

BACKYARD ASTRONOMY

A PHYSICS PROFILE PROJECT ON THE TOPIC OF ASTROPHYSICS
PRESENTED WITH THE USE OF L^AT_EX

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Abstract

Having built a telescope and walked through every part of its construction we understood the basic functionality of a telescope but felt like we wanted a better and more in depth understanding of its workings and capabilities. This profile project presents two investigations: The first being an investigation to the mass of Jupiter. The results indicate that the mass is around $1,7969 \cdot 10^{27}$ This result around an 5% difference to the accepted Value for the mass of Jupiter. The second being: Measuring the depth of a moon crater. The depth we hypothesized was 3.4km, with a margin of error of 2km. The depth we calculated is 5.1km, this is within the measurement uncertainty. The final subject we included is to display an in depth understanding in the topic by simulating a telescope in Python. Displaying the path light rays will follow before they reach a viewer's eye.

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Introduction

The universe, a cold and infinitively large space. Mankind has been amazed time and time again by the most amazing sight it has to offer. From planets, distant stars and galaxies like our own milky way all hidden somewhere and waiting to be found. So when given the opportunity to construct a telescope for our profile project, resisting wasn't an option and we grabbed this opportunity with both hands.

Although constructing a telescope and looking at some pretty dots in the sky would have been enough for some people we wanted more out of it. When looking up into the night-sky, what is the most prominent object visible? The moon and what a beauty it is, when viewed through our very own telescope various features of its marked surface can be distinguished. When gazing upon the lunar surface the opportunity of conducting some measurements had to be taken. Farther in our solar system there is a gas giant. Although visible with the naked eye as a bright speck on the night-sky, Jupiter is the most massive planet of them all. This makes one wonder: How massive exactly would this giant be? So we took it upon ourselves to accurately calculate the mass of Jupiter.

While collecting all the data, sitting and waiting one has time to wonder: How does this mechanism work? What happened to a light beam between entering our telescope and reaching our eyes. Finding the answer to this question we programmed a Python based simulation. Replicating what happens to this light beam while showing the path it followed.

To summarize: the goal of this profile project is to find out the depth of a lunar crater, Jupiter's mass, understand the workings of a telescope and simulate them in Python.

Chapter 1

Scientific cosmology through the ages

1.1 The history of scientific cosmology point. [1]

1.1.1 Object location in the sky

The human awareness of the universe began when people realised that they could observe objects in the sky, and saw that the position of these objects changed over days, months, and years. To an observer who is looking at the sky with the naked eye, the sky appears to be a spherical bowl, "a celestial sphere" that extends from all points along the horizon to the zenith, the point directly overhead. Since the objects seen in the sky are so far away and without the aid of any specialised equipment other than the human eye, ability to see depth is lost. This lack of depth perception causes everything to appear at equal distances on the night-sky. To determine the location two coordinates are needed on the "celestial sphere". (figure: 1.1) These coordinates determine the object's horizontal and vertical position. The vertical coordinate is determined by measuring an altitude angle upwards from the nearest point on the horizon, and the horizontal coordinate is found by measuring an azimuth angle from the rotating eastwards along the horizon to that nearest

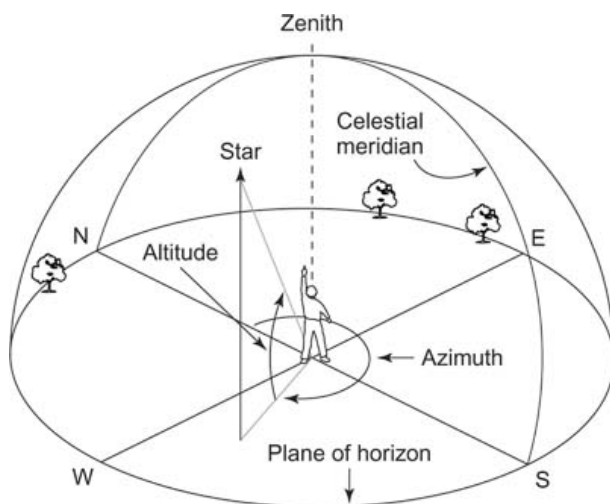


Figure 1.1: Naked-Eye Astronomy [1]

1.1.2 Early tools

Early astronomers used many kinds of instruments to study the heavens. All were basically tools for measuring or calculating the positions of objects in the sky. With the help of these



Figure 1.2: Ptolemy using a quadrant.

instruments, astronomers mapped the stars and made timetables to accurately predict the future positions of the planets, sun and moon. This knowledge was then used for calendar making or for navigational purposes on the sea.

The Quadrant [2] is an example of one of the many instruments used to measure angles. (1.3) It can be used to measure angles up to 90° degrees. As a general rule the larger the instrument, the more precisely it could measure angles. One of the largest is a mural ("wall") quadrant a large 90-degree arc attached to a north-south wall, with a sighting tool to measure

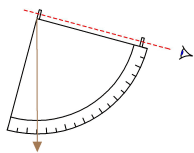


Figure 1.3: Geometric quadrant.

the altitudes of stars and planets. The most famous mural quadrant was "The great mural quadrant" of Thycho Brahe, built in Denmark as part of a grand observatory supported by the king. [3]

1.2 The telescope

Back in the early 17th century, a new instrument for studying the heavens was introduced. The so-called telescope revolutionised cosmology, it became the astronomer's most essential tool to observe the night sky. Objects previously invisible suddenly became visible. Suddenly the universe was no longer limited to what the human eye could see.

1.2.1 The invention

Manufacturing lenses and the properties of them were known since the time of ancient Greece. However, they were only introduced in Europe in the 13th century. With the first being available in cities like Venice and Florence. It is not

clear why it took so long for the first person to request a patent for a telescope, this was done by a Dutch eyeglass maker named Hans Lippershey in the year 1608. [4]

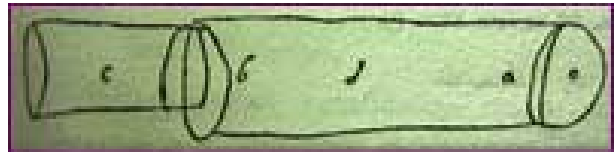


Figure 1.4: The earliest illustration known of a telescope

1.2.2 Galileo's optic tube

The news of this revolutionary invention spread very rapidly throughout Europe. The first telescope made by Galileo was a simple tube containing two lenses. After this, he made improvements on his initial design and presented his device to the Venetian Senate.



Figure 1.5: A 1754 painting by H.J. Detouche shows Galileo Galilei displaying his telescope to Leonardo Donato and the Venetian Senate.

The Senate, in turn, set him up for life as a lecturer at the University of Padua and doubled his salary. While Galileo did not invent the telescope, he did design and built telescopes with higher magnifying power for private use and others. He was a skilled instrument maker and his telescopes were known for their high quality.

1.2.3 Pointing the telescope skyward

Galileo became the first person to point a telescope skyward [5]. Although the telescope he used was small and the images were fuzzy, Galileo was able to make out mountains and craters on the moon's surface and discovered that Jupiter had its own set of moons: "The Galilean satellites". After Galileo's and Sir Isaac Newton's time, astronomy flourished, much larger and more complex telescopes were invented. With their advancement in technology, astronomers discovered many new objects in the sky and were able to do a more accurate calculation.

Chapter 2

The working of a telescope

A telescope is an instrument that collects the radiation emitted by distant sources. The name "telescope" covers a wide range of instruments. All of the telescopes collect and detect electromagnetic radiation, but the way they do it differs a lot. Every telescope can detect a specific part of the electromagnetic spectrum, the collective term for all known frequencies of light. Electromagnetic radiation differs in frequencies, the higher the frequency the more energy the light wave has and the smaller the wavelength is. The electromagnetic spectrum is divided into 6 main categories depending on their energy level.

2.0.1 Electromagnetic radiation

These 6 categories are from lowest to highest energy level: [6]

- **Radio wave radiation** is electromagnetic radiation with the lowest energy level and frequency. With the wavelengths varying between a meter to several kilometers.
- **Micro wave radiation** is a form of electromagnetic radiation with wavelengths ranging from as long as one meter to as short as one millimeter; with frequencies between 300 MHz (0.3 GHz) and 300 GHz.
- **Infrared radiation** is also invisible electromagnetic radiation with just longer wavelengths than those of visible light. Most of the thermal radiation emitted by objects near room temperature is infrared.
- **Visible light radiation** is the part of the electrical magnetic spectrum that is visible to the human eye. Visible light is usually defined as having wavelengths in the range of 400–700 nanometers.
- **Ultra violet radiation (UV)** is an electromagnetic radiation with wavelengths from 10 nanometers - 400 nanometers shorter than that of visible light.

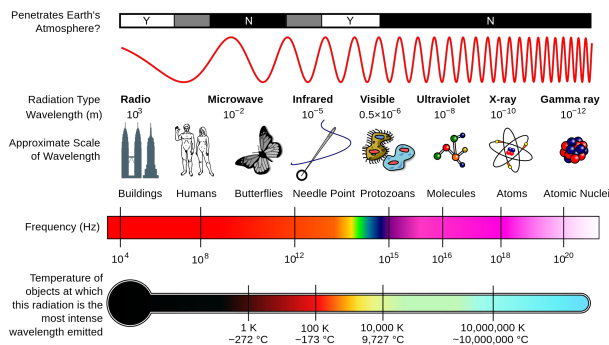


Figure 2.1: Spectrum of electromagnetic radiation

- **X-radiation** Most X-rays have a wavelength ranging from 0.01 to 10 nanometers. Due to their penetrating ability, hard X-rays are widely used to image the inside of objects.
- **Gamma radiation** is penetrating electromagnetic radiation arising from the radioactive decay of atomic nuclei, black holes and supernovae. These light ray consists of the highest range of energy.

2.0.2 Collecting electromagnetic radiation

The atmosphere is impenetrable for most of the electromagnetic spectrum. Only a small portion of UV spectrum, visible light, some portions of the infrared spectrum and radio waves are able to reach the earth surface. (figure: 2.4)



Figure 2.2: Artist's impression of the Fermi gamma-ray space telescope

For this reason, X-ray, gamma, Infrared and microwaves have to be observed from outside the earth his atmosphere. We shall quickly describe a few of the telescopes used to observe these types of electromagnetic radiation.

An example of this is the Fermi Gamma-ray Space Telescope. [7] (figure: 2.2) It is used to map the gamma radiation emitted

The Planck space observatory (figure: 2.3) [8] is another example of a telescope in orbit around the Earth. It was operational from 2009 to 2013. During this period one of its main tasks was mapping the microwave background radiation in greater detail than was previously possible.

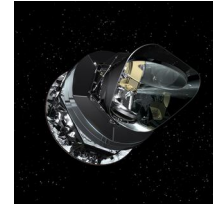


Figure 2.3: Artist's impression of the Planck spacecraft.

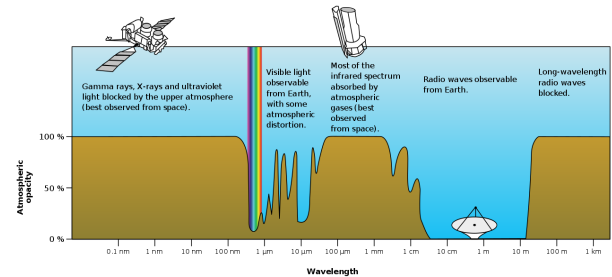


Figure 2.4: A diagram of the electromagnetic spectrum with the Earth's atmospheric transmittance (or opacity) and the types of telescopes used to image parts of the spectrum.

Because getting a telescope in orbit is a much harder task than making a ground telescope ground telescopes are way more abundant.

This includes professional observatory, DIY backyard telescopes, radio aerial antenna and giant radio dishes like the Five-hundred-meter Aperture Spherical Telescope (FAST).

