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The Complexity Continuum in the Radex and Hierarchical Models of Intelligence*

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The hierarchical and radex models of ability organization are shown to be parallel. Both models suggest a complexity continuum along which cognitive performance tasks can be arrayed. In our revised radex model, the complexity continuum from the center to the periphery is shown to correspond to the general-to-specific dimension in factor analyses, or to test correlations with the general factor; complexity is redefined as apparent processing complexity. Examination of the theoretical and empirical bases for this continuum indicates its central importance for theories of intelligence.

The purpose of the present paper is to show the theoretical and the empirical parallels between the radex and hierarchical models of ability organization. It is suggested that the complexity continuum is fundamental in both models and that correlational theorizing about the complexity continuum can serve as an important guide for process analyses of intelligence.

THE HIERARCHICAL MODEL AND COMPLEXITY

Some sort of hierarchical model of ability organization has been endorsed by many theorists (e.g., Cattell, 1971; Cronbach, 1970; Horn, 1976; Snow, 1978; Vernon, 1950, 1965). Hierarchical models have seemed to provide the most promising and parsimonious way to think about mental ability factors. At the top of the hierarchy is general ability or *G*—a broad general factor that accounts for performance in a great variety of intellectual tasks. Tests that correlate highly

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with this factor are "complex" tests, requiring abstract problem-solving analysis and rule inferring. Examples of such tests are Raven Progressive Matrices (Raven, 1962), Letter Series (Thurstone, 1938), Necessary Arithmetic Operations (French, Ekstrom, & Price, 1963), and Verbal Analogies (Terman, 1950). At the first level beneath G are two or three major group factors, which different theorists have defined in somewhat different ways; these ways appear superficially similar, though the details of interpretation are not necessarily interchangeable. Vernon (1950) distinguished verbal-educational and practical-mechanical-spatial abilities as major group factors; whereas Cattell (1971) and Horn (1976) identified crystallized ability (G_c) with verbal comprehension and knowledge emphasis, fluid ability (G_f) with nonverbal and analytic reasoning emphasis, and visualization ability (G_v) with figural and spatial emphasis. Sometimes a quantitative ability factor has also been distinguished and placed just below this level, with cross connections to more than one major group factor (Vernon, 1965; Snow, 1978). Each of these factors can, in turn, be subdivided into narrower, more sharply focused factors at the next lower level. At the lowest level, are test-specific factors that usually show low correlations with one another and with other measures in the universe of ability and achievement tests.

Somewhat different hierarchical factor structures can be produced simply by varying test construction or sampling, so the particular character of factors obtained in any given study may not be particularly important (Humphreys, 1981). What is important, however, is that tests that tend to load factors higher in such hierarchical models as Vernon's and Cattell's tend to be more complex tests; they tend to correlate more highly with G. It is this correlation that should be focused upon primarily, rather than the particular factors with which a test may be associated at lower levels. A secondary interest, to be sure, is the horizontal spectrum that spreads tests out among group factors at various hierarchical levels. Thus, tests can be located along two main dimensions of such hierarchies: the vertical or complexity dimension ranging from general to specific; and the horizontal dimension, which at intermediate levels of the hierarchy often expresses content distinctions, such as that between spatial, mathematical, and verbal content.

In this view, the complexity dimension can be defined as an ordering of ability tests along a continuum according to their correlation with G. More complex tests (e.g., Raven Matrices, Verbal Analogies) show high correlation with G, while apparently simpler tests (e.g., Memory Span, Perceptual Speed, Visual Memory) show only low or small correlation with G. The actual correlation between a test and G approximates the apparent complexity of its required cognitive operations. Objective manipulation of task complexity has confirmed this hypothesis with both figural and verbal content tasks (Lohman, 1979a; Marshalek, 1981). And, in stating this hypothesis, Jensen (1970, p. 147) added an important clarification:

> This complexity continuum is not the same as difficulty *per se*. Repeating a series of 10 digits, for example, is a difficult task if judged by the percentage

of the population who can do it, but in a more fundamental psychological sense it is a less complex task than answering the question: "In what way are a banana and an orange alike?" An echo chamber or a tape recorder can repeat a 10-digit series, but a relatively complex computer would be required to "infer" the correct superordinate category, given two subordinates, as in the banana-orange question.

Many theorists have simplified complexity into dichotomous terms. Spearman (1923) wrote of intellective vs. nointellective cognitive tasks. Jensen (1970) chose to contrast Level II (intellective) vs. Level I (associative memory) abilities, even though hypothesizing a continuous range of complexity in mental tasks displayed by their correlations with *G*. And Guttman, even though originating a continuous view of test complexity (Guttman, 1954), moved in later writings to distinguish rule inferring vs. rule applying processes, or analytic ability vs. achievement (Guttman, 1965, 1970). However, complexity seems best thought of as a continuum and, in the hierarchical view of ability organization, it seems best measured by correlation with *G*.

At lower levels of the hierarchy, other facets also determine the clustering among tests. But while tests can be classified on many other facets and dimensions, the complexity continuum and the content facet appear to be most important in determining the correlations among tests, particularly in the upper and intermediate levels of the hierarchy.

