n + 1 - \mathrm{SHIFT}(\text{pattern})$, and the part B2 computes the values of $D[j]$ for both Cases (3) and (4).

$$f1[1] = 0 \quad \text{and} \quad \text{for } 1 < j \leqq t',$$

$$f1[j] = \max\{i \mid 1 \leqq i < j \text{ and pattern } [j-i+1] \cdots \text{pattern } [j-1] = \text{pattern } [1] \cdots \text{pattern } [i-1]\}.$$

The correctness of the part B2 follows from the following: If ALLSHIFTS(pattern) = $\{i_1, i_2, \cdots, i_k\}$ and $i_1 = \mathrm{SHIFT}(\text{pattern})$ and $i_1 < i_2 < \cdots < i_k$ and $t_1 = n + 1 - i_1$, $t_{p+1} = f1[t_p]$ for $p = 1, 2, \cdots, (k-1)$ then $i_{p+1} = i_p + t_p - t_{p+1}$ for $p = 1, 2, \cdots, (k-1)$.

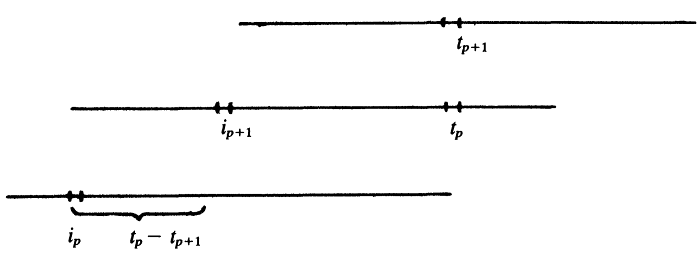


FIG. 1. *The graphical representation of the computation of* i_{p+1}.

Remark 1. The same table space can be used for f and $f1$.

Remark 2. The tables f and $f1$ are related in the following way: Let pattern′ be the string resulting from reversing the string pattern and $f1$ be computed for the string pattern and f be computed for pattern′.

Then

$$f1[i] = n - f[n - i + 1] + 1 \quad \text{for } i = 1, 2, \cdots, (n+1).$$

Remark 3. Denote OVR(pattern) = n − SHIFT(pattern). So OVR(pattern) gives the maximum overlap of the pattern with itself. The difference in the time complexity of Algorithms A and B is proportional to OVR(pattern) which can be linear with respect to n. However, on the average it is very small for alphabets of the size greater than 1. Let $V(n, k)$ denotes the average value of OVR(pattern) taken over the set of all patterns of the length n over the same alphabet of the size k.

The rounded values of $V(n, 2)$ for $n \leqq 14$ computed on B6700 are shown in Table 1.

TABLE 1

n	1	2	3	4	5	6	7
$V(n, 2)$	0	0.5	0.75	1.0	1.125	1.281	1.375
n	8	9	10	11	12	13	14
$V(n, 2)$	1.453	1.500	1.545	1.574	1.595	1.607	1.618

LEMMA. 1. *If* $k>1$ *then* $V(n,k)<k/(k-1)^2$.
2. $V(n,2)<2$.
3. $V(n,k)<1$ *for* $k>2$.

Proof. Fix n and k and assume that $k>1$. Let a_j be the number of patterns such that OVR(pattern) $=j$ for $j=1,2,\cdots,(n-1)$. Every pattern with OVR(pattern) $=j$ is determined by its prefix of the length $n-j$. So $a_j \leqq k^{n-j}$. Hence $V(n,k)=(\sum_{j=1}^{n-1} j\cdot a_j)/k^n \leqq \sum_{j=1}^{n-1} j\cdot(1/k)^j \leqq \sum_{j=1}^{\infty} j\cdot(1/k)^j = k/(k-1)^2$. Parts 2 and 3 of the lemma follow from 1. This ends the proof.

REFERENCES

[1] R. S. BOYER AND J. S. MOORE, *A fast string searching algorithm*, Comm. ACM, 20 (1977), pp. 762–772.

[2] D. E. KNUTH, J. H. MORRIS, JR. AND V. R. PRATT, *Fast pattern matching in strings*, this Journal, 6 (1977), pp. 323–350.