

Równanie	Rozwiązanie
$f_{n+1} = a_n f_n$	$f_n = f_1 \prod_{k=1}^{n-1} a_k$
$f_{n+1} = f_n + b_n$	$f_n = f_1 + \sum_{k=1}^{n-1} b_k$
$f_{n+1} = a f_n + b_n$	$f_n = a^{n-1} f_1 + \sum_{k=1}^{n-1} b_k a^{n-k-1} =$ $= a^{n-1} (f_1 - B_0) + B_n,$
	gdzie $B_n = \Delta_a^{-1} b_n$
$f_{n+1} = a_n f_n + b$	$f_n = f_1 \prod_{k=1}^{n-1} a_k + b \sum_{j=1}^{n-2} \prod_{i=j+1}^{n-1} a_i + b$
$f_{n+1} = a_n f_n + b_n$	$f_n = f_1 \prod_{k=1}^{n-1} a_k + \sum_{j=1}^{n-2} b_j \prod_{i=j+1}^{n-1} a_i + b_{n-1}$

Sumowanie operatorem Δ_z^{-1}

$$\Delta_z^{-1}(\alpha f_n + \beta g_n) = \alpha \Delta_z^{-1} f_n + \beta \Delta_z^{-1} g_n, \quad \text{gdzie } \alpha, \beta \text{ to stałe}$$

$$\Delta_z^{-1}(v_n \Delta_z u_n) = u_n v_n - \Delta_z^{-1}(u_{n+1} \Delta_1 v_n)$$

$$\Delta_z^{-1} 0 = C z^n$$

$$\Delta_z^{-1} A = \begin{cases} \frac{A}{1-z} + C z^n & A \text{ stała, } z \neq 1 \\ An + C & A \text{ stała, } z = 1 \end{cases}$$

$$\Delta_z^{-1} a^n = \begin{cases} \frac{a^n}{a-z} + C z^n & a \text{ stała, } z \neq a \\ na^{n-1} + Ca^n & a \text{ stała, } z = a \end{cases}$$

Sumowanie operatorem Δ_z^{-1}

$$\Delta_z^{-1} n^{(k)} = \begin{cases} \sum_{j=0}^k (-1)^j \frac{k^{(j)} n^{(k-j)}}{(1-z)^{j+1}} + Cz^n & k \text{ naturalne, } z \neq 1 \\ \frac{n^{(k+1)}}{k+1} + C & k \text{ naturalne, } z = 1 \end{cases}$$

$$\Delta_z^{-1} (a^n n^{(k)}) = \begin{cases} \sum_{j=0}^k (-1)^j \frac{k^{(j)} n^{(k-j)} a^{n+j}}{(a-z)^{j+1}} + Cz^n & k \text{ naturalne, } z \neq a \\ \frac{a^{n-1} n^{(k+1)}}{k+1} + Ca^n & k \text{ naturalne, } z = a \end{cases}$$