THE RADEX MODEL AND COMPLEXITY

Guttman's radex model (e.g., Guttman 1954, 1956, 1970) is the best known representation of ability test intercorrelations that is not based exclusively on factor analytic methods. It was derived using nonmetric scaling techniques and a faceted definition of the universe of ability tests. In a multidimensional scaling, tests are represented as points in two (or three) dimensional space; the higher the correlation between the tests the greater their relative proximity in the space. Guttman observed tha ability tests *within* a content area (spatial, verbal, or numerical) tend to form a *simplex*, a straight line array in scaling representation, on which tests are ordered from simple to complex. Tests of comparable complexity sampled from separate content areas tend to form a circumplex, a circular array in scaling representation. The covariation of the simplex and circumplex structures was hypothesized to form a *radex*—a disc in two dimensional space or a sphere in three dimensions, divided into verbal, numerical, and figural content areas.

Figure 1 shows schematic representations of one form of the radex and also of the hierarchical model. The hierarchy is designed to combine the Vernon and Cattell constructions, but using Cattell's terminology, in a manner following Snow (1978). Such a construction need not imply the adoption of the details of any particular theory, however. The radex model shown is our revision of

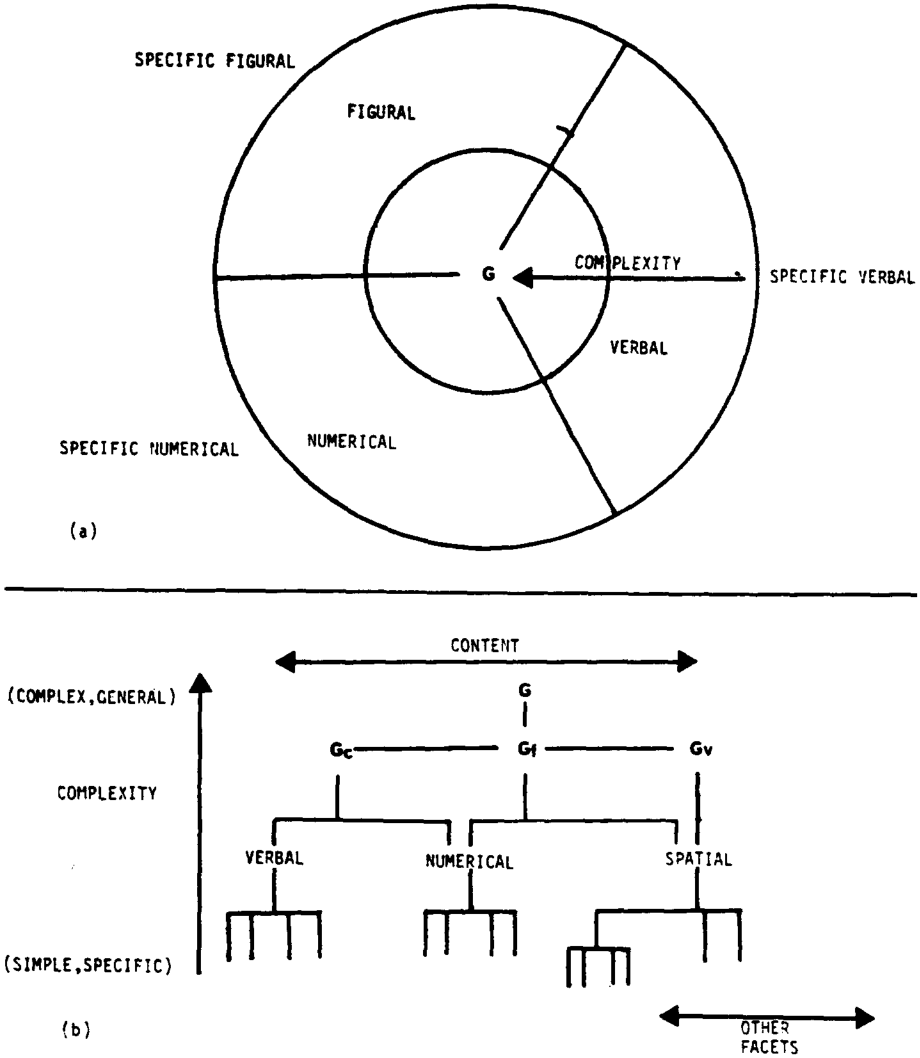

FIG. 1. Schematic representation of the radex (a) and hierarchical factor (b) models.

Guttman's view, designed to emphasize our hypothesis that complexity increases from periphery to central regions in the radex, just as it does from specific to general levels of the hierarchical model.

Guttman's (1954) earlier formulations of the radex model viewed complexity as a continuum in which tests differed in the number of performance components they required; more complex tests required the same components as simpler tests, plus additional components. Complexity, therefore, was a direct function of the number of components involved in solving test items. Guttman (1954) predicted that complex tests would fall on the periphery of the radex, since complexity could result from many different combinations of various components; he assumed, in other words, that complex tests would have fewer components in common with each other than would simple tests, and thus would show lower intercorrelations than would simpler tests. After applying his scaling techniques to portions of the data from Thurstone (1938) and Thurstone and Thurstone (1941), however, Guttman (1965, p. 34) remarked:

> When first discussing the radex for intelligence tests some dozen years ago, I hypothesized it would express a radial expansion of complexity; simplicity would be in the center and expand outwardly into complexity. Complex tests in different areas would tend to be less correlated with each other because they go off in different directions of complexity. The present data show that quite the opposite may be true, although complexity is not the same thing as analyticity (the distinction between being complex or simple is not the same as the distinction between analysis and achievement) . . . It is the analytic tests that tend to correlate more with each other, as is shown by their greater mutual proximities.

