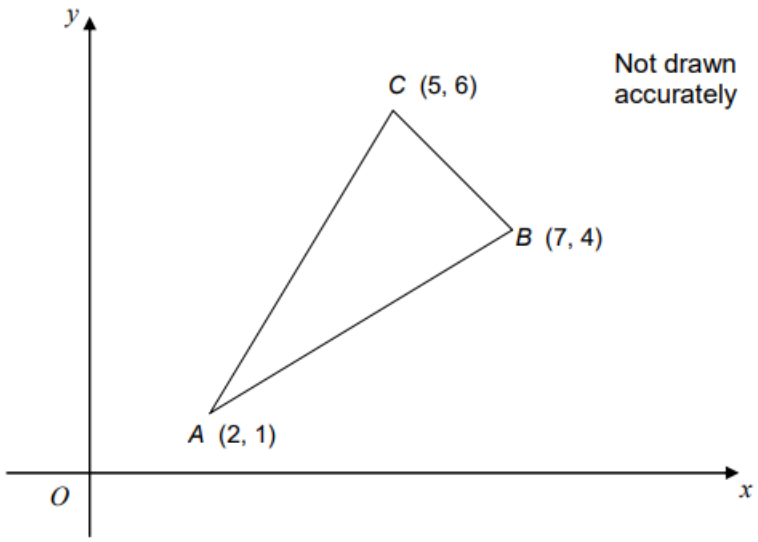
All questions are taken from AQA Level 2 Further Maths papers that are free to find online and are questions that could be found on a non-calculator GCSE Maths exam.

This resource is designed for students revising GCSE maths and not Level 2 Further Maths. The paper numbers are given for refrence only.

Practice Paper Set 1 Paper 2

4 The diagram shows an isosceles triangle *ABC*, with *AB* = *AC*.



Work out the area of the triangle.

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Answer ..... units^2 *(5 marks)*

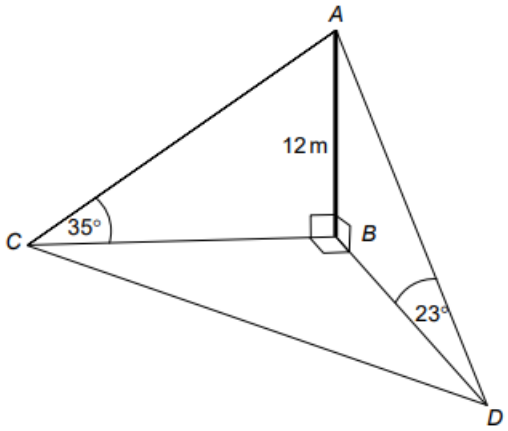
7 The diagram shows a vertical mast, *AB*, 12 metres high.
Points *B*, *C* and *D* are on a horizontal plane.

Point *C* is due West of *B*.

The angle of elevation of *A* from *C* is 35°.

Point *D* is due South of *B*.

The angle of elevation of *A* from *D* is 23°.



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7 (a) Calculate the distance *CD*.

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Answer ..... metres *(6 marks)*

7 (b) Calculate the bearing of *D* from *C*.

Give your answer to the nearest degree.

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Answer ..... ° *(3 marks)*

11 You are given that $x = 5^m$ and $y = 5^n$

11 (a) Write 5^{m+2} in terms of x.

Give your answer in its simplest form.

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Answer ..... *(2 marks)*

11 (b) Write 5^{m-n} in terms of x and y.

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Answer ..... *(1 mark)*

11 (c) Write 5^{3n} in terms of y.

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Answer ..... *(1 mark)*

11 (d) Write $5^{\frac{m+n}{2}}$ in terms of x and y.

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Answer ..... *(2 marks)*

4	$(BC =)\ \sqrt{8}$	B1	
	Midpoint of BC attempted $(\frac{7+5}{2}, \frac{4+6}{2})$	M1	(6, 5)
	AM^2 attempted (their 6 – 2)^2 + (their 5 – 1)^2	M1	32 or $AM = \sqrt{32}$
	$\frac{1}{2}$ × their $\sqrt{8}$ × their $\sqrt{32}$	M1	
	8	A1 ft	ft From B0 M3

Alt 4	Surrounding 5 × 5 square drawn	M1	
	Any one of $\frac{1}{2}$ × 3 × 5 or $\frac{1}{2}$ × 2 × 2 or $\frac{1}{2}$ × 5 × 3	M1	
	All three of $\frac{1}{2}$ × 3 × 5, $\frac{1}{2}$ × 2 × 2, $\frac{1}{2}$ × 5 × 3	A1	
	25 – their 7.5 – their 2 – their 7.5	M1	
	8	A1	