2.1 Types of optical telescopes.

In this thesis, we shall focus on the optical telescopes. These telescopes are able to collect and focus the radiation in the visible light range of the electromagnetic spectrum. They create a

magnified image that can be directly observed by the human eye. In understanding how a telescope works we shall first discuss the workings of the elements that form a telescope. We define 3 main types of optical telescopes: [9]

- **Reflectors:** Make use of a single or combination of curved mirrors to reflect and combine light to a single point.
- **Refractors:** Make use of a single or combinations of lenses to combine light beams to a single point.
- **Catadioptric telescopes** Make use of a combination of lenses and mirrors to combine light beams to a single point.

2.1.1 Optical elements

In order to understand how telescopes work we need to understand the basic principles of curved mirrors and lenses. Once a surface is shaped like the part of a larger sphere it is called a spheroidal surface. Once a surface has this shape it obtains the optical ability to focus light. Although the focusing property's of a spheroidal surface are not perfect it costs much less effort hence less money to create a spheroidal surface. The surface that does has a perfect focusing property is a parabolic (non-spherical) surface although this is much harder to create. Both mirrors and lenses used in telescopes can be shaped as spheroidal surfaces, parabolic surfaces or perfectly flat.

Refraction

Refraction is the bending of the path of a light wave as it passes from one material into the other material. Refraction is caused by a change in the speed of light wave as it crosses the boundary of

2 different transparent materials. For example, light travels through a vacuum at its maximum speed of $3.0 \cdot 10^8 m/s$ when the light wave hits a different material such as glass its speed will go down upon crossing the boundary. The amount of slowing down is measured in the materials refractive index. The refractive index of a medium is the ratio of the speed of light in a vacuum to the speed of light in the medium.

$$n = \frac{c}{v} \quad (2.1)$$

where c is the speed of light in a vacuum, v is the phase velocity of light in the medium and n is the refractive index. (table: 2.1)

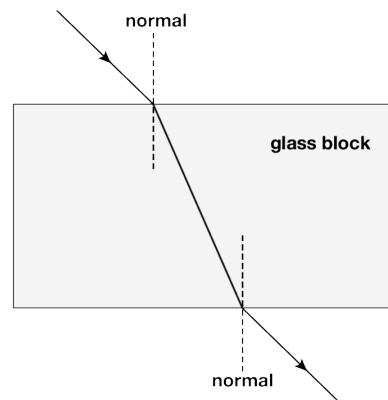


Figure 2.5: Light leaves the vacuum and enters the glass which has a higher refractive index bending it towards the normal. When it leaves the glass it will bend away from the normal.

When light enters the glass at an angle, not only the speed but also its direction will change. The light will bend towards the normal when travelling into a medium with a higher index of refraction, and away from the normal when travelling into a medium with a lower refractive index. [11] (figure: 2.5)

Table 2.1: Index of Refraction for Some Common Substances. [10]

Substance	Index of Refraction
Vacuum	1
Air	1.0003
Water	1.3
Ethyl alcohol	1.4
Ice	1.3
Glass	1.5
Diamond	2.4

Lens

In short, a lens is a transmissive optical device that can disperse or focus beams of light by means of refraction. Lenses are made of clear materials like glass or plastics and are molded or ground into the desired shape. Materials that can disperse or focus other radiation than light are also called lenses. For example microwave lenses and electron lenses. We define 3 main type of lens forms: [12]

1. **Convex** Bulging outwards from the lens.
2. **Concave** Depressed in to the lens.
3. **Planar** Flat side.

With these 3 basic combinations, every type of lens shape can be constructed. Two types of simple lenses are the biconvex and biconcave lens. The biconvex lens (figure: 2.6) is a lens consisting of 2 convex sides both with the same curvature. If beams of light travel through this lens they will converge into a single point: The focal point. The distance from the center of the lens to this focal point is called: The focal length. The focal length depends on the curvature of the convex sides of the lens.

The second simple lens type is the biconcave lens.(figure: 2.7) This lens consists of two con-

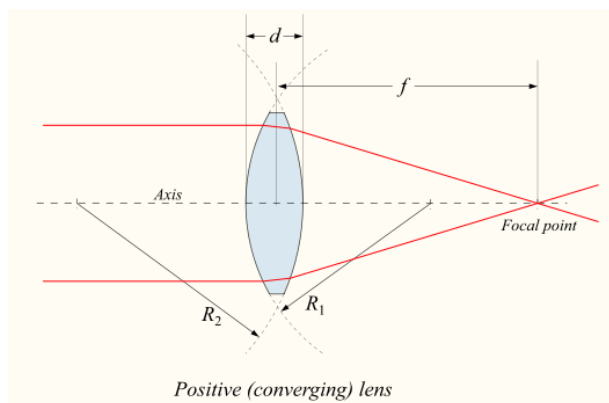


Figure 2.6: A illustration of a biconvex lens

cave sides both having the same spheroidal surface. When light beams enter the lens they get diverged.

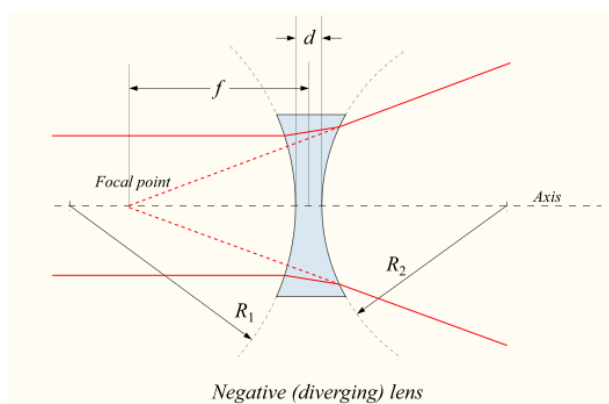


Figure 2.7: A illustration of a biconcave lens

To calculate the focal length of a lens in air. We use the lensmaker's equation.

$$\frac{1}{f} = (n - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n - 1)d}{nR_1R_2} \right] \quad (2.2)$$

In this equation: (figures: 2.6, 2.7) f is the focal length of the lens (in m, n is the refractive index of the lens material, R_1 is the radius of

curvature of the lens surface closest to the light source, R_2 is the radius of the lens surface farthest from the light source and d is the thickness of the lens.

Mirror

Like lenses mirrors are able to focus light, the main difference is the way they do it. Mirrors make use of the law of reflection. [13] When a light-ray is approaching a mirrored surface it is called the incident ray. When a light ray is leaving the mirrored surface after reflecting is called the reflected ray. The point of incidence is the spot where the ray strikes the mirror. From this point a line can be drawn perpendicular to the mirror surface, this line is called the normal line. The normal line always divides the angle between the incident- and reflected ray into two equal angles. The angle between the normal line and the incident ray is called the angle of incidence, the angle between the normal line and the reflected ray is known as the angle of reflection. To summarise the law of reflection states that when a ray of light reflects off a mirrored surface, the angle of incidence is equal to the angle of reflection.

$$\Theta_i = \Theta_R \quad (2.3)$$

In this equation: Θ_i is the angle of incidence and Θ_R is the angle of reflection.

The curvature of a mirror has the opposite effect on the bending of the light than lenses have. This means that concave mirrors converge light to a single point in front of the mirror whereas convex lenses converge light in to a single point behind the lens. (figure: 2.9)

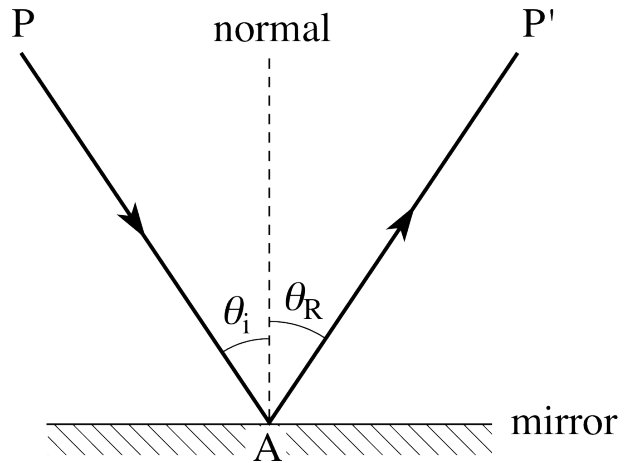


Figure 2.8: An illustration of the mirror law that states angle of incidence Θ_i is equal to Θ_R

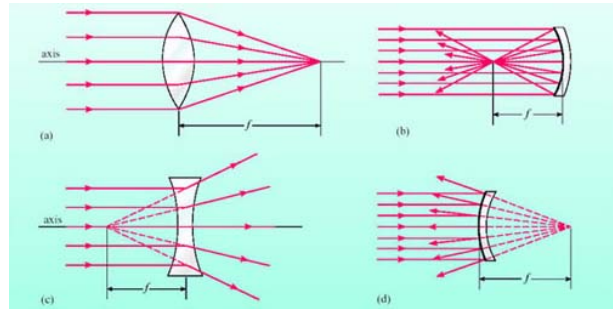


Figure 2.9: An illustration of the behavior of lenses and mirrors when light strikes it.

Spherical vs parabolic

Both spheroidal shaped mirrors and lenses share a problem. Both suffer from spherical aberration.(figure: 2.10) This means that all the light beams that passed through a spherical lens or have been reflected by a spherical mirror, aren't converging to a single point but instead are forming many smaller focal points close together. This will result in a blurred image.

The Solution to spherical aberration, is changing the spheroidal surface to a parabolic surface.

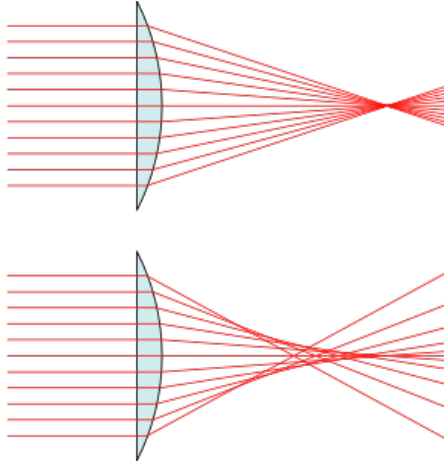


Figure 2.10: The top illustration is a perfect lens without spherical aberration, the bottom lens is a spherical lens with spherical aberration.

As can be seen in figure: 2.11 a parabolic mirror has 1 focal point and no spherical aberration. In figure 2.11 the focal point is referred to as F. This can be mathematically proven as following: We have a light-beam that comes from a star very far away, at these distances we can assume it will be traveling in a perfectly straight line when it hits the mirror. This light-beam hits the mirror in point P. We can extend the light-beam with line PV. Both the light-beam and PV are parallel to the axes of symmetry S. Since the definition of parabola is equal distance from a focal point to the directrix $|FP| = |PV|$. If we draw the bisector PM $\angle FPM = \angle VPM$

$$\triangle FPM = \triangle VPM = \begin{cases} |FP| = |PV| \\ \angle FPM = \angle VPM \\ |FP| = |PV| \end{cases} \quad (2.4)$$

$\angle MPV = \angle$ between the light-beam and the mirror. (vertical angles). Since the normal line is perpendicular to the mirror in point P and a

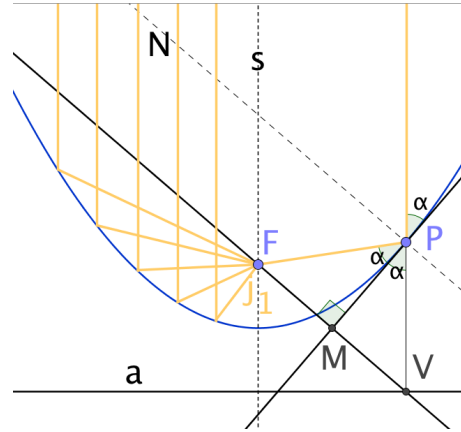


Figure 2.11: An illustration of the workings of a parabolic mirror

straight angle is 180°

Magnification

The size of an image produced by a lens or mirror is proportional to the focal length. The longer the focal length the larger the image. The brightness of an image projected by a telescope depend on how much light is being collected by the telescopes lenses or mirrors. Larger lenses or mirrors means that the telescope can collect more light, doubling the diameter increases the light gathering power by a factor 4. It might seem that magnification is the most important aspect of a telescope, but there are limits to how sharp an image a telescope can produce. Magnifying a blurred image makes it bigger but not clearer, so the priority when building a telescope is having the greatest light gathering power possible. Gathering more light makes it easier to see faint details. Magnification can be displayed in the following equation:

$$M = \frac{f_o}{f_e} \quad (2.5)$$

Where M is the magnification factor, f_o the focal length of the objective lens and f_e the focal length of the eyepiece.