The failure of his initial prediction apparently led Guttman to abandon his earlier ideas about a complexity continuum in favor of the analytic vs. achievement, or rule-inferring vs. rule-applying vs. achievement distinction (see Guttman, 1970; Schlesinger & Guttman, 1969). The failure, however, actually supports another view of the complexity continuum, one related both to hypotheses about test complexity based on test correlation with *G* and to hypotheses about executive assembly and control processes growing out of cognitive process analyses of individual differences (Marshalek, 1977; Snow, 1978, 1980a, 1981). We shall return to these interpretations later in this paper.

THE RADEX AND HIERARCHICAL MODELS IN PARALLEL

It can be shown that the radex model is a simple and objective scaling representation of the hierarchical organization of abilities. Specifically, if data conform to a radex structure, one should expect, on theoretical grounds, that complex tests with high loadings on *G* will scale in the center of the radex, simple tests reflecting mainly specific factors will scale in the periphery, and tests of intermediate complexity with high loadings on major or minor group factors will fall in the intermediate region.

In the context of the geometry of a circle, it can be proven mathematically that the farther a point X is from the center of a circle, the greater its average distance from all other points within the circle. Thus, the center has the shortest average distance from all the points in the circle.

In a radex, the shorter the average distance of a test from all other tests in the universe, the higher its correlation with all these tests, and hence the higher its loading on the general factor, and the smaller its group factor and specific variance. Tests in the center of the radex are thus highly loaded on the general factor, while tests in the periphery contain mostly specific variance. Tests in the middle range show highest correlations with the group factors. In other words, the vertical complexity dimension in the hierarchical factor model parallels the dimension that radiates out from the center in the radex representation, even though the mathematics of the factor and radex models are quite different. The horizontal dimension in the hierarchical factor model parallels the radex content facet. Just as in the lower levels of the hierarchical model, so also in the radex periphery; other facets gain importance in predicting the correlations among tests. Thus, tests on the periphery of the radex can share the same content area and the same level of complexity and still be quite distant.

While Guttman's content facet is clearly represented in other nonhierarchical ability models, such as the Structure of Intellect (Guilford, 1967), the complexity dimension is not. However, there is some evidence that the complexity continuum parallels an ordering of product × operation cells within content areas in Guilford's model (Cronbach & Snow, 1977; Snow, 1978), and there are other initial steps toward a hierarchical construction for the Guilford data (see Guilford, 1982; Haynes, 1970; Lohman, 1979b).

Both the radex and the hierarchical representations are parsimonious, and neither face the difficulty of specifying how many abilities or factors exist—the answer simply depends on the level of generality or breadth of the factors (see Coan, 1964; Humphreys, 1981). However, the radex representation has a clear edge in other respects. While there are procedures for extracting a hierarchical factor structure from a correlation matrix (Schmid & Leiman, 1957; Wherry, 1959), a combination of substantive reasoning and objective technique is required to perform the many transformations on the original correlation matrix involved in reaching a clear solution. Also, the factor model does not specify how to define and sample from the universe of ability tests. In contrast, the radex theory approach suggests a faceted definition of the universe of observations, and its implied sampling scheme. The complexity and content facets are derive from Guttman's work with Thurstone's data, but it has long been clear that additional facets could be defined and systematically investigated within this model; Humphreys (1962; 1981) suggested how such an approach might proceed, and showed how the hierarchical model quickly breaks down as facets are added. Although a radex structure might be obtained by other methods, multidimensional scaling best accommodates the study of additional facets. There is direct transformation of the original correlation matrix, and simplicity, objectivity, and uniqueness to the solution technique; one stays close to the original data.

A DEMONSTRATION STUDY

A study reported by Snow, Lohman, Marshalek, Yalow, and Webb (1977) provides data that demonstrate some of these points. That study was originally conducted to provide a battery of reference tests of cognitive and affective aptitude dimensions, so that ensuing laboratory experiments could be empirically related to external aptitude constructs. The reference battery was administered to 241 high school sophomores and juniors, and 123 college freshman and sophomores in a series of group and individual sessions. The cognitive ability tests included are listed in Table 1. For detail on method and results, and the original correlation matrices, see Snow et al. (1977).

These data allowed us to compare empirically the hierarchical model based on factor analysis, and the radex model based on multidimensional scaling, in a battery of tests differing in hypothesized complexity. They also allowed us to compare the Wechsler Adult Intelligence Scale (WAIS), an individually administered test, with a variety of widely used group tests, and to derive three alternative operational definitions of G: WAIS full scale score; the centroid of the correlation matrix; and a composite of complex tests from the three content areas (Raven Matrices, Necessary Arithmetic Operations, and Thurstone Letter Series). Finally, because of the dual sample of subjects, it was possible to compare solutions in a restricted college population with a more representative high school population.

The Hierarchical Factor Results

Table 1 provides the unrotated principal factor solution and the varimax-rotation based on the correlation matrix of the high school sample. Two broad central factors that can be labelled G_f and G_c were identified, as were three special ability factors: Perceptual Speed (PS); Memory Span (MS); and Closure Speed (CS). Two singleton factors were also produced. The G_c and G_f labels seem reasonable, even though these factors vary somewhat from the Cattell-Horn definitions. Note that the G_f factor combines tests thought to require analytic reasoning with tests of spatial visualization. Such tests often correlate highly and are not distinguished in orthogonal factor analyses. Also, as Lohman (1979a) has demonstrated, complex spatial tests are often susceptible to nonspatial analytic solution strategies. The combined label, G_{fv}, is thus sometimes used (see Snow, 1980a).