7(a)	$\tan 35 = \frac{12}{BC}$ or $\tan 23 = \frac{12}{BD}$	M1	$\tan 55 = \frac{BC}{12}$ or $\tan 67 = \frac{BD}{12}$
	$(BC =)\ \frac{12}{\tan 35}$ or $(BD =)\ \frac{12}{\tan 23}$	M1	(BC=) 12 tan 55 or (BD=) 12 tan 67
	(BC =) 17.13(7...) or 17.14	M1	
	(BD =) 28.27(02...)	A1	
	$\sqrt{\text{their } 17.14^2 + \text{their } 28.27^2}$	M1	
	33.05(...) or 33.1	A1 ft	ft their BC and their BD
7(b)	$\tan\theta = \frac{\text{their } 28.3}{\text{their } 17.1}$	M1	
	58.7° to 58.9°	A1	
	149°	A1 ft	ft their 58.7° if M1 earned

11(a)	$5^m \times 5^2$	M1	
	$25x$	A1	
11(b)	$\frac{x}{y}$	B1	oe
11(c)	y^3 or $y \times y \times y$	B1	
11(d)	$x^{\frac{1}{2}}\ y^{\frac{1}{2}}$	B2	oe eg, $\sqrt{xy}$ B1 $x^{\frac{1}{2}}$ or $y^{\frac{1}{2}}$ or $\sqrt{x}$ or $\sqrt{y}$ or $5^{\frac{m}{2}} \times 5^{\frac{n}{2}}$

Practice Paper Set 2 Paper 2

11 The nth term of sequence X is $an + b$

The nth term of sequence Y is $bn + a$

11 (a) Show that the sequences have the same first term.

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(1 mark)

11 (b) The 2nd term of sequence X is equal to the 3rd term of sequence Y.

Show that $a = 2b$

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(2 marks)

11 (c) Prove that

$$\frac{n\text{th term of sequence } X}{n\text{th term of sequence } Y} = \frac{2n+1}{n+2}$$

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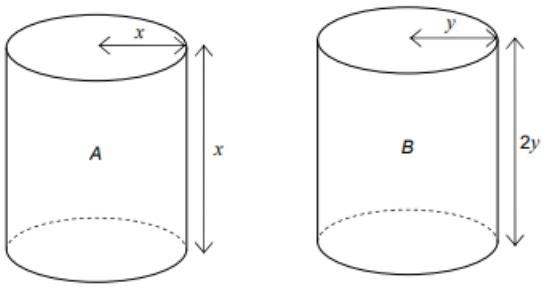
(3 marks)

13 Cylinder *A* has radius x cm and height x cm.
Cylinder *B* has radius y cm and height $2y$ cm.

Not drawn accurately



You are given that

total surface area of cylinder *A* = **total** surface area of cylinder *B*

13 (a) Show that $x^2 = \frac{3}{2}y^2$

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(4 marks)

13 (b) Which cylinder has the greater height?
You **must** show your working.

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Answer Cylinder ..... *(2 marks)*

17 The population of Japan, J, is 1.30×10^8
The population of Brazil, B, is 1.95×10^8

17 (a) You are given that $J : B = x : x + 5$

Work out the value of x.

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Answer $x =$ ..... *(3 marks)*

17 (b) The population of Pakistan is P.
You are given that $J : P = x : x + 4$

Work out the population of Pakistan.
Give your answer in standard form.

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Answer ..... *(2 marks)*

11(a)	$a \times 1 + b = a + b$ **and** $b \times 1 + a = b + a$	B1	
11(b)	$2a + b$ **or** $3b + a$	M1	
	$2a + b = 3b + a$ (leading to $a = 2b$)	A1	
11(c)	$\frac{2bn + b}{bn + 2b}$	M1	Allow for correct numerator **or** denominator correct
	$b(2n + 1)$ **or** $b(n + 2)$	M1	Factorises either their numerator **or** their denominator
	$\frac{b(2n + 1)}{b(n + 2)}$ **and** shows simplification	A1	

13(a)	$\pi x^2 + \pi x^2 + 2\pi x^2$ $(= 4\pi x^2)$	M1	oe
	$\pi x^2 + \pi x^2 + 4\pi x^2$ $(= 6\pi x^2)$	M1	oe
	Their $4\pi x^2 =$ their $6\pi y^2$	M1	Allow if equating curved surface areas
	Shows working leading to $x^2 = \frac{3}{2}y^2$	A1	
13(b)	$x = \sqrt{\frac{3}{2}}y$	M1	oe
	B **and** $x = 1.2\ldots y$ which is less than $2y$	A1	oe

17(a)	$\frac{x}{x + 5} = \frac{1.3(0) \times 10^8}{1.95 \times 10^8}$	M1	oe eg, $\frac{x + 5}{x} = \frac{3}{2}$
	$1.95x = 1.3(0)(x + 5)$	M1	oe eg, $2(x + 5) = 3x$
	10	A1	
17(b)	$\frac{\text{their } 10 + 4}{\text{their } 10} \times 1.3(0) \times 10^8$ $(= 182\,000\,000)$	M1	$\frac{\text{their } 10 + 4}{\text{their } 10 + 5} \times 195 \times 10^8$ $(= 182\,000\,000)$
	1.82×10^8	A1ft	ft Their 10 if answer in standard form

Practice Paper Set 3 Paper 2

2 h is 60% of m.

2 (a) Write an equation connecting h and m.