For example a telescope with a focal length of 120 cm and a eye piece with a focal length of 4.0 cm. Will give a magnification of 30 times the original image (30x).

2.2 Refracting telescopes

The earliest as well as many amateur telescopes uses lenses to focus light and make distant objects appear bright an magnified. This type of telescope is called a refracting telescope. [14] There two main designs in use: Galilean refracting telescope and the Keplerian refracting telescope:

The Galilean refracting telescope

The design Galileo Galilei used in 1609 is commonly called the Galilean telescope. This design uses a converging plano-convex lens as its objective lens. This lens consist of one convex side and one flat side. And a plano-concave converging lens as the eyepiece. This lens consist of one concave side and one flat side. (figure: 2.12)

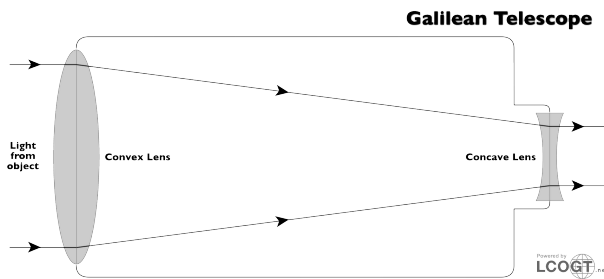


Figure 2.12: Optical diagram of Galilean telescope

Galileo managed to create telescopes with 30x magnification.

Keplerian telescope

The keplerian telescope invented by Johannes Kepler (figure: 2.13, is an improvement on Galileo's original design. This design uses a convex lens as the eye eyepiece lens instead of the plano-concave lens. The advantage of arranging the lenses in this way is that the light entering the eyepiece is converging. This allows for a much wider field of view. A downside to this technique is that the projected image is inverted and much longer focal points are needed to project a sharp image.

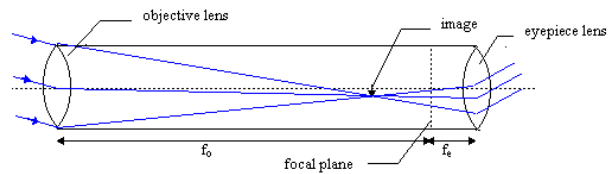


Figure 2.13: Optical diagram of the keplerian telescope

2.2.1 Reflecting telescopes

The reflector telescope is an optical telescope which uses a single or combination of curved mirrors to reflect and combine light to a single point. [15] The idea that curved mirrors behave like lenses dates back to the 11th century. But they were only put to use in telescopes in the year 1663.

There are 3 popular reflecting telescope designs.

Newtonian telescope

The Newtonian telescope is a type of reflecting telescope invented by Sir Isaac Newton. This design uses a primary concave mirror and a secondary flat mirror placed at an angle. (figure: 2.14)

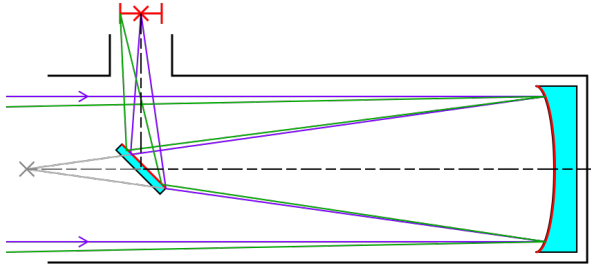


Figure 2.14: Light path in a Newtonian telescope

Gregorian telescope

The Gregorian telescope (figure: 2.15, described by James Gregory in his book *Optica Promota*). This reflecting telescope design features a concave primary mirror and a concave secondary mirror. The special thing to this design is that the secondary mirror reflects the light from the primary back through a hole in the center of the primary mirror.

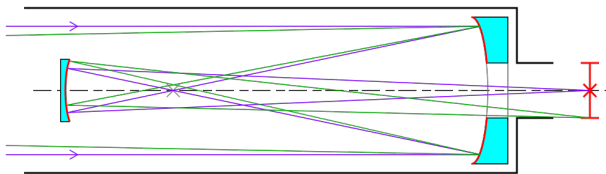


Figure 2.15: Light path in a Gregorian telescope

Cassegrain telescope

The Cassegrain telescope was designed in 1672 by Laurent Cassegrain. This reflecting telescope design features a concave primary mirror and convex secondary mirror. The secondary mirror reflect the light back through a hole in the center of the primary mirror. The convex secondary mirror allows the telescope to have a long focal length while having a short tube length.

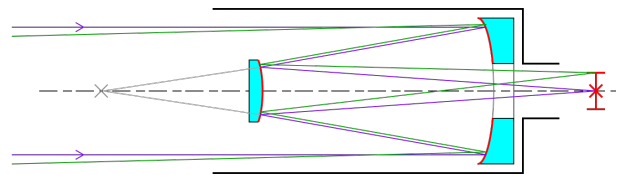


Figure 2.16: Light path in a Cassegrain Telescope

Chapter 3

Building a telescope

3.1 introduction

In early 2016 we got the offer from our physics teacher mr. Ockhorst to construct a telescope as our profile project. Without hesitation we accepted his offer and came in to contact with mr. Grootaerd from univeristy Gent. He guided us step by step through the building process and answered questions where needed.

3.2 Gathering the materials

When building something you need materials. To get our materials we went to university Gent to meet our co-mentor mr. Grootaerd in person. The materials we received:

- Two circular glass plates
- Wood for the telescope frame
- Informative books about astrophysics
- Carbon grain for grinding(in various sizes)
- Polishing equipment

While we were visiting university Gent, we got a quick course on how to grind the glass for a



Figure 3.1: The materials in the back of the car.

spherical mirror and a tour of the facility. Having thanked mr. Grootaerd we transported the materials back to the Netherlands and started the construction of our very own telescope.

3.3 Mirror

3.3.1 Grinding table

To begin grinding of the mirror glass we needed a grinding table. Not having one already at our school we had to improvise by modifying a 100 liter oil drum. Equipping it with a circular wooden top with a clamping mechanism for the glass plates build in to it. One important fac-

tor was that the grinding table had to be placed level.



Figure 3.2: Our grinding table.

3.3.2 Grinding

The grinding of the mirror glass is done by means of moving two circular glass plates over one another with fine carbon abrasive grains in between. Persistent grinding of the glass plates makes one concave-shaped and the other sphere-shaped, It also makes the glass plates less coarse as the size of the carbon grain decreases. The result is a concave-shaped circular glass plate with a predetermined depth, Which was given to us

by our co-mentor. After the right depth is met the carbon grain size is so small it barely has an effect on the depth anymore, The carbon grain will only affect the plate coarseness. The grinding process took 11 hours to complete.



Figure 3.3: Grinding the 2 glass plates.

When the glass plate was completely smooth seen through a microscope we started the polishing process. We polish the glass plate to fill the holes that we might have missed with the microscope. The polishing starts with heating up black tar and pressing it onto the concave-shaped plate to create a sphere-shaped plate that will not grind the glass plate.

When the black tar was the right shape we started moving the concave-shaped glass plate over the sphere-shaped black tar with dissolved polishing powder in between. The polishing process took 5 hours to complete.

3.3.3 Coating

To make the mirror glass into an actual mirror the mirror glass is put into a vacuum-chamber. In the vacuum-chamber a layer of aluminium is sublimated onto the mirror which gives the mirror it's reflective properties, then a protective



Figure 3.4: Standing on top of the black tar.

layer of silicon dioxide is sublimated on the mirror.

3.4 Telescope woodwork

When we received the wooden planks they were bland (figure 3.1). Therefore, we coated the outer side of the planks with a glossy black lacquer. We had to recoat it 3 times to make the coating as even as possible. To absorb light not originating from the object we are looking at in the sky we covered the inside with matte black paint.

3.5 Result

3.5.1 Mirror

Our goal was to get a spherical mirror, however, we accidentally ended up with a parabolic mirror. This didn't actually cause any problems and it actually benefits us because a parabolic mirror reduces the spherical aberration. What resulted

in a better image. On the back of the mirror we mounted three screws to be able to adjust the angle at which the mirror points.

3.5.2 Putting everything together

Once all parts of the telescope were finished, construction could begin. It was just like a big and somewhat complicated puzzle. First we constructed the tube and left one side open. Then we had to put our mirror at a certain height in the telescope tube to make sure the focal point falls with the ocular. We had some difficulties with placing the mirror in the right spot but in the end we got it just right. We put the second mirror in place and the clamp for the ocular was screwed in tight. Everything at the right spot we closed up the tube and attached the stand.

In the end we were very pleased with the end result and the image the telescope created.



Figure 3.5: The finished product.

Chapter 4

The mass of Jupiter

4.1 Introduction

Jupiter is the fifth and largest planet in our solar system. It has over 50 moons and an enormous magnetic field surrounding it. Jupiter's beautiful stripes and wild swirls are caused by its dense atmosphere of hydrogen and helium gas. Its signature red spot is a colossal storm bigger than the earth that has been raging on for hundreds of years.[16]

Mesmerised by Jupiter we wondered if we could determine the mass of this giant. So in this report, we shall try to determine the mass of Jupiter with help of the Galilean satellites and Kepler's Laws of planetary motion.

4.1.1 Galilean satellites

As interesting as the planet are its moons. With 53 confirmed moons and 14 provisional Jupiter forms its own miniature orbiting system. This miniature system is home to some of the largest moons in our solar system.

The largest four are known as the Galilean satellites [17] all of which were discovered by the famous astronomer Galileo Galilei and named in honor of him. The Galilean satellites are Io, Europa, Ganymede, and Callisto. Together they

contain 99.999% of the total mass that is in orbit around Jupiter.

Ganymede

Ganymede is the largest moon in our solar system. It is larger than all the dwarf planets, Mercury, and almost three-quarter the size of Mars. It is the only moon to have its own internally generated magnetic field and could easily be qualified as a planet if it weren't in orbit around Jupiter. The moon has an atmosphere of oxygen although it is far too thin to support any form of life. It completes one full orbit around Jupiter every 172 hours.



Figure 4.1: Ganymede.

Callisto

With a diameter of over 4800 kilometers, Callisto is one of the largest moons in our solar system. Since it



Figure 4.2: Callisto.

has hardly any geological activity on its surface for the last 4 billion years, it is heavily cratered and oldest one our solar system. Callisto is the outermost of the Galilean satellites with an orbital duration of 17 days.

Io

Io named after a lover of Zeus is the innermost of the four Galilean satellites.

With an orbital period of just 42 hours and a diameter of 3642 kilometers, it is the fourth-largest moon in the solar system. Io is the most geologically active object in our solar system. Its surface is scattered with over 400 active volcanoes.

As described by NASA: Looking like a giant pizza covered with melted cheese and splotches of tomato and ripe olives.

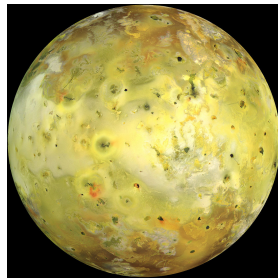


Figure 4.3: Io.

Europa

Europa, the second closest moon to Jupiter is the smallest of all Galilean satellites at 3121.6 kilometers in diameter. In perspective, this is slightly smaller than Earth's moon. Europa's surface is smooth and made out of water ice. It's lacking the many impact craters as can be seen on Callisto.

