MATHEMATICS LEVEL 2 TEST

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding circle on the answer sheet.

Notes: (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator.

(2) For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. The range of f is assumed to be the set of all real numbers $f(x)$, where x is in the domain of f.

(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding Question 1.

USE THIS SPACE FOR SCRATCHWORK.

1. If $f(x) = x^{-\frac{3}{2}}$, what is the value of $f(9)$?

   (A) 0.0007
   (B) 0.0370
   (C) 0.2311
   (D) 4.3267
   (E) 6.8587

2. If $0 < x < 1$, which of the following has the greatest value?

   (A) x (B) x^2 (C) x^3 (D) $\frac{1}{x}$ (E) $\frac{1}{x^2}$

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3. The midpoint of the segment in space with endpoints $(1,2,3)$ and $(9,-2,9)$ is

   (A) $(5,0,6)$
   (B) $(5,2,6)$
   (C) $(5,-2,6)$
   (D) $(10,0,12)$
   (E) $(4,-2,3)$

4. If $j = x^2$, then $x^4 + 4x^2 - 5 = 0$ is equivalent to which of the following?

   (A) $j^2 + 4j - 5 = 0$
   (B) $j^2 + j - 5 = 0$
   (C) $j^4 + 4j^2 - 5 = 0$
   (D) $j^8 + 4j^4 - 5 = 0$
   (E) $j^8 + 4j - 5 = 0$

5. A quiz game contains two types of questions, some worth 3 points and others worth 8 points. Which of the following must be true about the score for a player in this game?

   I. The total score must be a multiple of 3 or a multiple of 8.
   II. A score of 13 can be achieved.
   III. Both even scores and odd scores can be achieved.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) II and III

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5. Which of the following is an equation of the line that contains the origin and is parallel to the line $2x - y = 1$?

(A) $2x - y = 0$
(B) $2x + y = 0$
(C) $2x + y = 1$
(D) $x + 2y = 0$
(E) $x - 2y = 0$

7. Which of the following numbers belongs to the arithmetic sequence 1, 8, 15, 22, . . . ?

(A) 77
(B) 80
(C) 92
(D) 105
(E) 150

8. If $0 \leq x \leq \frac{\pi}{2}$ and $\sin x = 0.56$, then $\sin 4x =$

(A) 0.531
(B) 0.692
(C) 0.784
(D) 2.125
(E) 2.240

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9. A right rectangular prism has length m, width w, and height h. If $h = 3m$ and $w = 4h$, what is the volume of the prism in terms of m ?

(A) $9m^3$
(B) $12m^3$
(C) $16m^3$
(D) $36m^3$
(E) $48m^3$

10. For positive integers n, the nth term of a given sequence is $\frac{1}{n^2+2}$. For what value of n is the value of the nth term closest to 0.01 ?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

11. What is the range of the function f defined by $f(x) = 3x^2 - 2$?

(A) $f(x) \geq -2$
(B) $f(x) \geq 0$
(C) $f(x) \geq 2$
(D) $f(x) \geq 3$
(E) All real numbers

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MATHEMATICS LEVEL 2 TEST—*Continued*

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What negative integer is 42 less than its square?

(A) -8
(B) -7
(C) -6
(D) -5
(E) -3

The distance, measured on level ground, from the base of a flagpole to a point P is 10 feet. If the angle of elevation from P to the top of the pole is 66°, which of the following is an expression for the height, in feet, of the flagpole?

(A) $10\sin 66°$

(B) $10\cos 66°$

(C) $10\tan 66°$

(D) $\dfrac{\sin 66°}{10}$

(E) $\dfrac{10}{\tan 66°}$

USE THIS SPACE FOR SCRATCHW

14. If $\log_2 ab = 5$ and $\log_3 b = 4$, then $a =$

(A) $\frac{4}{81}$

(B) $\frac{32}{81}$

(C) $\frac{4}{5}$

(D) $\frac{5}{4}$

(E) $\frac{32}{5}$

15. If $f(x) = r^x$ where $r > 0$, and if $f(6) = 125$, what is the value of $f(7)$?

(A) 2.2
(B) 15.7
(C) 127.2
(D) 145.8
(E) 279.5

16. A straight line ℓ contains the points $(0,0)$ and $(3,4)$. What is the cosine of the acute angle that ℓ makes with the x-axis?