Answer ..... *(1 mark)*

2 (b) Also, r is 75% of m.

Work out h as a percentage of r.

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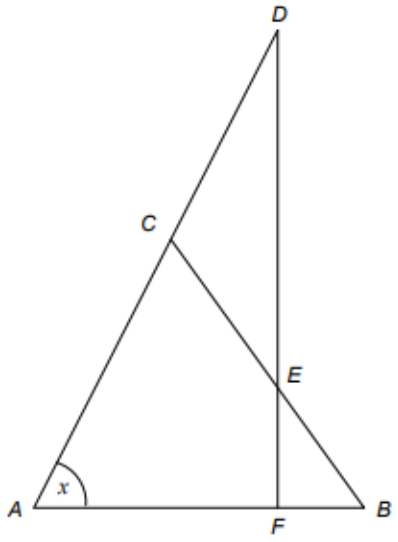
Answer ..... % *(2 marks)*

7 Triangle ABC is isosceles with $AC = BC$
Triangle CDE is isosceles with $CD = CE$
ACD and DEF are straight lines.



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7 (a) Prove that angle $DCE = 2x$

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(2 marks)

7 (b) Prove that DF is perpendicular to AB.

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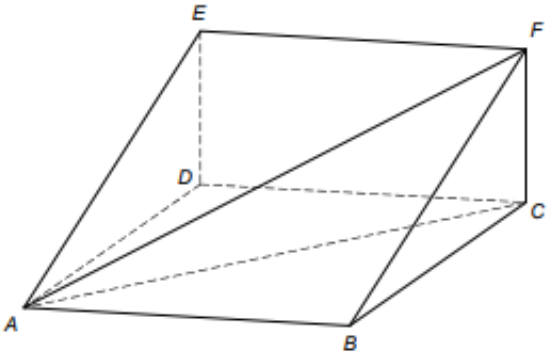
(3 marks)

11 The diagram shows part of a skate ramp, modelled as a triangular prism.

ABCD represents horizontal ground.
The vertical rise of the ramp, *CF*, is 7 feet.
The distance *BC* = 24 feet.

Not drawn accurately



You are given that $\text{gradient} = \dfrac{\text{vertical rise}}{\text{horizontal distance}}$

11 (a) The gradient of *BF* is **twice** the gradient of *AF*.

Write down the distance *AC*.

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AC = ... feet *(1 mark)*

11 (b) Greg skates down the ramp along *FB*.

How much further would he travel if he had skated along *FA*?

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Answer ..... feet *(4 marks)*

12 $f(x) = x^2 + px + q$ for all values of x.

p and q are integers.

f(0) is an odd number.

f(1) is an odd number.

Show that p is an odd number.

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(3 marks)

18 $C = \frac{3x + 7}{x + 1}$ and $D = \frac{4x - 11}{2x + 3}$

Work out the value of x when $C + D = 5$

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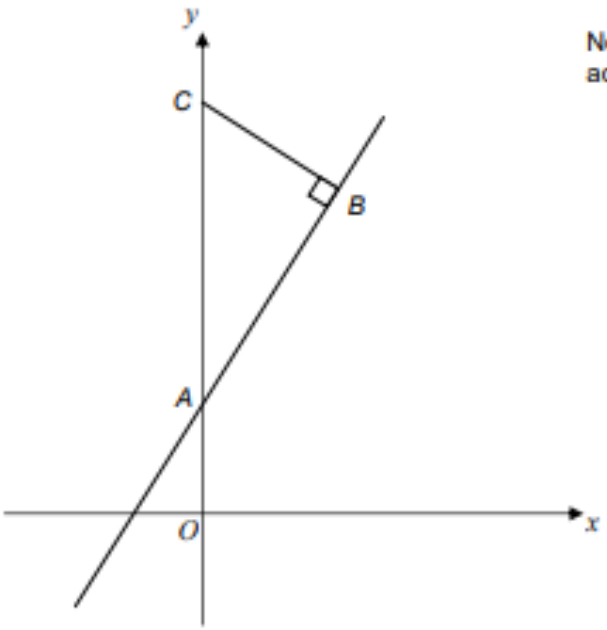
$x =$ ..... *(5 marks)*

21 On the diagram

- *A* and *C* are on the y-axis
- *C* is (0, 7)
- the equation of the line through *A* and *B* is $y = 2x + 1$
- *BC* is perpendicular to *AB*.



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Work out the area of triangle *ABC*.

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Answer ..... *(6 marks)*

25 $x^3 + ax^2 + bx + 150$ factorises to $(x + c)^2(x + d)$

a, b, c and d are positive integers and $c \neq 1$

Work out the values of a, b, c and d.

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Answer $a =$ .....

$b =$ .....

$c =$ .....