The factor solutions obtained in the high school and college samples were similar. As expected, the correlations in the college sample tended to be lower due to some restriction in range. Hence, as shown at the bottom of Table 1, the seven factors, particularly the broad ones, account for a smaller percent of the variance in the college solution than in the high school solution.

While a parallel factor solution of this sort identifies the main clusters of tests, the overall structure is not easily grasped. Certainly the tests that are highly loaded on G_f and G_c are highly correlated with each other. There is a strong G factor but it remains concealed in the rotated factor solution. Thus, a hierarchical model should fit the data best and provide a parsimonious representation.

To obtain a hierarchical factor solution, we first altered the correlation matrix in two ways. The two achievement composites, constructed originally to afford a

TABLE 1
Factor Solution for Ability Tests in the High School Sample (N = 241)

Test	Unrotated							Varimax Rotated							
	I	II	III	IV	V	VI	VII	G_c	G_f	MS	PS	CS	X_1	X_2	h_2
1 Street Gestalt	27	18	−14	02	03	34	−07					43			24
2 Harshman Gestalt	30	29	−17	05	15	34	−18					52			37
3 Film Memory III	22	10	30	−04	13	−18	12				35				21
4 Auditory Letter Span	39	−27	32	36	09	03	−20			64					51
5 Visual Number Span	55	−25	28	34	−03	01	03			64					56
6 Identical Pictures	53	36	35	−17	18	00	−12		38		59				60
7 Finding A's	45	04	39	−25	−02	11	08				60				44
8 Number Comparison	38	04	53	−21	−06	−02	−10				67				49
9 Paper Folding	17	33	−17	−21	−07	−19	01		77						71
10 Paper Form Board	58	41	−08	09	04	−10	−14		66						55
11 Surface Development	70	39	−07	20	−06	−08	−02		75						69
12 Word Transformations	73	−04	16	−06	12	05	−04	43			40				58
13 Camouflage Words	50	−03	22	01	−01	41	04				34	42			47
14 Hidden Figures	62	29	04	−03	−03	00	13		54		32				49
15 Word Beginnings and Endings	58	−19	11	05	16	16	07	39		38					44
16 Uses	31	05	−02	−13	51	−20	−11						59		43
17 Necessary Arithmetic Operations	76	−06	01	−04	−16	−10	−19	54	41		30				65
18 Thurstone Letter Series	72	05	−05	−01	−13	−03	21	47	48						58
19 Terman Concept Mastery	80	−23	−23	−13	01	−01	−11	77							77

20 WAIS Information	64	–28	–28	–16	07	00	04	73							60
21 WAIS Comprehension	56	–21	–22	–10	20	–12	14	60							49
22 WAIS Arithmetic	66	–22	–15	–01	–21	–15	–17	63	30						60
23 WAIS Similarities	55	–17	–31	–09	07	–07	02	60							44
24 WAIS Digit Span Forward	36	–28	16	50	16	–07	15			69					54
25 WAIS Digit Span Backward	45	–26	17	22	01	08	–01			49					35
26 WAIS Vocabulary	72	–39	–22	–15	12	–01	04	81							75
27 WAIS Digit Symbol	45	07	56	–23	–08	–19	06				74				62
28 WAIS Picture Completion	40	19	–07	–08	–02	19	25		31					30	31
29 WAIS Block Design	66	35	–18	13	05	–06	10		70						62
30 WAIS Picture Arrangement	36	08	–17	–07	–02	04	–01								17
31 WAIS Object Assembly	54	38	–14	07	13	04	–04		58						48
32 Raven	78	19	00	05	–20	–01	13	39	62						70
33 Achievement Verbal[a]	79	–40	–04	–15	–02	11	01	78		30					81
34 Achievement Quantitative[a]	80	–12	–06	–03	–36	00	–13	63	41						80
% Variance accounted for	34	7	7	4	4	4	3	20	16	8	10	4	3	2	63
% Variance accounted for in the Stanford sample	24	8	6	6	5	4	4	13	14	7	6	11	4	4	57
% Variance accounted for in the combined sample	37	8	6	4	4	3	3	18	17	10	9	4	4	2	64

Decimals omitted. Rotated loadings smaller than ± .30 not shown.

[a]Achievement Verbal is the sum of the standard scores of four verbal subtests (Reaading Vocabulary, Reading Comprehension, Language Expression, and Language Spelling) and Achievement Quantitative is the sum of standard scores of three arithmetic subtests (Arithmetic Computation, Arithmetic Concepts, and Arithmetic Application) of the California Test of Basic Skills.