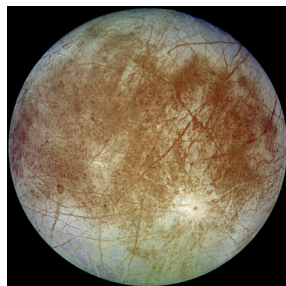


Figure 4.4: Europa.

The reason is a crust only 40 to 90 million years old, youthful in geologic terms. The surface of the moon hides a planet-wide ocean. Europa orbits Jupiter every 3.5 days.

4.2 Theory

In the early 1600s Johannes Kepler the famous astronomer proposed his three laws of planetary motion. [18] He did this after carefully studying the data collected by his mentor Tycho Brahe. These three laws of planetary motion describe the motion of planets in a sun-centered solar system. Even after 4 centuries, the laws are still considered to be an accurate model for the motion of planets and satellites.

The three laws of Kepler can be described as follows:

- **The law of Ellipses:** The path of the planets about the sun is elliptical in shape, with the center of the sun being located at one of the two foci.
- **The Law of Equal Areas:** An imaginary line drawn from the center of the sun to the center of the planets will sweep out equal areas in equal intervals of time.
- **The Law Harmonies:** The squares of the periods of any two planets in relation to the cubes of their average distance from the sun will be a constant.

4.2.1 The law of harmonies in-depth

To calculate the mass of Jupiter we only need Kepler's third law and can be described in an equation as following:

$$\frac{T^2}{r^3} = K \quad (4.1)$$

In this equation, the variables are: T the orbital duration (in s), r : The average radius (in m) and K Kepler's constant (in s^2m^{-3}) As an example, we shall use the orbital period and average distance from the sun for Earth and mars.

$$Earth : \frac{3.156 \cdot 10^{14}}{1.4957^{33}} = 2.977^{-19} s^2 m^{-3} \quad (4.2)$$

Planet	Period 10^7 s	radius 10^{11} m	$K_S 10^{-19} s^2 m^{-3}$
Earth	3.156	1.4957	2.977
Mars	5.93	2.278	2.975
Venus	1.941	1.082	2.975

As can be seen, the ratio is the same for Earth, Mars, Venus and all other planets orbiting around the sun. The period and distance can be described by Kepler's third law. Important for our research is the fact that Kepler's third law also describes the period and distance of a satellite.

4.2.2 Gravitation

The gravity is a natural phenomenon that causes all things with mass to be brought to one another.[19] This includes planets, stars and even a sugar grain. The force the two objects with mass pull on one another is equal for both objects. Even the sugar grain exerts the same amount of force on the earth as the Earth does on the sugar grain. Only the effect it has on the Earth is so infinitesimal it can't be noticed. Whereas the grain clearly moves towards the Earth.

The magnitude of the gravitational force depends on the mass of both objects and the distance between those objects. The force can be described with the following equation:

$$F_g = G \cdot \frac{m \cdot M}{r^2} \quad (4.3)$$

In this equation the variables are: F_g gravitational force (in N), m : the mass of object 1 (in

kg), M : the mass of object 2 (in kg), r : The average radius (in m)

4.2.3 Uniform circular motion

The Galilean satellites are pulled towards Jupiter by its gravity. The only thing preventing them from crashing into Jupiter is the constant speed that they are orbiting around Jupiter: Their Orbital velocity. Only at the right combination of speed and distance is a circular motion a uniform circular motion.

Orbital eccentricity

The orbital eccentricity of an object determines the amount by which its orbit around another body deviates from a perfect circle.

A value of 0 is a perfect circular orbit. A value between 0 and 1 give elliptical orbits. And a values of 1 and grater give escape orbits. As can be seen in (table: 4.1) the orbital eccentricity of several bodies in our solar system are displayed. Since the eccentricity of the Galilean satellites is very low, Europa

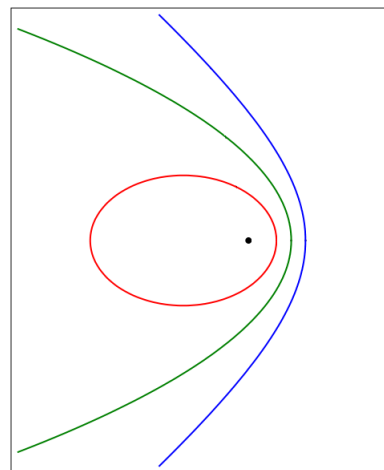


Figure 4.5: Red: Elliptic (eccentricity: 0.7), arabolic (eccentricity = 1), hyperbolic orbit (eccentricity = 1.3)

being the most eccentric. We can assume that they are following a uniform circular motion.

Table 4.1: Orbital Eccentricity [20]

Galilean Sattelite	Eccentricity
Ganymede	0.0013
Calisto	0.0074
Europa	0.0094
Io	0.0041
Earth	0.0167
Mars	0.0935
Jupiter	0.0167

Therefore the speed of the moons is constant. We shall keep this in mind when predicting our measurement uncertainty.

4.2.4 Centripetal force

To successfully perform a uniform circular motion a force is needed that is always perpendicular to the orbital velocity. This force is called centripetal force. The origin of the centripetal force working on the Galilean satellites is the gravity of Jupiter. The strength of the centripetal force depends on the mass, orbital velocity and the orbit radius. The force can be represented by the following equation:

$$F_{mpz} = \frac{m \cdot v^2}{r} \quad (4.4)$$

In this equation the variables are: F_{mpz} the centripetal force (in N), m the mass (in kg), v the speed (in ms^{-1}) and r the average radius (in m)

4.2.5 Orbital velocity

When objects are following a uniform circular motion the time needed for one full orbit is the same every rotation. In this time the object will cover a distance equal to circumference of a circle. So the orbital velocity will be given by the following equation:

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T} \quad (4.5)$$

In this equation the variables are: v the speed (in ms^{-1}), s the distance(in m), T the time (in s) and r The average radius (in m)

4.2.6 Rewriting the equation's

All the necessary theory has been explained we start by rewriting the equations First equate the gravitational force to the centripetal force.

$$F_{mpz} = \frac{m \cdot v^2}{r}$$

$$F_g = G \cdot \frac{m \cdot M}{r^2}$$

$$\frac{m \cdot v^2}{r} = G \cdot \frac{m \cdot M}{r^2}$$

Then we rewrite the equation for orbital velocity:

$$v = \frac{2\pi r}{T} \implies v^2 = \frac{4\pi^2 r^2}{T^2} \quad (4.6)$$

Then via substitution we get the following equation:

$$\frac{m \cdot 4\pi^2 \cdot r^2}{r \cdot T^2} = G \cdot \frac{m \cdot M}{r^2} \quad (4.7)$$

$$\frac{T^2}{r^3} = \frac{m \cdot 4\pi^2}{G \cdot m \cdot M} \quad (4.8)$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{G \cdot M} = K \quad (4.9)$$

As can be seen the only variables left are the radius, orbit duration and the mass of Jupiter.

4.3 Research question and hypothesis

The research question we have chosen: What is the mass of Jupiter? Our hypothesis: We expect the mass of Jupiter that we calculate to be around $1,898 \cdot 10^{27}$ kg based on a source from NASA [21].

4.3.1 Measurement uncertainty

For the measurement uncertainty in this experiment we expect the following sources to resolving power:

- **Quality of the telescope** Since we can't zoom in forever with our telescope. Therefore the distance from the moons to Jupiter has to be estimated. We expect here the measurement uncertainty to be around half the radius of the moons.
- **Quality of our camera** A camera has its limits, the camera we use is the ASI130 mm monochrome high frame rate camera with a pixel density of 1280 x 1080. We expect each pixel to be around 1/10 the size of the image of the moons and for every measurement we expect to be off by one or two pixels.
- **Elliptical orbit** As described in the theory the orbits of the Galilean satellites aren't perfectly spherical. When looking at the largest eccentricity we expect there to be a measurement uncertainty of 7500 km based on the periapsis and apoapsis (the closest and the furthest point in an orbit)

These uncertainty's combined we expect the final mass will be of by around 10 percent the mass of Jupiter.

4.4 Test assembly

1. Telescope, In our case a 150mm F8 Newtonian Dobson telescope
2. Camera, We used a ASI130 mm monochrome high frame rate camera
3. Computer with photo edit software and Excel

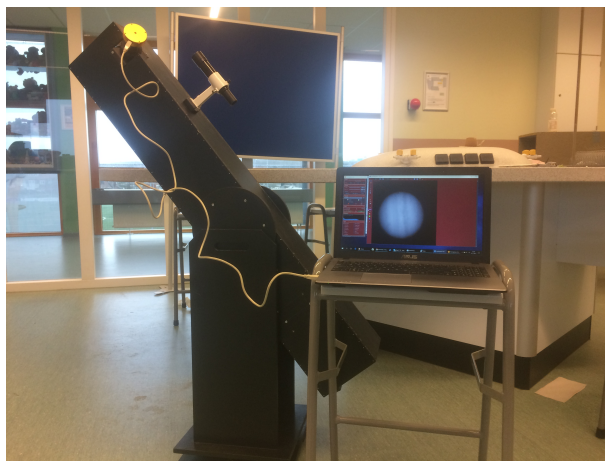


Figure 4.6: Test assembly.

4.5 Process

4.5.1 Known variables

Before performing this experiment one variable has to be known.

1. The diameter of Jupiter in meters. We assume it is: $1.42984 \cdot 10^8$ m [21]

Now to calculate the mass we performed the following procedures. First we took a set of photo's. We did this with the program Fire-capture a planetary capture tool. [22] (NOTE:

It is important that all 4 moons are visible on the same photo.) These photos have to be taken in a sequence of at least 9 days. This is because the longest orbital period of the Galilean Satellite Callisto is 17 days. For every photo we note the date and time it was taken. (4.1.1) An example picture is shown in 4.7



Figure 4.7: Jupiter and Galilean Satellites.

4.5.2 Distance from the moons to Jupiter

Once we had taken all the pictures, we had to calculate the scale of each picture. To get the scale we divided the real diameter of Jupiter as given with the diameter of Jupiter on the picture (in millimetre). Then we measured the distances from the center of the moons to the center of Jupiter (in millimetre) and multiply this value by the scale. Now we have the distance from the moons to Jupiter. Note that if the moons are on the left side of Jupiter the distance is negative, this is very important to get a good result. We have repeated these steps for all the pictures and written the distances in a table. The result should look somewhat like this:

Distance Galilean satellites to Jupiter

Days	Ganymede Distance	Callisto Distance	Io Distance	Europa Distance
Day 0	$3.2 \cdot 10^9 m$	$1.5 \cdot 10^9 m$	$3.0 \cdot 10^9 m$	$3.2 \cdot 10^9 m$
Day 1
Day 2

(NOTE: "Day 0" Is the first day of measuring. When we missed a measurement day we left the distance values empty.)

We collected the data and placed it in a graph. Then for each moon we let a computer program like Excel generate a sinusoid through the measurement points. 4.8

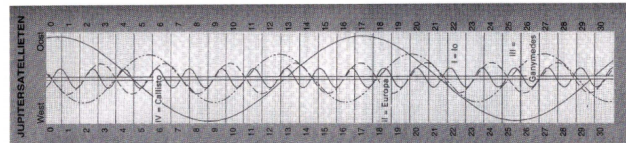


Figure 4.8: Sinusoid graph of the Galilean satellites.