$d =$ ..... *(6 marks)*

2(a)	$h = 0.6m$	B1	oe eg, $h = \frac{60}{100}m$
2(b)	$\frac{\text{their } 0.6m}{0.75m}$ $(= 0.8)$	M1	oe
	80	A1 ft	ft From their 0.6

7(a)	$\angle CBA = x$ **and** base angles of isosceles triangle (are equal)	M1	oe
	$\angle DCE = 2x$ **and** exterior angle = sum of interior opposite angles	A1	$\angle ACB = 180 - 2x$ **and** angle sum of triangle = 180 **and** $\angle DCE = 2x$ **and** (adjacent) angles on a straight line add up to 180 SC1 'Correct' solution without reasons
7(b)	$\angle CDE = (180 - 2x) \div 2$ **and** base angles of isosceles triangle (are equal)	M1	$\angle CED = (180 - 2x) \div 2$ **and** base angles of isosceles triangle (are equal)
	$90 - x$	A1	
	$\angle AFD = 180 - x - (90 - x)$ **and** angle sum of triangle = 180	A1	$\angle FEB = 90 - x$ **and** vertically opposite angles **and** $\angle EFB = 180 - x - (90 - x)$ **and** angle sum of triangle = 180 SC2 'Correct' solution without reasons

11(a)	48	B1	
11(b)	$7^2 + 24^2$ $(= 625)$ **or** $7^2 +$ their 48^2 $(= 2353)$	M1	
	$\sqrt{7^2 + 24^2}$ **or** $\sqrt{7^2 + \text{their } 48^2}$	M1	$\sqrt{625}$ $(= 25)$ **or** $\sqrt{2353}$ $(= [48.5, 48.51])$
	$\sqrt{7^2 + \text{their } 48^2} - \sqrt{7^2 + 24^2}$	M1	[48.5, 48.51] – 25
	[23.5, 23.51]	A1 ft	ft Their 48 and M3

12	q is odd (because f(0) = q)	B1	
	f(1) = $1 + p + q$	M1	
	$1 + q$ is even **and** even + odd = odd	A1	oe eg, odd + odd + odd = odd

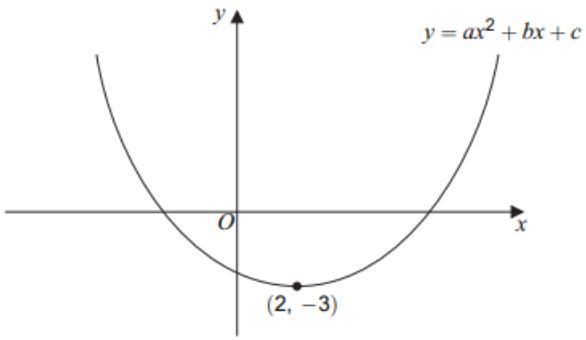
18	$\frac{(3x+7)(2x+3)+(x+1)(4x-11)}{(x+1)(2x+3)}(=5)$	M1	
	$6x^2+14x+9x+21$ **or** $4x^2+4x-11x-11$ **or** $(5)(2x^2+2x+3x+3)$	M1	oe 4 terms with any 3 correct
	$6x^2+14x+9x+21$ **or** $4x^2+4x-11x-11$ **or** $(5)(2x^2+2x+3x+3)$	A1	oe All 4 terms correct
	Their $6x^2+14x+9x+21$ + their $4x^2+4x-11x-11$ = 5 their $(2x^2+2x+3x+3)$	M1	
	$-\frac{5}{9}$	A1	

21	A (0, 1) **or** $AC = 6$	B1	
	Gradient BC is $-\frac{1}{2}$	B1	oe eg, equation BC $y=-\frac{1}{2}x+7$
	$2x+1=$ their $-\frac{1}{2}x+7$	M1	oe eg, $\frac{2x+1-7}{x}=$ their $-\frac{1}{2}$
	$2\frac{1}{2}x=6$ $(x=2.4)$	M1	oe eg, $5x=12$ ft From their $-\frac{1}{2}x+7$ or their $-\frac{1}{2}$
	$\frac{1}{2}\times$ their 6 $\times$ their 2.4	M1	
	7.2	A1 ft	ft From B1 B0 M3 or B0 B1 M3
Alt 21	Gradient BC is $-\frac{1}{2}$	B1	oe eg Equation BC $y=-\frac{1}{2}x+7$
	$\frac{2x+1-7}{x}=$ their $-\frac{1}{2}$	M1	oe eg $2x+1=$ their $-\frac{1}{2}x+7$
	$5x=12$ $(x=2.4)$	M1	oe eg $2\frac{1}{2}x=6$ ft From their $-\frac{1}{2}$ or their $-\frac{1}{2}x+7$
	$y=5.8$	A1	
	$\frac{1}{2}\times\sqrt{(\text{their } 5.8-7)^2+\text{their } 2.4^2}$ $\times\sqrt{(\text{their } 5.8-1)^2+\text{their } 2.4^2}$	M1	
	7.2	A1 ft	ft From B0 M2 A1 M1 or B1 M2 A0 M1