comparison with the SAT-V and SAT-Q scores of the college sample, were replaced by the original eight achievement subtest scores; it was hoped that the subtests might show somewhat different relations to various other tests and factors in the hierarchical solution. Also, WAIS forward and backward digit span were combined into a composite to reduce the number of narrow memory span variables in the matrix. This revised matrix was then submitted to a clustering program (Johnson, 1967) to sort the variables into hypothesized factor groups, and the multiple group method of factor analysis was applied to the clusters (see Thurstone, 1947). Correlations between the ten first-order factors were then computed and cluster analyses performed on them. These indicated that the ten first-order factors could be clustered into three second-order factors. The Perceptual Speed factor and the Picture Arrangement singleton did not cluster neatly with any of these second-order clusters, so they were left to stand alone. Thus, five second-order factors were extracted. The matrix of their intercorrelations defined one factor at the third level. The three factor structure matrices were then transformed into one orthogonal, hierarchical matrix by procedures developed by Wherry (1959). The method starts with the factor loadings at the third level and partials these out of the loadings at the second level. Loadings at the second level are then partialled out of the loadings at the first level. The multiple group factor and Wherry technique, though less sophisticated than the principle factor and Schmid-Leiman technique, was considered preferable for this demonstration because it eliminates the rotational dilemma and simplifies communality estimation. Other investigators using other methods might well obtain somewhat different factor structures, but the particular boundaries between factors are arbitrary, as noted earlier, particularly at intermediate hierarchical levels.

The final result is shown in Table 2, with tests rearranged from Table 1 to depict the hierarchical structure. Note that there is a general factor (G) and three broad group factors which we have labeled G_v, G_f, and G_c. Again, the G_f factor differs somewhat from the Cattell-Horn definition because it includes the arithmetic achievement tests. These do seem to involve a degree of analytic reasoning, however. There are also nine first-order factors, identified in the table as Closure Speed (CS), Spatial Relations (SR), Perceptual Speed (PS), Reasoning with Symbols (RS), Numerical Skill (NS), Verbal Comprehension (VC), Reading Comprehension (RC), Language Skill (LS), and Memory Span (MS). The general factor accounts for slightly more variance than the other 12 factors combined. The broad group factors come next, and together account for about 15% of the total variance. The remaining nine first-order factors account for only a small proportion of variance.

The hierarchical solution thus provides a satisfying structural model that is more or less consistent with those of other theorists. It rests, however, on a combination of complicated statistical machinery and informed, but subjective, researcher judgment.

Alternative Definitions of G

Table 3 shows the ordering of tests in the battery on the complexity continuum constructed according to three alternative definitions of *G*: the centroid, the WAIS full scale score, and the sum of the standard scores of three complex tests from three content areas (Raven Progressive Matrices, Necessary Arithmetic Operations, and Letter Series). The three alternative measures of *G* are highly correlated with one another: for the centroid with the WAIS, $r = .91$; for the centroid with the sum of the three tests, $r = .88$; for the WAIS with the sum of three, $r = .78$. Note that the simple tests that traditionally define CS, PS, and MS show relatively low correlations with *G*. On the other hand, complex tests requiring abstract problem solving, analysis, and the inference of rules show the highest loadings on *G*. Tests that define G_f in the hierarchical solution seem somewhat more central to *G* than the tests that define G_v or G_c. It is true, of course, that those correlations in Table 3 that relate a test to a composite in which the test is included are spuriously high. This is only a significant problem for the correlations between Raven, NAO, and Letter Series, and the composite they define. However, to exclude a test from the composite when computing the test-composite correlation is to change the construct represented by the composite. For the purpose of constructing this rough continuum, it was decided to report the uncorrected correlations.

The Radex Multidimensional Scaling Results

Multidimensional scaling analyses were based on methods developed by Shepard (1962) and Kruskal (1964a, b). The computer program used was KYST (Kruskal, Young, & Seery, 1973). We first discuss the scaling of the WAIS subtests and then report the results for the full battery of tests.

No WAIS subtest had more than 46% of its variance accounted for by *G*, even though the total score combining all subtests is used as one of the definitions of *G*. The subtests seem to be either simple or of intermediate complexity. This result suggested that the WAIS subtests taken alone might scale as a radex. The scaling solution is shown in Figure 2. This two-dimensional solution consists of two circumplexes arrayed around the definitions of *G* in the center. The WAIS subtests that have 40% to 46% of their variance accounted for by *G* form the inner circumplex, while the simple tests for which *G* accounts for less than 30% of the variance are in the peripheral circumplex. The subtests are also spaced according to the spatial, verbal, and numerical content areas.

The complexity continuum is elaborated in Figure 3, which is a two-dimensional scaling representation of the tests in the reference battery as listed in Table 1. Complex tests that have more than 50% of their variance accounted for by *G* are marked by squares and, as predicted, fall in the center. The simple tests that have less than 25% of their variance accounted for by *G* are marked by cir-

TABLE 2
Orthogonal Hierarchical Matrix Produced by Transforming Factor Structure Matrices Using Wherry's (1959) Procedure

	Factors[a]													
Test	G	G_v	G_f	G_c	CS	SR	PS	RS	NS	VC	RC	LS	MS	h^2
1 Street Gestalt	34	47	–16		38									50
2 Harshman Getsalt	37	52	–14		36									56
28 WAIS Picture Completion	44	42			28									46
9 Paper Folding	63	34	17			33								67
11 Surface Development	57	35	10			39								65
10 Paper Form Board	66	34	18			33								71
14 Hidden Figures	63	31				29	15				10			62
29 WAIS Block Design	61	39	12			32					–12			68
31 WAIS Object Assembly	54	42				36				10	–10			62
6 Identical Pictures	58	19				10	52							66
7 Finding A's	50						54					10		56
8 Number Comparison	47						62							61
27 WAIS Digit Symbol	52	–11					63							70
18 Thurstone Letter Series	69		35					20						65
32 Raven	76	14	31				11	16		–16	12			78
22 WAIS Arithmetic	61		38					22		12				62
17 Necessary Arithmetic Operations	72		34					17						66
12 Word Transformations	69		17	16			14	20				14		63
34 Arithmetic Computation[b]	70	–11	52						30					88