The last step is to extract the Orbital Radius and the Orbital duration from the sinusoids. The orbital radius is equal to the amplitude of the sinusoid and the orbital duration is equal to the wavelength of the sinusoid 4.9

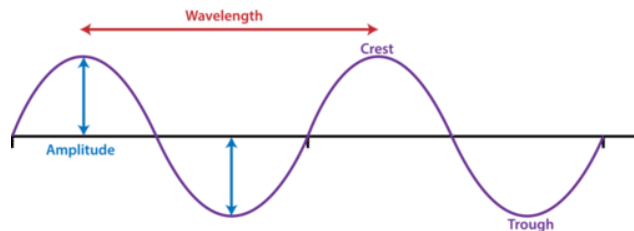


Figure 4.9: Sinusoidal example

4.6 Research results

4.6.1 Quick remark

Before discussing our results we have found, A.Q.R.: Since we are heavily dependent on clear weather it is challenging to observe Jupiter several days in a row. On top of that comes that on the date of writing and conducting research Jupiter is only visible from 3 AM till sunrise. This makes observing in a team hard. We had to fill in some of the blank data days with data we found on the online. [23]

4.6.2 Images

The first step in calculating the mass of Jupiter is taking a streak of images. As can be seen in figure: (4.10). In this image the center sphere is Jupiter and the surrounding dots are the Galilean satellites. We marked them with the first letter of their name.



Figure 4.10: The moons of Jupiter through our 15cm F8 Newtonian telescope. Date: 28-12-16, 7AM

4.6.3 Distances

After measuring the distance and calculating the scale in all the pictures the following table was constructed (table 4.3)

Table 4.2: Galilean satellites distance to Jupiter

Days	Ganymede $1 \cdot 10^9 \text{m}$	Callisto $1 \cdot 10^9 \text{m}$	Io $1 \cdot 10^9 \text{m}$	Europa $1 \cdot 10^9 \text{m}$
0	-0,44	0,12	0,20	-0,41
1	0,45	0,78	0,62	0,38
2	1,03	1,35	-0,37	-0,26
3	0,89	1,73	-0,43	0,17
4	0,08	1,87	0,60	-0,01
5	-0,75	1,75	0,24	-0,15
6	-1,03	1,40	-0,63	0,26
7	-0,54	0,87	0,07	-0,37
8	0,34	0,22	0,66	0,43
9	1,00	-0,48	-0,28	-0,36
10	0,94	-1,10	-0,50	0,29
11	0,35	-1,57	0,52	-0,07
12	-0,64	-1,81	0,35	-0,09
13	-1,03	-1,81	-0,61	0,26
14	-0,67	-1,53	-0,06	-0,34
15	0,21	-1,05	0,68	0,43

When transferring these points into a graph it leads to some very rough result as can be seen

Table 4.3: Galilean satellites

	Amplitude $1 \cdot 10^9 \text{m}$	Period in days
Ganymede	1.1	7,2
Calisto	1.9	16
Europa	0,67	3,6
Io	0,41	1,8

in figure: (4.12) We can't retrieve correct data from the graph in its current state. So the next step that had to be done was refining the data. To do this the data had to be loaded in to Excel and the MS Solver function was used to generate a sine function. This function is the average sinusoidal of all measurement points. Now the amplitude and period can be determined from the generated functions. (figure:4.13)

4.6.4 The mass of Jupiter

For the final step the mass of Jupiter shall be approximated.

In the previous step we determined the aptitude and period for the Galilean satellites. These are as following:

The last thing to do is fill in the equation as given in the theory (4.9) Ganymede is fully written out.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{G \cdot M} = K_s$$

For Ganymede the amplitude is $1,056 \cdot 10^9 \text{m}$ and the period is $6,1 \cdot 10^5 \text{s}$

$$M_j = \frac{r^3 \cdot 4\pi^2}{T^2 \cdot G} \quad (4.10)$$

$$M_j = \frac{1.178 \cdot 10^{27} \cdot 4\pi^2}{3.828 \cdot 10^{11} \cdot 6,67 \cdot 10^{-11}} \quad (4.11)$$

$$M_j = 1,8199 \cdot 10^{27} \text{kg} \quad (4.12)$$

When taking the average of all the moons the mass of Jupiter is:

Galilean satellite	Mass of Jupiter in kg
Ganymede	$1,82 \cdot 10^{27}$
Calisto	$1,78 \cdot 10^{27}$
Europa	$1,86 \cdot 10^{27}$
Io	$1,73 \cdot 10^{27}$
Average mass of Jupiter	$1,80 \cdot 10^{27}$

The final mass we found after studying the orbit of the Galilean Satellites is: $1,80 \cdot 10^{27}$ The mass for Jupiter found in BiNaS is $1.898 \cdot 10^{27}$. The result is a 5% difference with the accepted value for the mass of Jupiter.

After comparing the standard orbital radius of the Galilean Satellites with the orbital radius we found that all the orbits are about 15000 km too small. This corresponds to about 1mm on the photo's .

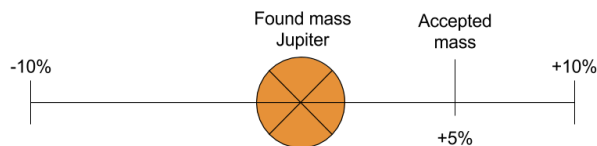


Figure 4.11: Measurement uncertainty of Jupiter

When displayed in an image (figure:) we can see that the accepted mass of Jupiter overlaps with our measurement uncertainty and thus we have succeeded in calculating the mass of Jupiter.

4.7 Conclusion

In this report, we tried to accurately calculate the mass of Jupiter with help of the Galilean satellites and Kepler's Laws of planetary motion. With the measurement data we were able to answer the research question: What is the mass of Jupiter. As result determined the mass to be $1,7969 \cdot 10^{27}$ a result with an 5% difference to the accepted Value for the mass of Jupiter. We can conclude from this that our hypothesis is correct. We expected a measurement uncertainty to be around 10% with only 5% percent deviation from the accepted value we can say that this was overestimated and that our measurements were more precise than we expected them to be.

4.7.1 Discussion

If we were to repeat this experiment again, we would do the following things different:

- Take the measurements in one row, if you are waiting for too long between measurements. Data can be misinterpreted and it becomes harder to construct an accurate sinusoid from the data points.
- Use a stable tripod, the one we used for our telescope had the tendency to move during our measurements

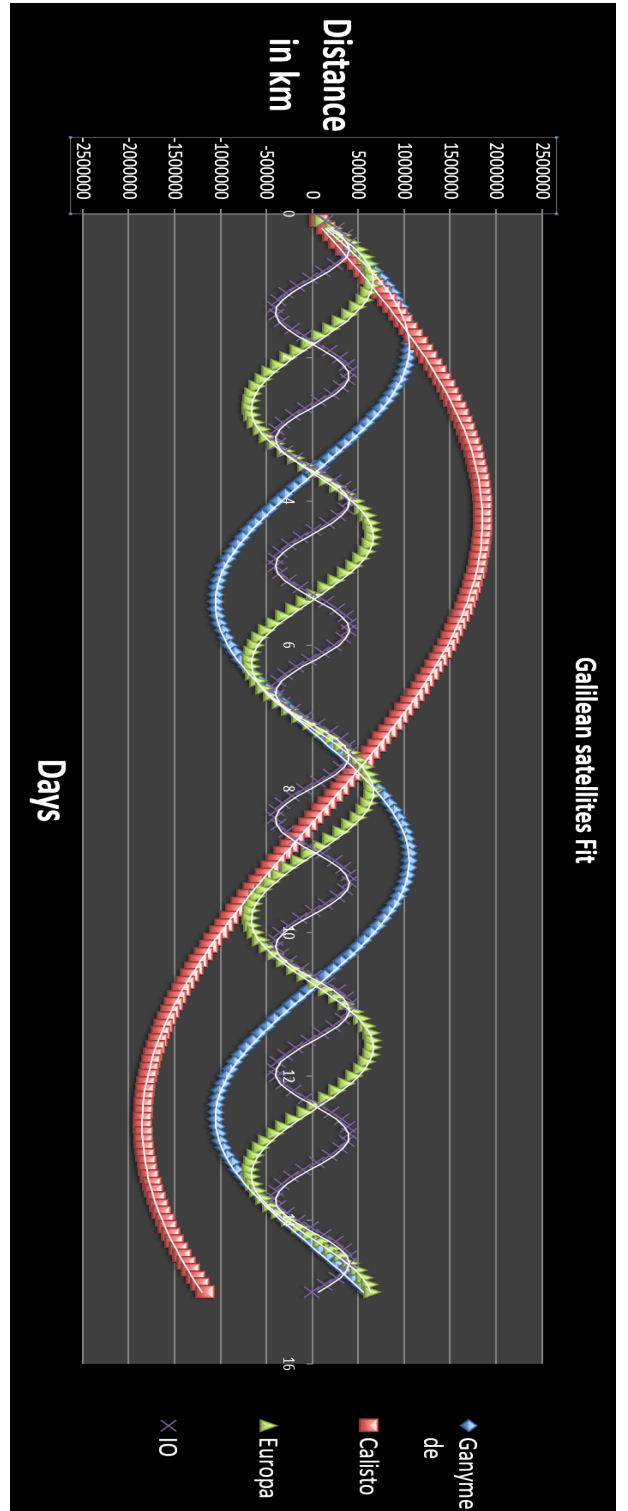
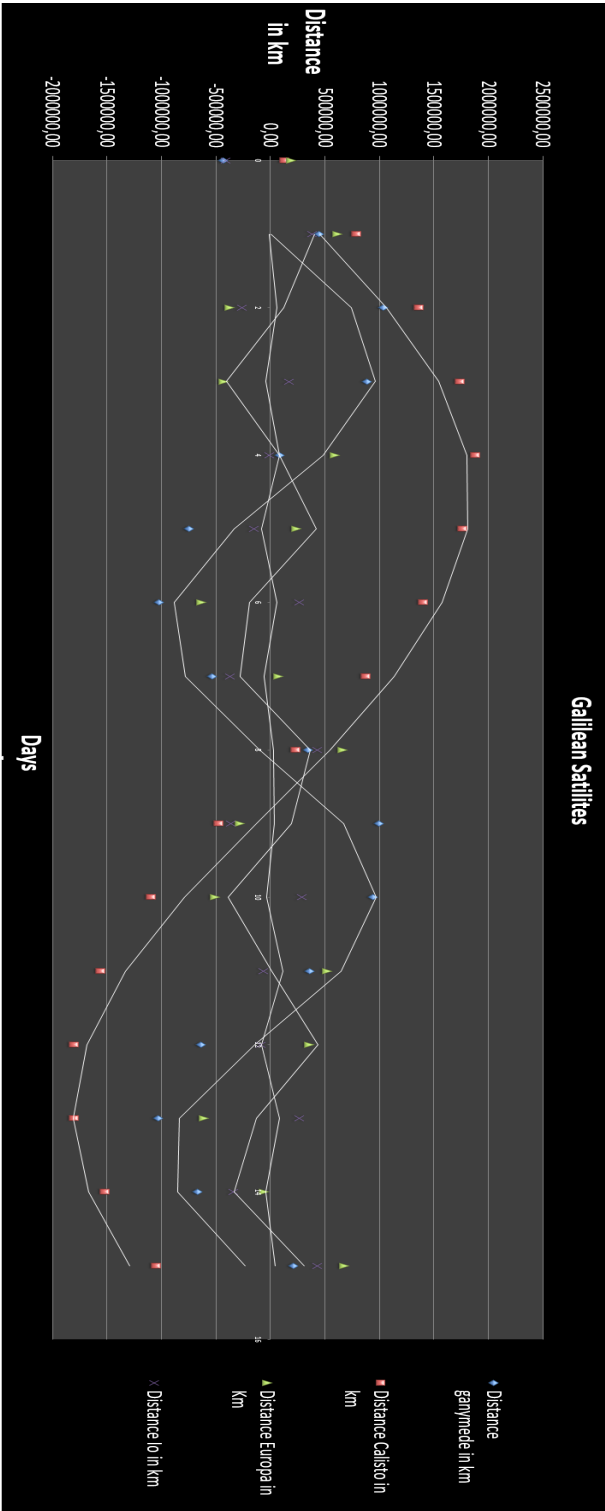


Figure 4.12: The distance of the Galilean satellites from Jupiter (rough data)

Figure 4.13: The distance of the Galilean satellites from Jupiter (fit)

Chapter 5

The depth of craters on the Moon

5.1 Introduction

Earth’s moon is the fifth largest moon of the Solar system and is, as of writing this, the only place beyond Earth where humans have set foot. The moon was likely formed after a Mars-sized body collided with Earth. The moon is the brightest and largest object in our night sky. It makes Earth a more livable planet by moderating our home planet’s wobble on its axis, leading to a relatively stable climate. It also causes tides, creating a rhythm that has guided humans for thousands of years.