25	Any 2 factors of 150 except 1 and 150	M1	2, 75 **or** 3, 50 **or** 5, 30 **or** 6, 25 **or** 10, 15
	$c = 5$ **and** $d = 6$	A1	
	x^2 (+) $5x$ (+) $5x$ (+) 25	M1	ft Their c 4 terms with at least 3 correct
	$(x^2 + 10x + 25)(x + 6) = x^3$ (+) $10x^2$ (+) $25x$ (+) $6x^2$ (+) $60x$ (+) 150	M1	ft their c and their d Allow one error or one omission
	$x^3 + 10x^2 + 25x + 6x^2 + 60x + 150$	A1 ft	Fully correct for their c and their d
	$a = 16$ **and** $b = 85$	A1 ft	ft Their expansion
Alt 25	$x^3 + dx^2 + 2cx^2 + 2cdx + c^2x + c^2d$	M1	Allow up to two errors or omissions
	Their $c^2d = 150$	M1	
	$c = 5$ **and** $d = 6$	A1	
	Their $d + 2c = a$ **or** their $2cd + c^2 = b$	M1	
	$a = 16$	A1 ft	ft Their $d + 2c$ and their c and their d
	$b = 85$	A1 ft	ft Their $2cd + c^2$ and their c and their d

June 2012 Paper 2

4 A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is $(2, -3)$.



For the sketch shown, circle the correct answer in each of the following.

4 (a) The value of a is

zero positive negative *(1 mark)*

4 (b) The value of c is

zero positive negative *(1 mark)*

4 (c) The solutions of $ax^2 + bx + c = 0$ are

both zero both positive both negative one positive and one negative

(1 mark)

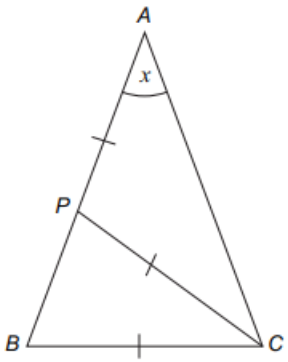
4 (d) The **number** of solutions of $ax^2 + bx + c = -6$ is

0 1 2 3 *(1 mark)*

4 (e) The equation of the tangent to $y = ax^2 + bx + c$ at $(2, -3)$ is

$x = 2$ $y = 2$ $x = -3$ $y = -3$ *(1 mark)*

5 *ABC* is a triangle.
P is a point on *AB* such that $AP = PC = BC$
Angle $BAC = x$



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5 (a) Prove that angle $ABC = 2x$

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(3 marks)

5 (b) You are also given that $AB = AC$

Work out the value of x.

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$x =$ ..... degrees *(3 marks)*

7 $1 \leqslant m \leqslant 5$ and $-9 \leqslant n \leqslant 2$

7 (a) Work out an inequality for $m + n$.

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Answer..... $\leqslant m + n \leqslant$ ..... *(2 marks)*

7 (b) Work out an inequality for $(m + n)^2$.

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Answer..... $\leqslant (m + n)^2 \leqslant$ ..... *(2 marks)*

11 A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\frac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

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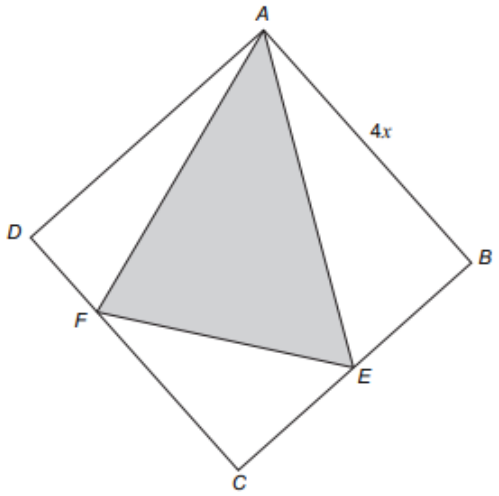
$\frac{y}{x} =$ ..... *(3 marks)*

17 *ABCD* is a square of side length $4x$.

E is the midpoint of *BC*.
$DF : FC = 1 : 3$



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You are given that

area of triangle $AEF = kx^2$

Work out the value of k.

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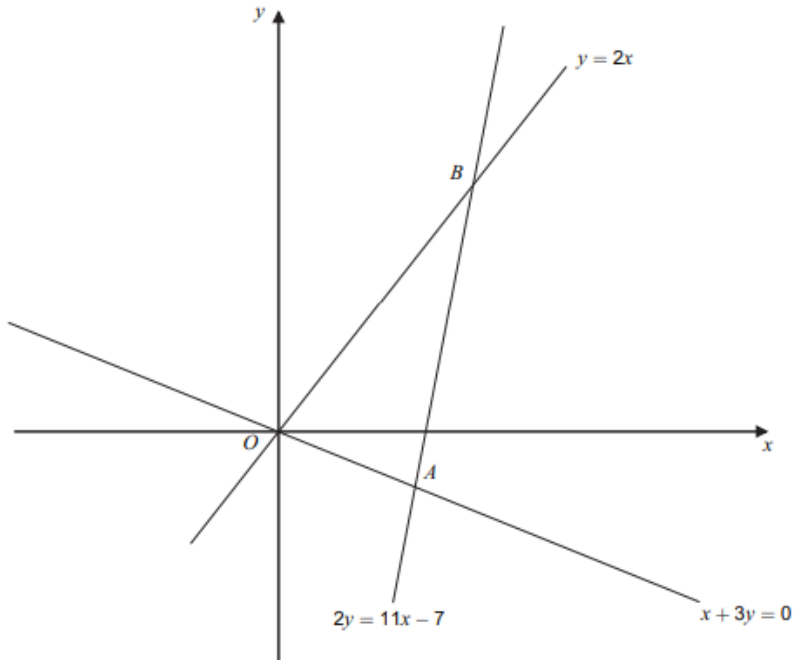
$k =$ ..... *(5 marks)*