34 Arithmetic Concepts[b]	76		51						24					90
34 Arithmetic Applications[b]	70		48				–13		26					82
26 WAIS Vocabulary	64			48						41	10			82
19 Terman Concept Mastery	72		15	31						30				74
20 WAIS Information	59			35			–11			40				66
23 WAIS Similarities	49			27			–13			54	–14			65
21 WAIS Comprehension	51			33						48				62
33 Reading Vocabulary[b]	68			51						13	34			87
33 Reading Comprehension[b]	70			43							43			87
33 Language Expression[b]	69			45							42			86
13 Camouflaged Words	53	12	–12	34			15					44		66
33 Language Mechanical[b]	60	–11	16	24			12			–16	13	24		57
33 Language Spelling[b]	53	–17		42						12		34		64
15 Word Beginnings and Endings	56			42								38		67
4 Auditory Letter Span	38			42									53	64
5 Visual Number Span	52			35									54	70
24–5 WAIS Digit Span	43		–10	48									58	78
30 WAIS Picture Arrangement	56													33
% of Total Variance	35	4	4	7	1	2	4	1	1	3	2	2	3	

Note. Decimals omitted. Loadings smaller than ± .10 not shown.
[a]Factor labels defined in text.
[b]Subscores of achievement composite; Comprehensive Test of Basic Skills
[c]Composite of WAIS digit span forward and backward.

TABLE 3

Ordering of Tests on the Complexity Continuum Defined by the Percent of Their Variance Accounted by G Showing Their Correlations with Three Alternative Measures of G

Apparent Complexity	Tests	% of Variance[a] accounted by G	Measures of G: WAIS Total Score	Centroid	Raven + NAO + Letter Series
Complex	Achievement Quantitative	60	72	79	82
	Raven	60	66	76	88
	Achievement Verbal	55	74	78	70
	Terman Concept Mastery	54	75	76	70
	Thurstone Letter Series	54	64	70	85
	Necessary Arithmetic Operations	54	66	74	80
Intermediate Complex	WAIS Vocabulary	46	76	68	59
	Surface Development	43	60	68	68
	Paper Folding	43	62	65	69
	Word Transformations	42	62	71	62
	WAIS Block Design	40	67	61	61
	WAIS Information	40	73	63	53
	WAIS Arithmetic	40	68	62	60
	Hidden Figures	35	54	63	59
	WAIS Comprehension	35	70	58	47
	WAIS Similarities	32	67	55	45
	Paper Form Board	30	52	60	52
	Word Beginnings and Endings	29	53	60	47
	WAIS Object Assembly	28	58	56	45
Simple	Visual Number Span	25	49	53	47
	WAIS Digit Span	22	52	48	38
	Camouflage Words	21	41	53	43
	Identical Pictures	21	41	53	44
	WAIS Picture Arrangement	18	48	43	34
	WAIS Picture Completion	18	45	45	37
	WAIS Digit Symbol	17	42	41	42
	Auditory Letter Span	16	33	52	30
	Finding A's	16	34	47	38
	Number Comparison	10	28	37	31
	Uses	07	29	30	22
	Harshman Gestalt	07	25	32	24
	Street Gestalt	06	23	27	23
	Film Memory III	04	17	23	23

Note. Decimals omitted.

[a]Average squared correlations with the three measures of G.