(A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

MATHEMATICS LEVEL 2 TEST—*Continued*

USE THIS SPACE FOR SCRATCHWORK.

17. A red ball, a blue ball, and a green ball are placed in an empty hat. If a blindfolded person takes them out one at a time, what is the probability that the person draws them in the order red, blue, green?

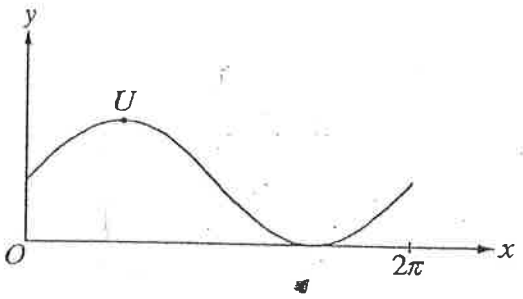
(A) $\frac{1}{27}$

(B) $\frac{1}{12}$

(C) $\frac{1}{9}$

(D) $\frac{1}{6}$

(E) $\frac{1}{3}$



8. The figure above shows one cycle of the graph of $y = 1 + \sin x$ for $0 \le x \le 2\pi$. If the maximum value of y occurs at U, what are the coordinates of U?

(A) $\left(\frac{\pi}{4}, 1\right)$

(B) $\left(\frac{\pi}{4}, 2\right)$

(C) $\left(\frac{\pi}{2}, 1\right)$

(D) $\left(\frac{\pi}{2}, 2\right)$

(E) $\left(\frac{\pi}{2} + 1, 1\right)$

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19. If $f(x) = \dfrac{x+2}{x-2}$, what value does $f(x)$ approach as x approaches 3.5 ?

(A) −1.00
(B) −0.43
(C) 0.27
(D) 2.07
(E) 3.67

20. In the coordinate plane, the circle whose equation is $(x-3)^2 + (y+1)^2 = 25$ is to be translated 5 units to the right and 2 units down. What is the equation of the translated circle?

(A) $(x-3)^2 + (y+1)^2 = 74$
(B) $(x+2)^2 + (y-1)^2 = 25$
(C) $(x+2)^2 + (y-1)^2 = 74$
(D) $(x-8)^2 + (y+3)^2 = 25$
(E) $(x-8)^2 + (y+3)^2 = 74$

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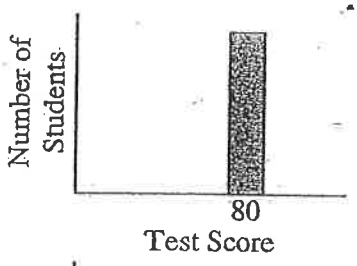
21. The graphs below show the scores on a test for five classes. In which graph does the set of scores have a standard deviation of 0 ?

(A)



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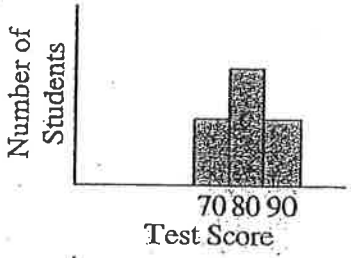

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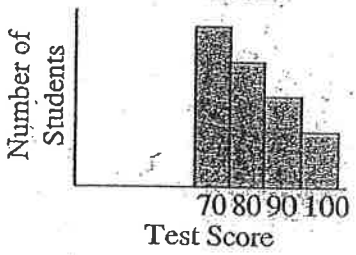
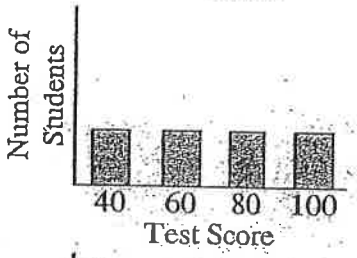


(D)



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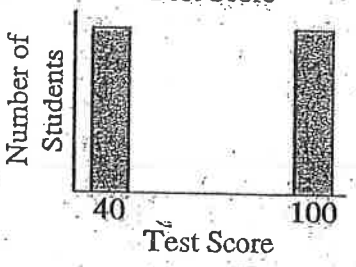

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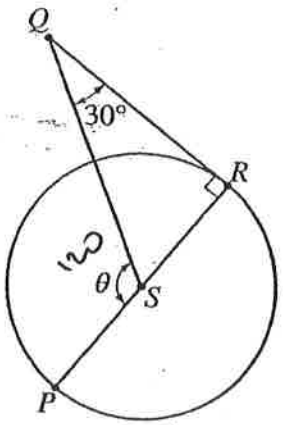
22. If $0° \leq \theta \leq 90°$, what are all values of θ for which $\sin\theta \leq \cos\theta$?