21 The equations of three straight lines are

$y = 2x$ $\quad x + 3y = 0$ $\quad 2y = 11x - 7$

The lines intersect at the points O, A and B as shown on this sketch.



Show that length OB = length AB

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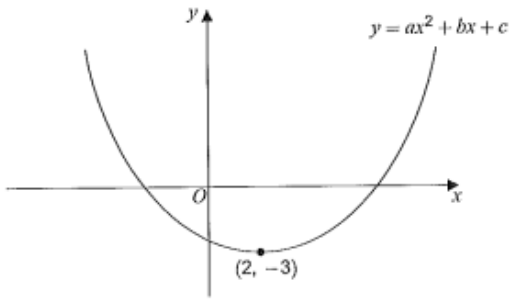
(6 marks)

4 A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is $(2, -3)$.



For the sketch shown, circle the correct answer in each of the following.

4 (a) The value of a is

U shape

zero **positive** negative *(1 mark)*

4 (b) The value of c is

y intercept

zero positive **negative** *(1 mark)*

4 (c) The solutions of $ax^2 + bx + c = 0$ are

both zero both positive both negative **one positive and one negative** *(1 mark)*

cross x-axis

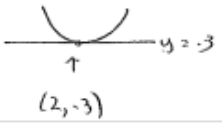
4 (d) The **number** of solutions of $ax^2 + bx + c = -6$ is

0 1 2 3 *(1 mark)*

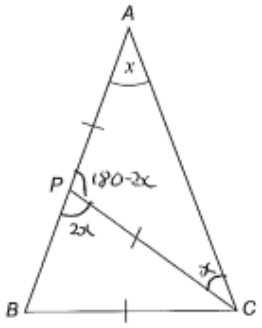
Draw line $y = -6$!

4 (e) The equation of the tangent to $y = ax^2 + bx + c$ at $(2, -3)$ is

$x = 2$ $y = 2$ $x = -3$ **$y = -3$** *(1 mark)*



5 *ABC* is a triangle.
P is a point on *AB* such that $AP = PC = BC$
Angle $BAC = x$



Not drawn accurately

5 (a) Prove that angle $ABC = 2x$

ACP = x (isosceles Δ)

APC = 180 - 2x (angles in Δ = 180)

BPC = 180 - (180 - 2x) = 2x (Angles on straight line = 18

ABC = 2x (isosceles triangle)

(3 marks)

5 (b) You are also given that $AB = AC$

Work out the value of x.

x + 2x + 2x
= 5x

5x = 180

x = 180/5

$x =$ 36° degrees *(3 marks)*

7 $1 \leqslant m \leqslant 5$ and $-9 \leqslant n \leqslant 2$

7 (a) Work out an inequality for $m + n$. Add together

m: 1 + -9 = -8

5 + 2 = 7

Answer $-8 \leqslant m + n \leqslant 7$ *(2 marks)*

7 (b) Work out an inequality for $(m + n)^2$.

(-8)² = 64, 7² = 49

But smallest value is if m = 1 and n = -1

→ $(m+n)^2 = 0^2 = 0$

Answer $0 \leqslant (m + n)^2 \leqslant 64$ *(2 marks)*

11 A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\frac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

$\frac{4}{3} \div 2 = \frac{2}{3}$

[X] $V = \frac{4}{3}\pi x^3$ [Y] $V = \frac{2}{3}\pi y^3$

Volumes are equal $\rightarrow$ $\frac{4}{3}\pi x^3 = \frac{2}{3}\pi y^3$

$\div \pi$ $\quad \frac{4}{3}x^3 = \frac{2}{3}y^3$ $\quad \sqrt[3]{\ }$ $\quad \sqrt[3]{2} = \frac{y}{x}$

$\times 3$ $\quad 4x^3 = 2y^3$ $\quad \rightarrow \frac{y}{x} = 2^{\frac{1}{3}}$

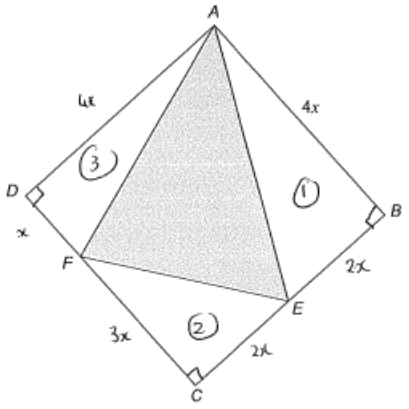
$\div x^3$ $\quad 4 = \frac{2y^3}{x^3}$

$\div 2$ $\quad 2 = \frac{y^3}{x^3}$

$\frac{y}{x} = \frac{y}{x} = 2^{\frac{1}{3}}$ *(3 marks)*

17 *ABCD* is a square of side length $4x$.