5.2 Measuring the depth in theory

The depth of a crater on the moon is easily measured when you are standing on the moon with a giant ruler. However, our school does not have a large enough budget for a manned mission to the moon, so we used our telescope and selenography techniques. Selenography is the study of the surface and physical features of the moon.

5.2.1 The shadow

The moon does, like all other star orbiting bodies, have a light and a dark side. The line that can be drawn between the two sides is called the terminator. A crater that is on the light side casts a shadow on its self. As shown in the diagram (5.1), a crater with a depth D , casts a shadow with a length (as seen from above) L . The angle θ is the angle of the Sun above the horizon. Displayed as an equation:

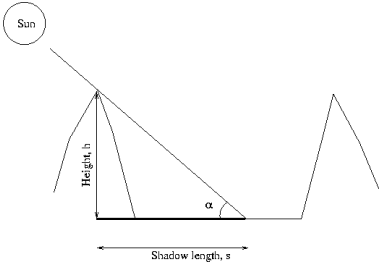


Figure 5.1: moon crater.

$$\tan \theta = \frac{D}{L} \tag{5.1}$$

The length of the shadow we observed is not the true length as seen from above. The shadow is foreshortened, the length we measured is shorter than the true length. To compensate we divided the measured length of the shadow by the cosine of the crater’s selenographic longitude to get the true length of the shadow. Displayed as an equation:

$$L_c = \frac{L_m}{\cos(YC)} \tag{5.2}$$

In this equation the variables are: L_c the corrected length, L_m the measured length of the crater and YC the selenographic longitude.

5.2.2 The angle

To calculate the angle θ we used spherical trigonometry. The figure 5.2 shows the lunar disc.

- Point C is the crater.
- Point E is the point closest to earth, also called sub-earth point.
- Point P is the pole.
- Point S is the point closest to the Sun, also called the sub-solar point.
- Point T is the point on the Terminator in line with point C and S.

The angular distance from S, through the crater C, to the terminator at T, is 90° . (If you were standing on the terminator then the Sun would be on the horizon). We are interested in the angle of Sun above the horizon at C, which we called θ .

The angle CY is the selenographic latitude of the crater, and angle EY is the selenographic longitude of the crater. Similarly, angle SX is the selenographic latitude of the sub-solar point, and angle EX is its selenographic longitude. We used the cosine equation for a spherical triangle to get:

$$\cos(CS) = \cos(PC) \cos(PS) + \sin(PC) \sin(PS) \cos(CPS) \quad (5.3)$$

This equation is rewritten as:

$$\angle CS = \arccos(\cos(PC) \cos(PS) + \sin(PC) \sin(PS) \cos(CPS)) \quad (5.4)$$

- CPS: Is the vertex angle, and is the difference in longitudes between the sub-solar point and the crater.
- PS: Is the angle between point P and S.
- PC: Is the angle between points P and C.
- CS: Is the angle between points C and S

The last step in the calculation is:

$$\theta = 90^\circ - CS \quad (5.5)$$

Using equation 5.1:

$$D = \tan(\theta) \cdot L_c \quad (5.6)$$

In this equation the variables are: D the depth of the crater, θ the angle and L_c the corrected length of the shadow.

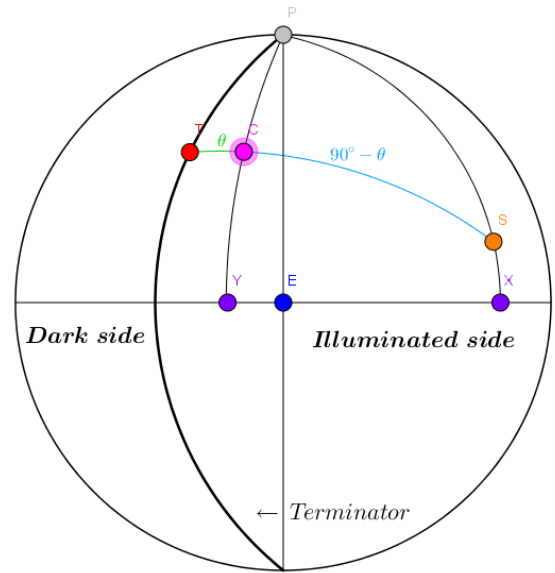


Figure 5.2: Lunar disc

5.2.3 Using external data

A picture of the moon does not include the coordinates of the sub-solar point. Therefore we used an astronomer's program called Ephemeris to determine the coordinates of the sub-solar point at the moment of photographing the moon. We used these coordinates to calculate the angle between the sub-solar point and the crater.

5.3 Research question and Hypothesis

The research question we chose: What is the depth of a crater on the moon? We have measured the depth of a crater called Neander. Our hypothesis: We expect the depth of the Neander crater to be 3.4km. Our Hypothesis is based on the NASA Lunar Nomenclature database. From this Database we also gathered its latitude(31.3°S), longitude(39.9°E) and Colongitude(321° at sunrise).

5.3.1 Measurement uncertainty

For the measurement uncertainty in this experiment we expect that the following components could cause measurement uncertainty:

- **Quality of the telescope** Since we cannot zoom in forever with our telescope, the length of the shadow has to be estimated. Since we know the diameter of the crater we can use this to calculate the scale of the image and so the length of the crater. Because of the image getting blurry the exact border of the crater can be a bit off so will the scale be. Therefore we have a measurement uncertainty of 1 kilometer.

- **Quality of our camera** A camera has its limits, the camera we use is the ASI130 mm monochrome high frame rate camera with a pixel density of 1280 x 1080. We expect each pixel to be around 1/50 the size of the crater and for every measurement we expect to be off by three or four pixels which results in a measuring uncertainty of 1 kilometer.

- **Atmospheric distortions** When light rays travel through the atmosphere the image gets distorted by the earth's atmosphere. It bends light in random directions. When the telescope which causes a blurry image, to remove this blur we need to stack multiple images and this causes the shadow to change in length.

Together these uncertainties result in a margin of error of 2 kilometers.

5.4 Test assembly

1. Telescope, In our case a 150mm F8 Dodson telescope
2. Camera, We used a ASI130 mm monochrome high frame rate camera
3. Computer with photo edit software and Excel

5.5 Process

Variables

Before performing this experiment a few variables have to be known:

- **Sub-solar point:** These two values can be found on: [24] Enter the correct date and time and the data will roll out.

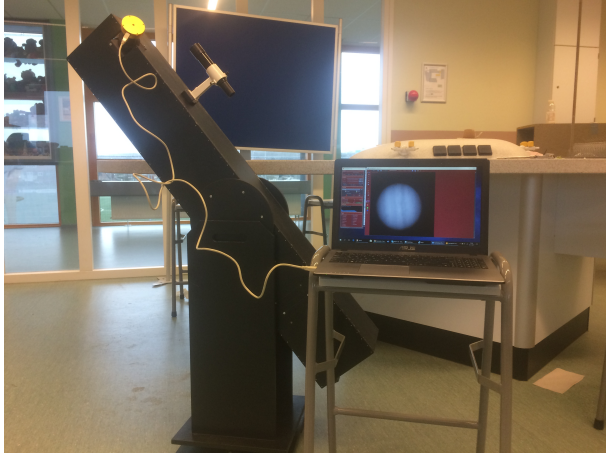


Figure 5.3: Test assembly.

- **Diameter of the crater in question** To find this value loop up your crater in this database: [25] It will state the diameter in the lower left corner of the screen.
- **Selenographic longitude of the crater** This data can be found in the database of NASA: [26]

Taking photo's

When photographing the moon we used picture overlapping software, AutoStackert [27], to combine multiple photo's into one. We did this to rule out any deformations in the photo's caused by the Earth's atmosphere.

Calculate angle θ

To find the angle θ . Make use of spherical trigonometry and equation (5.3)

5.5.1 The depth of the crater

The last step is to correct the length of the shadow in the photo with equation (5.2) and

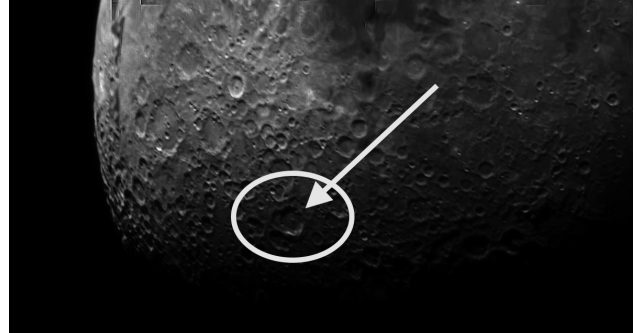


Figure 5.4: The lunar crater Neander on the our photo

then fill in the variables in equation (5.6)

5.6 Research results

We recorded the moon with our telescope. Then we used Autostackert to get a clear picture of the part of the moon we wanted to observe. From the Ephemeris program we gathered the latitude($0.9^{\circ}N$) and longitude($61.1^{\circ}W$) of the sub-solar point. From this we calculated the vertex angle between S and C:

$$\angle CPS = 61.1^{\circ} + 39.9^{\circ} = 100^{\circ} \quad (5.7)$$

From the NASA Lunar Nomenclature database we know the Neander crater is 50 kilometers in diameter, we used pixel counting to determine that the measured shadow length was 6.5 kilometers. The true length of the shadow is the measured length divided by the cosine of the Selenographic longitude of the crater:

$$L_c = \frac{6.5}{\cos(39.9)} = 8.47km \quad (5.8)$$

Filling in the cosine equation for a spherical tri-

angle 5.9

$$\angle CS = \arccos(\cos(PC) \cos(PS) + \sin(PC) \sin(PS) \cos(CPS)) \quad (5.9)$$

$$\angle CS = \arccos(\cos(58.7) \cos(-0.9) + \sin(58.7) \sin(-0.9) \cos(100)) = 58.88^\circ \quad (5.10)$$

From $\angle CS$ we calculate $\angle \theta$:

$$\angle \theta = 90^\circ - \angle CS = 90^\circ - 58.88^\circ = 31.12^\circ \quad (5.11)$$

Using the shadow's length and $\angle \theta$ we calculated the depth of the crater:

$$D = \tan(\theta) \cdot L_c = \tan(31.12^\circ) \cdot 8.47 \text{ km} = 5.1 \text{ km} \quad (5.12)$$

By our calculations the depth of the Neander crater would be 5.1 kilometers deep.

5.6.1 Conclusion

We answered the research question using our telescope, data from NASA and Ephemeris, tangent and spherical trigonometry: What is the depth of a crater on the moon? The depth we hypothesized was 3.4km, with a margin of error of 2km. The depth we calculated is 5.1 kilometers, this is within the measurement uncertainty.

5.6.2 Discussion

We would like to make the calculated depth to be closer to the depth calculated by NASA. We could stabilize the telescope, and if it is possible we could enhance the image quality.

If we had used the exact coordinates of where the shadow ended we could have possibly gotten closer to the depth of the crater.