(A) $0° \leq \theta \leq 45°$
(B) $0° \leq \theta \leq 60°$
(C) $0° \leq \theta \leq 90°$
(D) $30° \leq \theta \leq 45°$
(E) $45° \leq \theta \leq 90°$

23. For which of the following values of a and c does the equation $ax^2 + 3x + c = 0$ have no real roots?

(A) $a = 1, c = 2$
(B) $a = 1, c = -2$
(C) $a = -1, c = 2$
(D) $a = -3, c = 1$
(E) $a = -3, c = -1$



24. In the figure above, $\overline{PR}$ is a diameter of the circle. What is $\tan\theta$?

(A) −1.73
(B) −0.57
(C) 0.57
(D) 1.00
(E) 1.73

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25. If today a country's population is 5.7 million people and is growing by roughly 1.5 percent per year, in how many years will the population first exceed 6 million?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

26. If $x < 0$, then $-|x| =$

(A) $-x$
(B) $-\frac{1}{x}$
(C) 0
(D) $\frac{1}{x}$
(E) x

USE THIS SPACE FOR SCRATCHW

Questions 27-28 refer to the following table and information.

SURVEY RESULTS

h (height in inches)	w (shoulder width in inches)	$\frac{w}{h}$ (ratio of shoulder width to height)
68	15.5	0.228
74	18	0.243
66	17.5	0.265
70	19	0.271
75	19	0.253
64.5	16	0.248
70	17	0.243

In order to find realistic proportions for a sketch of a standing human figure, an art student took a survey, measuring the height and the shoulder width of seven people. The results of her survey are shown in the table above, along with the ratio of shoulder width to height, $\frac{w}{h}$, of each person she measured.

27. What is the median of the $\frac{w}{h}$ ratios?

(A) 0.243 (B) 0.248 (C) 0.250 (D) 0.251 (E) 0.271

28. If the art student uses the arithmetic mean of the $\frac{w}{h}$ ratios to determine realistic proportions, what would the art student determine to be the height of a person whose shoulder width is 48 centimeters?

(A) 11.6 cm
(B) 12.0 cm
(C) 191.9 cm
(D) 193.5 cm
(E) 197.5 cm

USE THIS SPACE FOR SCRATCHWORK.

29. How many distinct four-digit numbers contain the digits 1, 2, 3, and 4 without repetition, if the digit 2 is always immediately followed by the digit 3 ?

(A) Four
(B) Six
(C) Twelve
(D) Eighteen
(E) Twenty-four

30. The graph of which of the following functions has exactly three distinct x-intercepts?

(A) $f(x) = (x^2 - 4)(x + 2)$

(B) $f(x) = (x^2 + 4)(x + 2)$

(C) $f(x) = (x^2 + 4)(x + 2)^2$

(D) $f(x) = (x^2 + 4x + 4)(x + 2)$

(E) $f(x) = (x^2 + 4x - 4)(x + 2)$

31. One method for finding a given number that is in an ordered list of numbers requires a computer to repeatedly split the list in half until the number is found. For a list of n numbers, the maximum number of splits is the least integer greater than or equal to $\frac{\log n}{\log 2}$. What is the maximum number of splits needed to find a given number in a list of 300,000 numbers?

(A) 3 (B) 6 (C) 15 (D) 18 (E) 19

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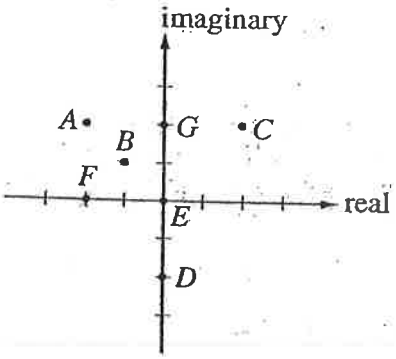
32. If $n^{\square}$ is defined by the equation $n^{\square} = n(n+1)(n+2)$, which of the following is equivalent to $\frac{8^{\square}}{4^{\square}}$?