E is the midpoint of *BC*.
DF: *FC* = 1 : 3



Not drawn accurately

You are given that

area of triangle $AEF = kx^2$

Work out the value of k.

Area of square $= 4x \times 4x = 16x^2$

Area of (1) $= \frac{1}{2} \times 2x \times 4x = 4x^2$

Area of (2) $= \frac{1}{2} \times 2x \times 3x = 3x^2$

Area of (3) $= \frac{1}{2} \times x \times 4x = 2x^2$

Area of (1) + (2) + (3) $= 9x^2$

$\therefore$ Area of shaded triangle $= 16x^2 - 9x^2$

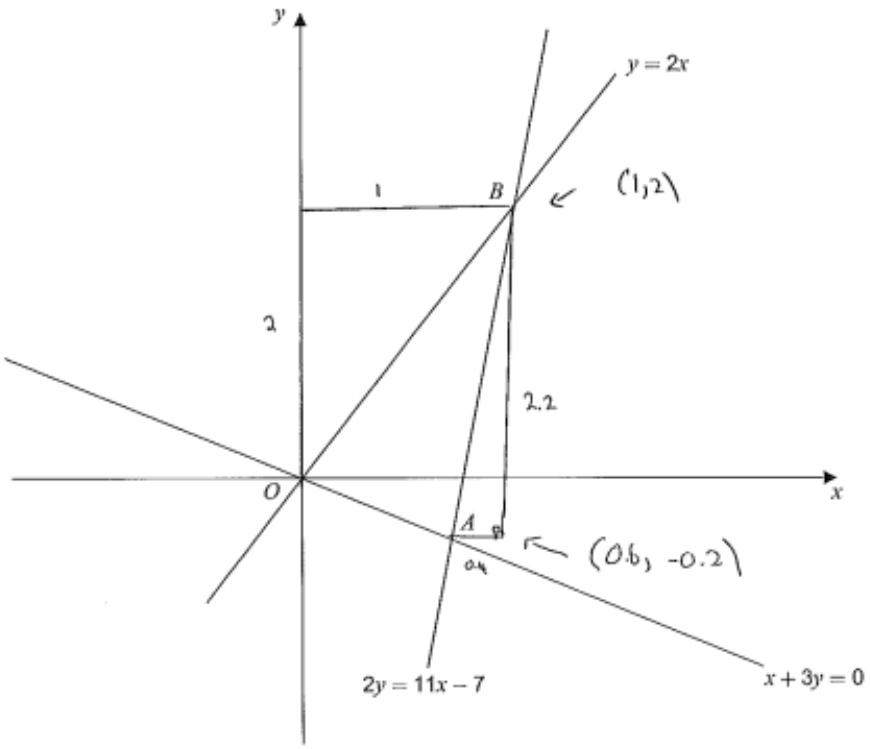
$= 7x^2$

$k =$ 7 *(5 marks)*

21 The equations of three straight lines are

$y = 2x$ $\quad x + 3y = 0$ $\quad 2y = 11x - 7$

The lines intersect at the points O, A and B as shown on this sketch.



See next page for working out

Show that length OB = length AB

Find A where $2y = 11x - 7$ ② meets $x + 3y = 0$

$\rightarrow x = -3y$ ①

Sub ① into ② $\rightarrow 2y = 11(-3y) - 7$

$\rightarrow 2y = -33y - 7$

$\rightarrow 35y = -7$

$\rightarrow y = -7/35 = -0.2$

① $x = -3y = -3(-0.2) = 0.6$

$\therefore A = (0.6, -0.2)$

Find B where $y = 2x$ meets $2y = 11x - 7$

$\therefore 2y = 4x$ ① $\quad 2y = 11x - 7$ ②

Put ① = ② $\rightarrow 4x = 11x - 7$

$\rightarrow 0 = 7x - 7$

$\rightarrow 7 = 7x \rightarrow x = 1$

① $y = 2x \rightarrow y = 2(1) = 2$

$\therefore B = (1, 2)$

See Diagram! $OB = \sqrt{1^2 + 2^2} = \sqrt{5}$

$AB = \sqrt{0.4^2 + 2.2^2} = \sqrt{5}$

$\therefore OB = AB$

(6 marks)