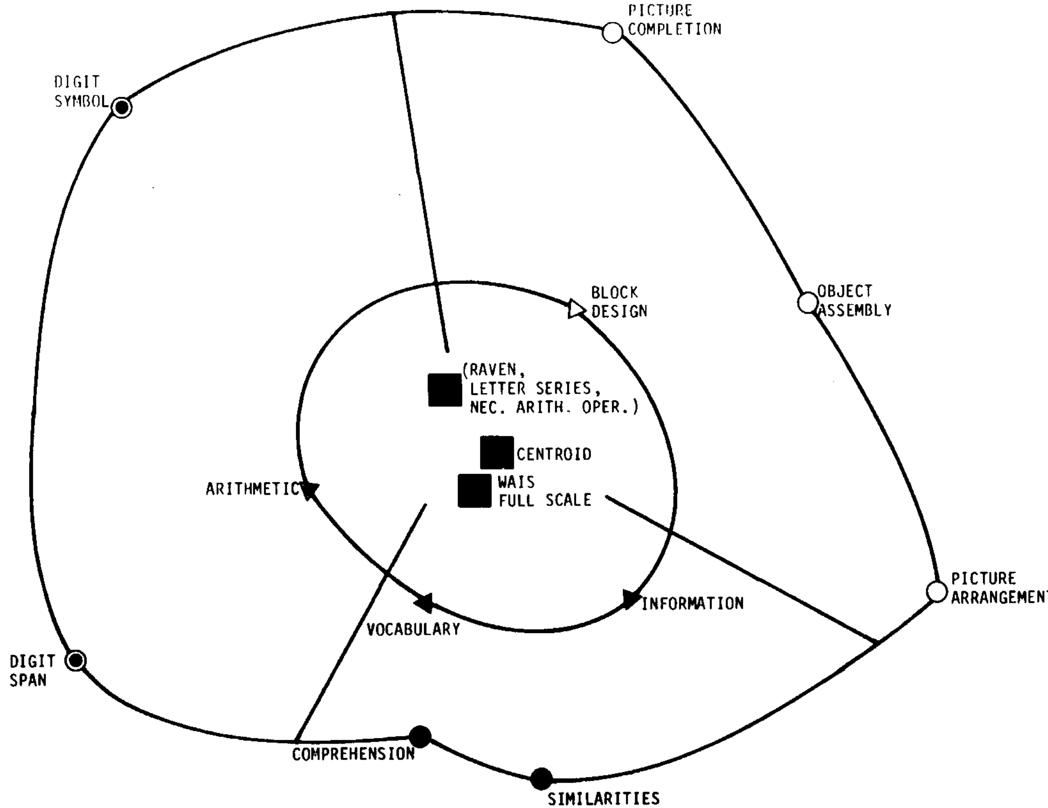

FIG. 2. A multidimensional scaling of the WAIS showing two circumplexes of subtests around three definitions of G (high school sample, $N = 241$). Complex, intermediate, and simple tests are indicated as black (verbal), dotted (numerical), and white (figural-spatial) squares, triangles, and circles, respectively.

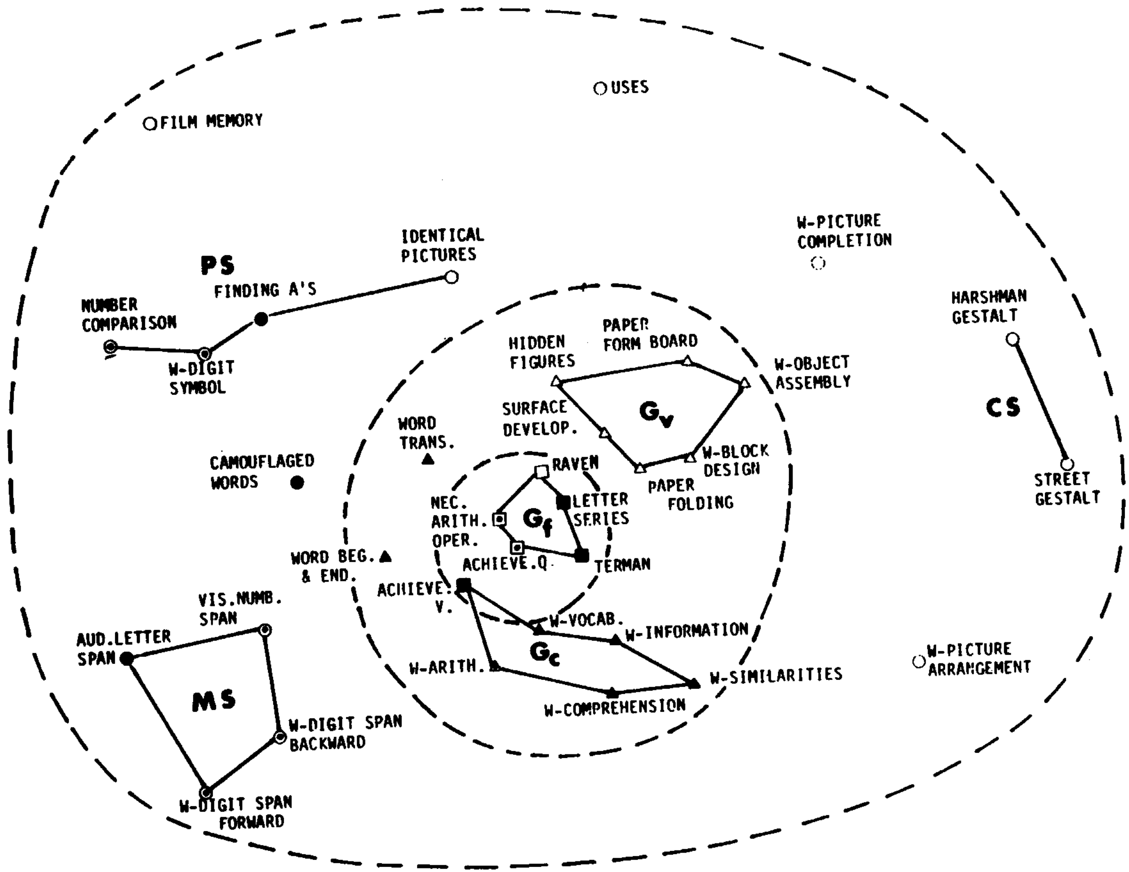

FIG. 3. A multidimensional scaling of the full test battery showing three levels of complexity and three content areas (high school sample, $N = 241$).Complex, intermediate, and simple tests are indicated as black (verbal), dotted (numerical), and white (figural-spatial) squares, triangles, and circles, respectively.

cles and, as expected, fall in the periphery. Tests of intermediate complexity, those having 26% to 50% of their variance accounted for by G, are marked by triangles and, as expected, fall between the squares and the circles. The complexity continuum that radiates out from the center of the radex closely approximates the ordering of the tests in Table 3. While the verbal tests (black symbols) and numerical tests (dotted symbols) are clearly distinguished from the figural and spatial tests (white symbols), verbal and numerical content tests appear interspersed. This suggests that the distinction is unimportant at a simple, symbol-processing level. It may be due also to the lack of numerical tests of intermediate complexity in the battery. Note also that the Film Memory Test and the Uses Test are here considered to involve figural-spatial processing, though they include verbal content.