(A) $1^{\square}$
(B) $2^{\square}$
(C) $3^{\square}$
(D) $4^{\square}$
(E) $6^{\square}$

33. An insurance company has found that the proportion of claims that are resolved within t days is given by $p(t) = \left(\frac{t}{t+10}\right)^2$. How many days does it take to resolve 75 percent of the claims?

(A) 1 (B) 13 (C) 30 (D) 65 (E) 75



34. In the figure above, points F and G represent two complex numbers. Which point represents the sum of these two numbers?

(A) A (B) B (C) C (D) D (E) E

USE THIS SPACE FOR SCRATCHWORK.

35. In the xy-plane, in how many points does the graph of the curve $xy^5 - 2x = 3y + x^2$ intersect the x-axis?

(A) None
(B) One
(C) Two
(D) Three
(E) Five

36. In which of the following is y a function of x?

I.

x	y
-2	-10
0	-1
1	0
-1	1
0	2

II.

x	y
-10	-2
-1	0
0	1
1	-1
2	0

III.

x	y
-2	$\frac{1}{2}$
-1	$\frac{1}{2}$
0	$\frac{1}{2}$
1	$\frac{1}{2}$
2	$\frac{1}{2}$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

37. Point C is 6 inches from plane M. To the nearest integer, what is the area, in square inches, of the portion of M that contains all points that are not more than 12 inches from C?

(A) 33
(B) 113
(C) 339
(D) 452
(E) 565

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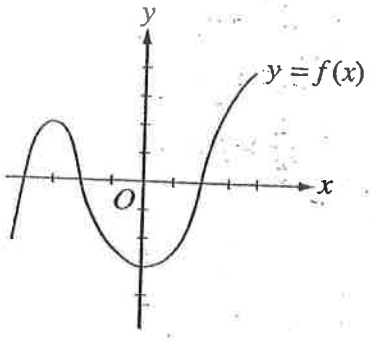
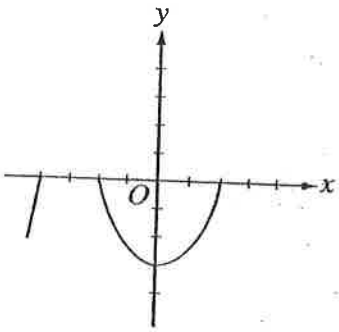


38. A portion of the graph of $y = f(x)$ is shown above. Which of the following is the graph of $y = -|f(x)|$?

(A)



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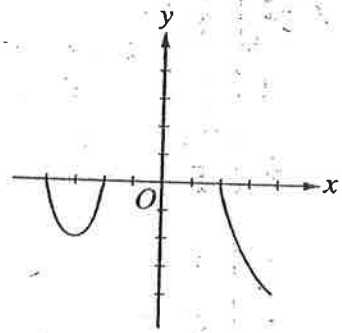

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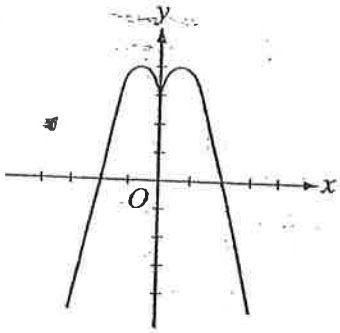

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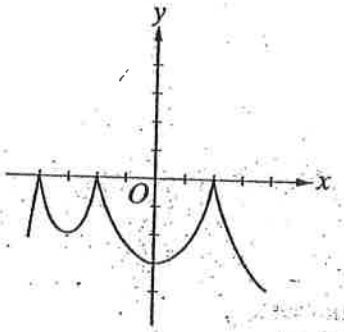

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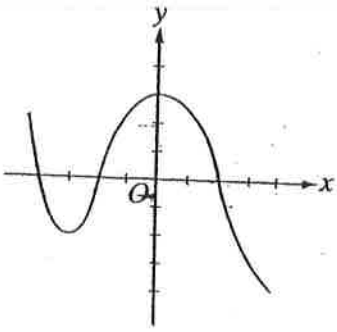

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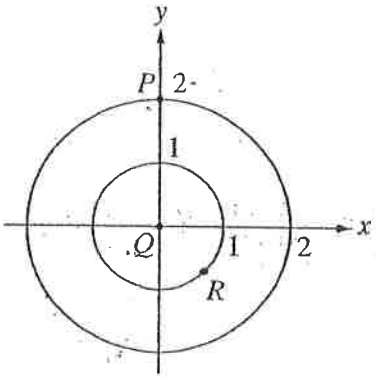
39. If $f(x) = 3\sin(\pi x) + \cos(2\pi x)$, what is the period of the function f?

