The Basis of Necessity and Possibility

November 8, 2016

1 Preliminaries

1.1 Epistemic and metaphysical modalities

1.2 Relative and absolute modalities

$\Box(\Gamma \to p))$ for: It is necessary that p *relative to* Γ
$\Box p = \forall q(q \mathrel{\Box\!\!\to} p)$ for: It is *absolutely* necessary that p

1.3 The logic of absolute necessity

K $\quad \Box(A \to B) \to (\Box A \to \Box B)$

T $\quad \Box A \to A$

S4 $\quad \Box A \to \Box\Box A$

B $\quad A \to \Box\Diamond A$

S5 $\quad \Diamond\Box A \to \Box A$

2 Problems

Dummett ‘The philosophical problem of necessity is twofold: What is its source, and how do we recognize it?’

Blackburn’s Dilemma Suppose $\Box p$ because q. Then either (a) $\Box q$ or (b) $\neg\Box q$ (i.e. $\Diamond\neg q$). If (a), we merely explain one necessity by assuming another – “bad residual ‘must”’; but if (b), then p isn’t really necessary after all.

3 Some unsatisfactory answers – reductive theories

3.1 Conventionalism and the linguistic theory of necessity

3.2 Worlds and combinatorial theories of possibility

4 Essence and modality

4.1 Basics

The general pattern of essentialist explanations of necessities:

> Necessarily . . . because it is true by the nature of __ that —

Examples:

> Necessarily *Aristotle is a man* because it is true by *Aristotle*'s nature that *he is a man*
> Necessarily *whales are mammals* because it is true by the nature of *whales* that *they are mammals*
> Necessarily *gold is an element* because it is true by the nature of *gold* that *it is an element*
> Necessarily *everything is self-identical* because it is true by the nature of *identity* that *it is a reflexive relation*
> Necessarily $a + b = b + a$ for all whole numbers a, b because it is true in virtue of the nature of *addition* that *it is commutative*

4.2 An essentialist theory of necessity and possibility

(*Necessity*) $\Box p$ iff, and because, $\exists x_1 ... x_n$ *it is true by the nature of* $x_1 ... x_n$ *that* p

(*Possibility*) $\Diamond p$ iff, and because, $\neg\exists x_1 ... x_n$ *it is true by the nature of* $x_1 ... x_n$ *that* $\neg p$

4.3 The essentialist theory further explained – questions and difficulties

4.3.1 What are essences/natures?

Essence and definition. The necessity of essence: $\Box_x \phi(x) \rightarrow \Box\Box_x \phi(x)$

4.3.2 Blackburn's dilemma revisited, and a related difficulty

Transmissive and non-transmissive explanations of necessity

4.3.3 What kind of explanation does the essentialist theory provide?

4.3.4 Contingency

Iterated necessities

(a) $\Box\forall z(z = a \rightarrow Ha)$

(b) $\exists x\Box_x\forall z(z = a \rightarrow Ha)$

(c) $\Box\Box\forall z(z = a \rightarrow Ha)$

(d) $\Box\exists x\Box_x\forall z(z = a \rightarrow Ha)$

(**) $\Box(\Box p \leftrightarrow \exists x_1 ... x_n \Box_{x_1...x_n} p)$

($\Box_{\leftrightarrow} sub$) $\Box(A \leftrightarrow B), \Box A \vdash \Box B$

(e) $\Box(\Box\forall z(z = a \rightarrow Ha) \leftrightarrow \exists x\Box_x\forall z(z = a \rightarrow Ha))$

Contingent existents and non-existents Supposing some things which actually exist might not have done, does that mean that some propositions which are necessary as things stand might not have been necessary? And supposing that there might have been things distinct from any there actually are, does that mean that some propositions which are actually possible might not have been possible?

Necessary existence of purely general properties Let ϕ be any purely general property, p the proposition that ϕ exists, q the proposition that there exists a predicate standing for ϕ. Then:

(Abundance) $\Box(p \leftrightarrow \Diamond q)$

so $\Box p \leftrightarrow \Box\Diamond q$ (by K)

and $p \leftrightarrow \Diamond q$ (by the Law of Necessity, i.e. T)

but $\Diamond q \leftrightarrow \Box\Diamond q$ (by S5)

so $p \leftrightarrow \Box p$ (by transitivity of the biconditional)

Strong individual essences

Indiscernibles $\forall x\forall y(x \neq y \rightarrow \exists\phi(\phi x \leftrightarrow \neg\phi y))$

Indiscernibles$^+$ $\forall x\exists\phi\forall y(x \neq y \rightarrow (\phi x \leftrightarrow \neg\phi y))$