It is noteworthy that the achievement composites fall close to the center and correlate highly with G. This result suggests that Guttman's distinction between rule-inferring and rule-applying operations may be a more appropriate dichotomy than his distinction between analytic and achievement tests. Achievement tests can appear anywhere on the complexity continuum, depending on the extent to which they call for abstract and complex problem solving, requiring analysis and inference (see Snow 1980b).

Tests that define a factor in the factor solutions are connected by lines drawn into the radex representation in Figure 3 and the factor symbols are given. Thus, the scaling representation stays close to the raw correlations, yet allows the more highly processed, factor analytic results also to be incorporated into the radex. Complexity becomes defined within a continuous disc rather as a dichotomy, and both tests and factors can be compared in these terms. The radex structure in Figure 3 suggests that there exist not one, but many of what Jensen (1969) called Level I abilities, including MS, PS, CS, etc.

As noted earlier, the test battery was also administered to a college sample. A comparable solution in the college sample accounted for less variance than in the larger and more representative high school sample. Relative to high school students, the college sample could be expected to show restricted range on SAT-V and perhaps SAT-Q, and thus on G_c and the correlated G and G_f factors. Range restriction on G_c distorted the radex analysis even more than it did the factor analysis. Its effect was to "pinch off" variance on the verbal side of Figure 3, pulling tests associated with SAT-V, and thus the G_c factor, to the left of the periphery, moving G_f tests also somewhat left, and throwing CS, PS, and MS tests to the top, right, and bottom periphery respectively; G_v tests appeared to be least disturbed, probably because college admission decisions place little weight on spatial abilities. It is clear from such effects that the sampling of subjects for cognitive psychological research is an important consideration. External validation of experimental processing parameters (Sternberg, 1977) will be influenced by the structure of relations among the reference tests used, which, in turn, depend on the score ranges available in the subject population. The sampling of tests will also influence the structure observed, of course.

Where reasonably representative sampling of persons and tests is used, however, it can be expected that radex structures approximating that reported here will be obtained. Snow, Kyllonen, and Marshalek (in press) have shown essentially the same radex structure, and the same relation between it and test correlation with *G*, in several other ability and learning task correlation matrices. They also have discussed in more detail some of the substantive and methodological aspects of multidimensional scaling analyses relevant to future research in cognitive differential psychology. The agenda for further methodological research should include formal studies of the effects of variations in sampling, measurement error, and communalities on scaling solutions. It may also be possible to construct formal tests of the parallelism between radex and hierarchical structures that we have displayed here.

DISCUSSION

What is the complexity continuum? Can a process theory of the complexity continuum be constructed that explains variation as one moves from specific to general tasks, or peripheral to central tasks, or rule-applying to rule-inferring tasks? Sternberg (1981) has described the transition from hierarchical factor models and overlapping primary ability models to the radex model as a transition from Stage IIa and IIb to Stage III correlational theories of intelligence. But we have suggested that both the hierarchical model and the radex model yield the same message; it is that understanding the complexity continuum is the key to a theory of intelligence.

Guttman, as noted above, thought of complexity as arising from the number of different components involved in test performance, but he did not define these components. In continuing research, Sternberg's (1977, 1981) componential analysis approach might be used both to identify the components and to test this hypothesis. In this work, however, a second hypothesis about the nature of complexity must also be examined. Increases in apparent complexity from task to task may reflect the increased involvement of one or more centrally important components, rather than simply an increase in the number of components involved in performance. The research of Jensen (1981) and others with reaction time tasks that manipulate apparent processing complexity may help to test this alternative. Still a third hypothesis is possible. We have suggested elsewhere (Kyllonen, Woltz, & Lohman, 1981; Snow, 1978, 1980a, 1981) that more complex tasks may require more involvement of executive assembly and control processes that structure and analyze the problem, assemble a strategy of attack on it, monitor the performance process, and adapt these strategies as performance proceeds, within as well as between items in a task, and between tasks. The increasing correlations among superficially different tasks as one moves up the complexity continuum may thus reflect the increasing variance in performance due to these metacognitive, executive control, and adaptive functions. Simpler tasks, on the other hand, seem more to reflect limitations of specific parameters, such as speed of different kinds of processing, or visual or verbal memory storage and retrieval limitations. Performance programs for simple tasks are hypothesized to be more automatic and repetitious, and less in need of adaptation across items or trials. The specificity and automaticity of these task programs might account for the relatively low correlations among them. It is possible, of course, that apparent processing complexity reflects all three of these hypothesized sources: more components, more central components, and more adaptive or flexible organization of components and metacomponents.

In summary, there are theoretical as well as empirical parallels between the radex and the hierarchical models of ability organization, particularly with regard to the complexity continuum. The revised radex model presented here introduces two new features: we show theoretically and empirically that the continuum from the center to the periphery corresponds to the general-to-specific dimension in factor analyses, or to test correlations with *g*; we redefine complexity as apparent processing complexity and find it to correspond to test correlations with *g*. Thus, the continuum that radiates out from the center of the radex runs from general-complex to simple-specific. A process description of this complexity continuum will be essential for a theory of the nature of intelligence.

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