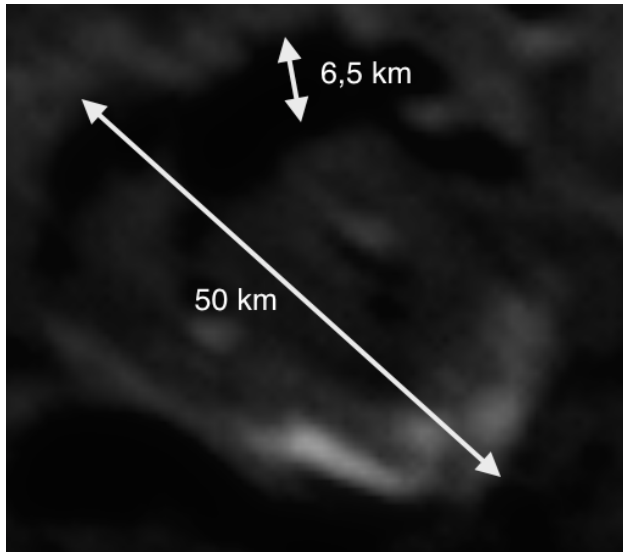


Figure 5.5: The lunar crater Neander we photographed with the measurements we took

Chapter 6

Computer simulation

6.1 Why

We wanted to create a simulation of a telescope to get a better understanding of how light behaves when it comes in contact with a reflective surface so we made our own program in Python.

6.2 Results

To be able to replicate a Newtonian telescope better we added walls and a second mirror. The second mirror reflects the light to the ocular which make the light beams go parallel to each other.

Figure 6.1 is an example of a possible outcome.

6.3 How does it work?

6.3.1 Light

To simulate light we look at it as if it behaves as individual particles and use our self defined object `light_particle`. This object has the following properties: `x_location`, `y_location`, `angle` and `speed`. In the `angle` property right would be 0, up 90, left 180 and down 270 degrees. The `speed` property is the step size for the light. We calculate the course of one light particle completely

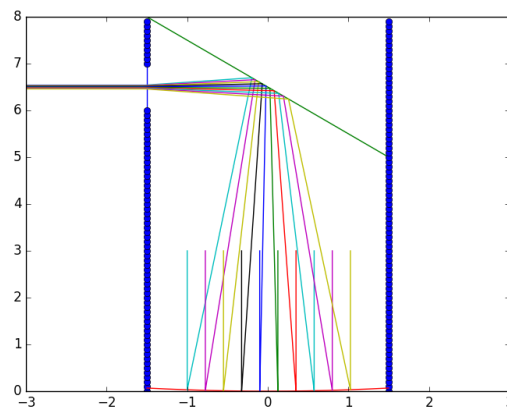


Figure 6.1: A standard outcome.

before we move on to the next light particle, this creates a collection of coordinates of which we make a plot and as result gets us a line in the graph that follows the course of a light beam and we do this for every light particle.

6.3.2 Reflection

When a light beam hits the mirror a new angle needs to be calculated, for this we use the law of reflection. It states that the angle of incidence is equal to the angle of reflection. How-

ever the surface on which the light reflects isn't always straight, for example when the mirror follows a parabolic function. This creates the problem in Python that the new angle can't be directly calculated. So to solve this we create a straight sloped line between two points in the graph of the mirror and change the orientation so that it's horizontal and apply that same change to the angle of the incoming light beam, then we calculate the angle of reflection and revert the change in orientation. This can all be put into one equation: $outwardangle = 2 * mirrorangle - lightangle$.

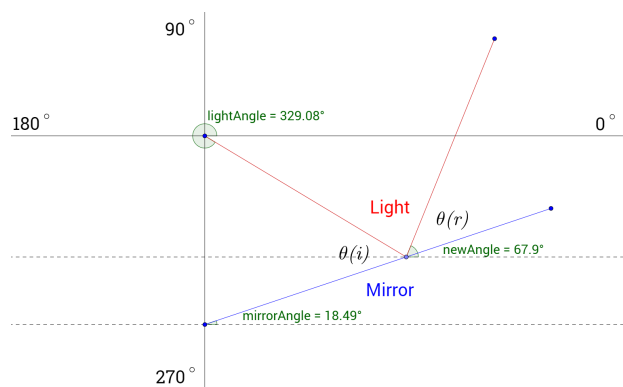


Figure 6.2: Angles in Python work in respect to the x axis.

6.4 Constants and variables

In figure 6.1 you can see a rather standard outcome of the program. There are some variables and constants we defined that changes the outcome significantly. By changing the variable steep you affect the steepness of the bottom mirror by changing the slope of the mirror function, when the bottom mirror is less steep the light diverges more and when it's more steep it converges more. In figure 6.4 you can see an example

of a lower steepness in comparison to figure 6.1.

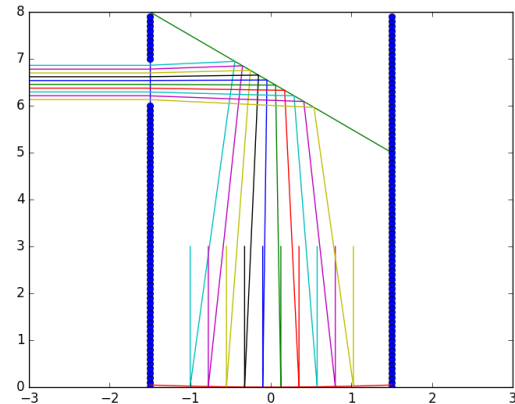


Figure 6.3: The mirror is less steep and as result the light diverges more than the standard outcome.

A real mirror isn't perfectly smooth, to replicate this in our simulation we added the variable coarse. At every y value of the mirror function it either adds or subtracts a random value between 0 and half of the coarse value, this results in a rough mirror. The higher the coarse is the more the angle in which the light reflects deviates.

It can also occur that a lot of light enters the telescope at once, for example when looking into the sun. To simulate this we set the amount of light beams created at a high value. What we can see is that all those light beams that were spread out at first are now all focused in a small area, if you were to look in the telescope now your eye would get damaged.

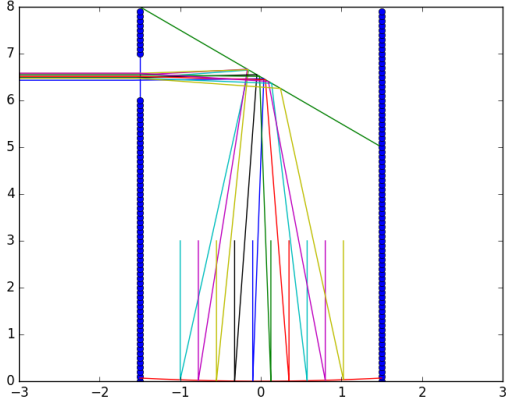


Figure 6.4: The mirror is rougher.

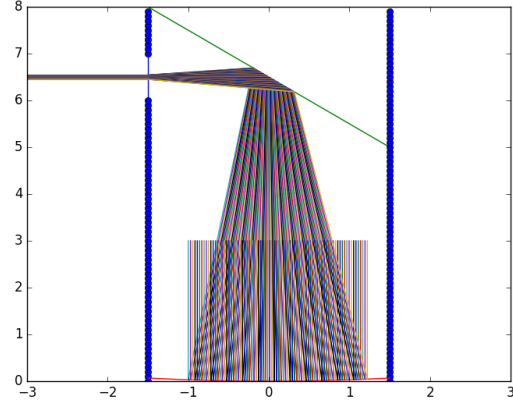


Figure 6.5: The amount of light beams is set to 80.

6.5 Mirror

The mirror in our code is a collection of points that follow a function that we define. We set the boundaries for x and every x value is separated by a predefined step size then we calculate the corresponding y value and store the coordinates in a list. The mirror is plotted by drawing lines between the collection of points, those lines are used as the reflection surface with which we can calculate the reflection angle. We detect if a light particle reflects on the mirror by comparing the y value of the light with the y value of the mirror. When the light is below the mirror the light reflects.

6.6 Point of intersect

6.6.1 The problem

Since the light only reflects when it has already passed the mirror, it reflected when it was below the mirror. But we wanted the light to reflect on the mirror instead of below it so we needed to

create a method of finding the point of intersect. Another benefit of finding the point of intersect is that it allows you to put in high step sizes which increases the speed and efficiency of the program dramatically.

6.6.2 Analytical method

In our code we have constructed an analytical method for calculating the intersect point between the light and mirror. We get two points on the trajectory of the light particle, one point (x_1, Lighty_1) with the x value of the current position of the light particle which we call x_1 and another point (x_2, Lighty_2) with $x_1 +$ the aforementioned step size, both points have the y value of the light corresponding to their respective x value. We also get two points on the mirror, since the mirror use the same x steps as light we can use the x values of the previous two coordinates with which we calculate the corresponding y values this results to the two points: (x_1, Mirroy_1) and (x_2, Mirroy_2) . With those four points we can draw two lines: the light

line and the mirror line. These cross each other at the point of intersect.

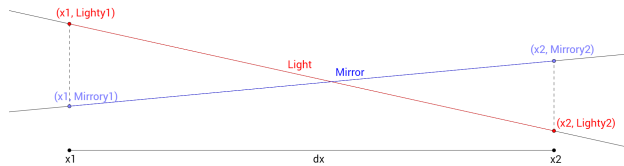


Figure 6.6: A representation of light passing the mirror and what the coordinates are.

There are four variables: a,b,c and d.

$$a = (\text{Lighty2}-\text{Lighty1})/(\text{x2}-\text{x1})$$

$$b = \text{Lighty1} - a * \text{x1}$$

$$c = (\text{Mirroy1}-\text{Mirroy2})/(\text{x2}-\text{x1})$$

$$d = \text{Mirroy1} - c * \text{x1}$$

You get two triangles that are similar to each other and what we're doing is using the ratio between the two triangles to calculate what the x position of the point of intersect is. The x value for the point of intersect would be: $(d-b)/(a-c)$. We put that x value into the function of the mirror which returns the y value of the intersect point then we assign the new coordinates to the light particle.

Discussion

Possible research subjects

Astrophysics is an extremely wide subject, if we would have had more time during this profile project we would've liked to explore a few more topics:

Spectral analysis

You could detect the spectrum of a hydrogen by dispersing the light of a star. In this research we would compare 2 different stars, for example Sirius and Betelgeuse. We would look at the differences in their spectrum, the locations of the hydrogen lines and the composition.

Parallax method

It's possible to calculate the distance to an object in space by using the parallax method. Due to time shortage we were not able to perform this experiment. Parallax is the phenomenon that a near object seems to move relative to its background when observed from different positions. To calculate the distance using the parallax method the earth needs to do exactly half an orbit. Then the angle can be measured on which the star has moved

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Appendix

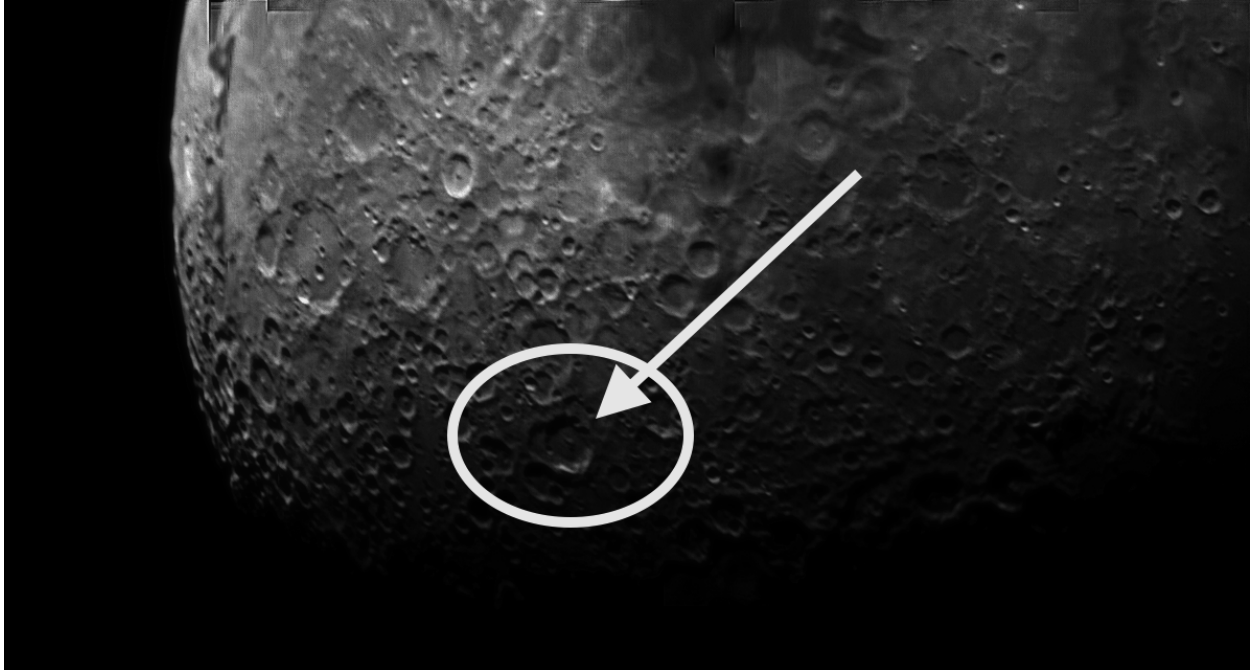


Figure 6.7: The lunar crater Neander on the our photo.