(A) 2 (B) 3 (C) 4 (D) 2π (E) 3π



40. In the figure above, which of the following could NOT be polar coordinates of points P, Q, or R?

(A) $(0,0)$

(B) $\left(1,-\frac{\pi}{4}\right)$

(C) $\left(1,\frac{7\pi}{4}\right)$

(D) $\left(2,\frac{\pi}{2}\right)$

(E) $\left(2,\frac{3\pi}{2}\right)$

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41. If the function f is symmetric with respect to the line $x = 3$ and if $f(-1) = -1$, what is the value of $f(7)$?

(A) -7 (B) -1 (C) 1 (D) 3 (E) 7

42. If $x = 3\cos\theta$ and $y = 3\sin\theta$, where θ is a real number, which of the following must be true?

(A) $x + y = 3$

(B) $x^2 + y^2 = 3$

(C) $x^2 + y^2 = 9$

(D) $x^2 - y^2 = 9$

(E) $-x^2 + y^2 = 9$

43. If $f(x) = x^4 - 4x + 1$ and $g(x) = f(x + 2)$, which of the following statements are true?

I. f and g have one common zero.
II. The graphs of f and g have one intersection point.
III. f and g have the same range.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

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USE THIS SPACE FOR SCRATCHWORK.

44 If $\sin(2x) = a$ and $0 < x < \frac{\pi}{2}$, then $2\sin x$ is equal to which of the following?

(A) a

(B) $\frac{a}{2}$

(C) $2a$

(D) $a\cos x$

(E) $a\sec x$

45. What are the coordinates of the point that is the reflection of the point $(3,2)$ through the line $y = -x$?

(A) $(-3,-2)$
(B) $(-3,2)$
(C) $(-2,-3)$
(D) $(-2,3)$
(E) $(3,-2)$

46. In $\triangle ABC$, $AB = 5$, $BC = 6$, and $AC = 7$. To the nearest degree, what is the measure of $\angle ABC$?

(A) 12°
(B) 48°
(C) 66°
(D) 78°
(E) 102°

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47. If p, s, and t are distinct positive primes, what is the least perfect cube that is divisible by $p^2s^5t^7$?

(A) $p^3s^3t^3$

(B) $p^3s^6t^9$

(C) $p^9s^9t^9$

(D) $p^6s^{15}t^{21}$

(E) $p^8s^{125}t^{343}$

48. On a 20-kilometer trip, Chris averaged 40 kilometers per hour for the first 10 kilometers and 80 kilometers per hour for the last 10 kilometers. What was her average speed, in kilometers per hour, for the 20-kilometer trip?

(A) 65

(B) $63\frac{1}{3}$

(C) 60

(D) $53\frac{1}{3}$

(E) 50

USE THIS SPACE FOR SCRATCHWORK.

49. If $4x^2 + 3y^2 = 16$ is equivalent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which of the following could be values of a and b?

(A) $a = 2, b = \frac{\sqrt{3}}{4}$

(B) $a = 2, b = \frac{4}{\sqrt{3}}$

(C) $a = 2, b = \frac{16}{3}$

(D) $a = 4, b = \frac{3}{16}$

(E) $a = 4, b = \frac{16}{3}$

50. A function f has the property that $f\left(\frac{x}{2}\right) = \sqrt{\frac{1 + f(x)}{2}}$ for $0 \le x \le 1$. If $f(a) = 0$, where $0 \le a \le 1$, what is the value of $f\left(\frac{a}{4}\right)$?

(A) 0
(B) 0.35
(C) 0.71
(D) 0.92
(E) 0.98

STOP

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST ONLY.
DO NOT TURN TO ANY OTHER TEST IN THIS BOOK.