6.7 Code

```

1 import random
import math
3 import matplotlib.pyplot as plt

5
#lightAmount is the amount of light particles you want to spawn in
7 #stepAmount is how long you want to draw the light particle
lightAmount = 10
9 stepAmount = 20001

11
#These lists are necessary for inserting the light
13 lightListX = []
lightListY = []
15

17 #Some global properties(globDegrees = starting angle of light)
globDegrees = -90 #The starting angle of the light
19 globMirrorx = -0.5 #The starting x position of the mirror
globMirrory = 0 #The starting y position of the mirror
21 globMirrordx = 0.001 #The stepsize in which the mirror is drawn
xintersect = 0 #Touching this doesn't change anything but it's important it's here
23

#Coarse adds roughness to the mirror and steep determines how steep the mirror is.
25 coarse = 0 * 10 **-5 #Recommended value = 0.5 * 10 **-5 or 0
steep = 0.03 #Recommended value = 0.03 but it depends on where you want the focal
point

27
#Ocular Properties
29 ocularCounter = 0 #Keep it 0
ocularHeightBottom = 6
31 ocularHeightTop = 7
plt.plot([globMirrorx-1,globMirrorx-1],[ocularHeightBottom,ocularHeightTop])
33 ocularRefraction = 3 #Not implemented
airRefraction = 1.000293 #Not implemented

35
#globMirrorLength determines the length of the mirror
37 globMirrorLength = 3

39
#globCounter is used to see if the reflection has already taken place(0 = not
reflected, 1 = reflected)
41 globCounter = 0
globCounter2 = 0
43 globCounter3 = 0
globCounter4 = 0
45 plt.gca().set_color_cycle(['blue','red','green']) #Is used to give the graph a
specific color #python3.5 only

```

```

47 #Function for defining the light particles
49 def initia_light_particles (aantal_light_particles):
    list_initia_light_particles=[]
51     for p in range(0,aantal_light_particles):
        light_particle={"x_location":-1 + p * (0.225),"y_location": 3 ,"angle": math.
            radians(globDegrees+0.00001),"speed": 0.001} #speed is the stepsize of the light
53         list_initia_light_particles.append(light_particle)
    return list_initia_light_particles
55
57 #Function for calculating the movement of light in the x axis
def movementPeriodX(x, speed, angle):
59     dx = speed * math.cos(angle)
    x = x + dx
61     return x
63
65 #Function for calculating the movement of light in the y axis
def movementPeriodY(y, speed, angle):
67     dy = speed * math.sin(angle)
    y = (y + dy)
69     return y
71
73 #Function for defining the function of the mirror: insert the function for mirror in
    the yVal variable.
def mirrorFunction(xVal):
75     yVal = steep*(xVal)**2 + (random.random()-0.5) * coarse
    return yVal
77
79 def secondMirrorFunction(xVal):
    global xintersect
    boundaryX1 = globMirrorx-1
    boundaryX2 = globMirrorx - 1 + globMirrorLength
81     if xVal > boundaryX1 and xVal < boundaryX2:
        y = -1*xVal + 6.5
83         return y
    return None
85
87 def secondMirrorPlotter():
    lx = globMirrorx-1
89     dlx = globMirrordx
    plotListX = []
91     plotListY = []
    for l in range (int(round(globMirrorLength/globMirrordx))):

```

```

93     if secondMirrorFunction(lx) != None:
94         plotListX.append(lx)
95         plotListY.append(secondMirrorFunction(lx))
96     lx += dx
97     plt.plot(plotListX, plotListY)
secondMirrorPlotter()
99
101 #This function makes the mirror visible as a line
def drawMirrorLine():
103     mirrorx = globMirrorx - 1
104     mirrordx = globMirrordx
105     mirrory = globMirrory
106     mirrorLineX = []
107     mirrorLineY = []
108     for l in range(int(round(globMirrorLength/globMirrordx))):
109         mirrory = mirrorFunction(mirrorx)
110         mirrorLineX.append(mirrorx)
111         mirrorLineY.append(mirrory)
112         mirrorx = mirrorx + mirrordx
113     plt.plot(mirrorLineX, mirrorLineY)
114
115 def drawTelescope():
116     global globMirrordx
117     global globMirrorLength
118     leftBound = globMirrorx - 1
119     rightBound = leftBound + globMirrorLength
120     plotListX = []
121     plotListY = []
122     density = 0.1
123     height = 8
124     for n in range(round(height/density)):
125         if ocularHeightTop <= n * density or n * density <= ocularHeightBottom:
126             plotListX.append(leftBound)
127             plotListY.append(n*density)
128     plt.plot(plotListX, plotListY, "bo")
129     plotListX.clear()
130     plotListY.clear()
131     for m in range(round(height/density)):
132         plotListX.append(rightBound)
133         plotListY.append(m*density)
134     plt.plot(plotListX, plotListY, "bo")
135 drawTelescope()
136
137 #Function that returns the angle of the mirror in a small area
def giveMirrorAngle(xVal):
139     dMirrory = mirrorFunction(xVal + globMirrordx) - mirrorFunction(xVal)
140     nAngle = math.atan(dMirrory/globMirrordx)
141     return nAngle

```

```

143 def giveSecondMirrorAngle(xVal):
144     global globMirrordx
145     if secondMirrorFunction(xVal) != None and secondMirrorFunction(xVal + globMirrordx
146         ) != None:
147         dY = secondMirrorFunction(xVal + globMirrordx) - secondMirrorFunction(xVal)
148         nAngle = math.atan(dY/globMirrordx)
149         return nAngle
150
151 def secondMirrorReflection(ry, mirrorAngle, lightAngle, rx):
152     global globCounter
153     global globCounter2
154     global globCounter3
155     global xintersect
156     global globCounter4
157     if movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle) >=
158         secondMirrorFunction(rx) and globCounter3 == 1 and secondMirrorFunction(rx) !=
159         None:
160         la = -math.radians(180) - mirrorAngle
161         ra = lightAngle + la
162         ba = (math.radians(90)-abs(ra))
163         rAngle = mirrorAngle - math.radians(90)-ba
164         globCounter = 1
165         a = (movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle)-ry)/((
166         movementPeriodX(lightBeamsX, lightBeamsSpeed, lightBeamsAngle)-rx)
167         b = ry - a * rx
168         c = ((secondMirrorFunction(rx)-secondMirrorFunction(movementPeriodX(lightBeamsX,
169             lightBeamsSpeed, lightBeamsAngle))))/(movementPeriodX(lightBeamsX,
170             lightBeamsSpeed, lightBeamsAngle)-rx)
171         d = secondMirrorFunction(rx) - c * rx
172         xintersect = movementPeriodX(lightBeamsX, lightBeamsSpeed, lightBeamsAngle)
173         globCounter2 = 1
174         globCounter3 = 0
175         globCounter4 = 1
176         return rAngle
177     if movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle) <=
178         secondMirrorFunction(rx):
179         globCounter = 0
180         globCounter2 = 0
181         return lightAngle
182
183 #Gives the light a new angle which corresponds with the law of reflection
184 def mirrorReflection(ry, mirrorAngle, lightAngle, rx):
185     global globCounter

```

```

185 global xintersect
186 global globCounter2
187 global globCounter3
188 #If the y of the light particle is lower than the y of the mirror it needs to
189 reflect
190 if movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle) <=
191     mirrorFunction(rx) and globCounter == 0:
192     rAngle = 2 * mirrorAngle - lightAngle
193     globCounter = 1
194     a = (movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle)-ry)/(
195     movementPeriodX(lightBeamsX, lightBeamsSpeed, lightBeamsAngle)-rx)
196     b = ry - a * rx
197     c = (mirrorFunction(rx)-mirrorFunction(movementPeriodX(lightBeamsX,
198     lightBeamsSpeed, lightBeamsAngle)))/(movementPeriodX(lightBeamsX,
199     lightBeamsSpeed, lightBeamsAngle)-rx)
200     d = mirrorFunction(rx) - c * rx
201     xintersect = (d-b)/(a-c)
202     globCounter2 = 0
203     globCounter3 = 1
204     return rAngle
205 if movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle) >=
206     mirrorFunction(rx):
207     globCounter = 0
208     globCounter2 = 0
209     return lightAngle
210
211 light_beams = initia_light_particles(lightAmount)
212 drawMirrorLine()
213
214 def refractiveIndex(angle):
215     tAngle = math.radians(90)-(math.radians(180) - angle)
216     cAngle = math.radians(90)-(tAngle+math.asin(airRefraction * math.sin(tAngle)/
217     ocularRefraction))
218     nAngle = angle + cAngle
219     return math.radians(180)
220
221 for c in range(0,lightAmount):
222     lightBeamsTemp = light_beams[c]
223     lightBeamsX = lightBeamsTemp["x_location"]
224     lightBeamsY = lightBeamsTemp["y_location"]
225     lightBeamsSpeed = lightBeamsTemp["speed"]
226     lightBeamsAngle = lightBeamsTemp["angle"]
227     for q in range(stepAmount):
228         lightBeamsAngle = mirrorReflection(lightBeamsY, giveMirrorAngle(lightBeamsX),
229         lightBeamsAngle, lightBeamsX)
230         if secondMirrorFunction(lightBeamsX) != None:
231             lightBeamsAngle = secondMirrorReflection(lightBeamsY, giveSecondMirrorAngle(
232             lightBeamsX), lightBeamsAngle, lightBeamsX)

```

```

225     if globCounter2 == 0 and (lightBeamsX > globMirrorx-1 or ocularHeightTop >
lightBeamsY > ocularHeightBottom):
227         lightBeamsX = movementPeriodX(lightBeamsX, lightBeamsSpeed, lightBeamsAngle)
lightBeamsY = movementPeriodY(lightBeamsY, lightBeamsSpeed, lightBeamsAngle)
229         if lightBeamsX <= globMirrorx-1 and ocularHeightTop > lightBeamsY >
ocularHeightBottom and ocularCounter == 0:
            lightBeamsAngle = refractiveIndex(lightBeamsAngle)
            ocularCounter = 1
        else:
231             lightBeamsX = xintersect
lightBeamsY = mirrorFunction(xintersect)
233             if globCounter4 == 1 and secondMirrorFunction(xintersect) is not None:
lightBeamsX = xintersect
235             lightBeamsY = secondMirrorFunction(xintersect)
            lightListX.append(lightBeamsX)
237             lightListY.append(lightBeamsY)
globCounter = 0
239             ocularCounter = 0
plt.plot(lightListX, lightListY)
241             del lightListX[:]
del lightListY[:]
243
245 plt.xlim(-3,3)
plt.ylim(0,8)
plt.show()

```

Github link to code: <https://github.com/Bloodworks13/AstrophysicsMirror/blob/master/Telescope>