Turn over for the next question

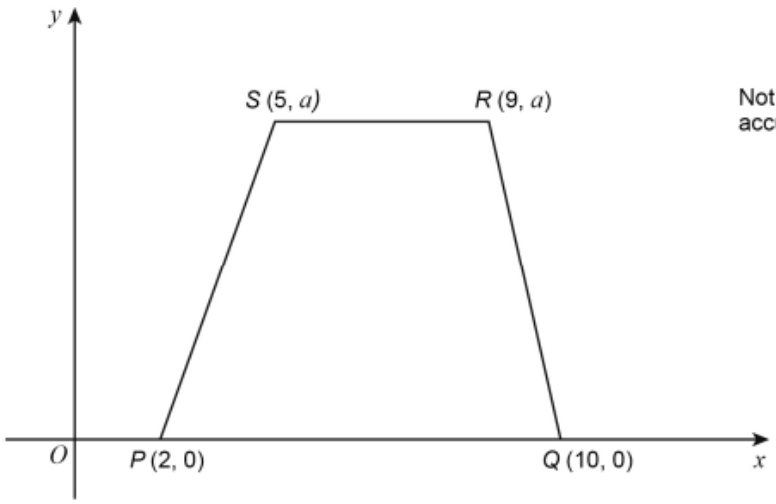
June 2018 Paper 2

2 P (–3, –10) and Q (a, b) are points on a straight line with gradient 12

Work out one possible pair of integer values for a and b.

[2 marks]

7 $PQRS$ is a trapezium.



The area of the trapezium is 63 square units.

Work out the value of a.

[2 marks]

12 (a) Write $\frac{7}{9x} + \frac{2}{3x^2}$ as a single fraction in its simplest form.

[3 marks]

Answer _____

12 (b) Show that $\frac{x^4}{x+4} \times \frac{x+2}{x} \div \frac{x^2}{3x+12}$

simplifies to the form $ax^2 + bx$ where a and b are integers.

[4 marks]

14 (a) Factorise fully $12pq^3r - 18pq^2r^2 + 24pq^2r$

[2 marks]

Answer _____

14 (b) Factorise fully $6(y+3)^5 + 4(y+3)^4$

Give your answer in its simplest form.

Do **not** attempt to expand $(y+3)^5$ or $(y+3)^4$

[3 marks]

Answer _____

14 (c) Factorise fully $48 - 75x^2$

[2 marks]

Answer _____

16 $A = 2 - 5x$ $B = 3x - 1$ $C = x^2$

Show that $(2A + 3B)^2 \equiv A + B + C$

[4 marks]

17 A circle has equation $x^2 + y^2 = 29$

P is the point (–5, 2)

17 (a) Show that P is on the circle.

[1 mark]

17 (b) The tangent to the circle at P intersects the x-axis at point Q.

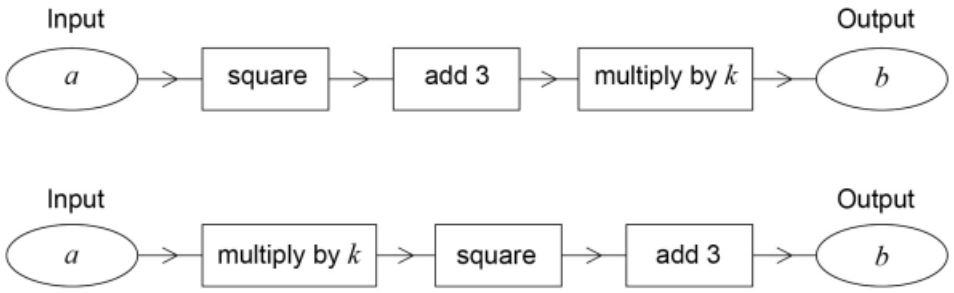
Work out the x-coordinate of Q.

You **must** show your working.

[4 marks]

23 For each of these two function machines, when the input is a the output is b.

$k > 0$ and $k \neq 1$ and $a > 0$



Work out an expression for a in terms of k.

Give your answer in its simplest form.

[6 marks]

24 Work out the value of p when

$$9^{0.5p} \times 81 = 27^{2p-1}$$

[4 marks]

Answer _____

2	Any pair of integer values for a and b for which $b = 12a + 26$	B2	B1 Correct equation in any form eg $\frac{b - -10}{a - -3} = 12$ or $b + 10 = 12(a + 3)$ or $\frac{y - -10}{x - -3} = 12$ or $y + 10 = 12(x + 3)$ or $b = 12a + c$ and $c = 26$ or $y = 12x + c$ and $c = 26$ or –3 + k and –10 + 12k where k is a non-zero integer
	Additional Guidance		
	Examples of B2 responses a = –4 and b = –22 or a = –2 and b = 2 or a = –1 and b = 14 or a = 0 and b = 26 or a = 1 and b = 38 or a = 2 and b = 50 or a = 3 and b = 62 or a = 4 and b = 74		B2
	a = –3 and b = –10 is point P so will not score B2 (B1 possible)		
	–3 + 1 and –10 + 12		B1
	–3 + 2 and –10 + 24		B1

7	$\frac{1}{2} \times (8 + 4) \times a$ (= 63) or $\frac{1}{2} \times 12 \times a$ (= 63) or $6a$ (= 63) or 63 ÷ 6	M1	any letter oe eg $12a$ = 126 or $\frac{1}{2} \times 3 \times a + 4 \times a + \frac{1}{2} \times 1 \times a$ (= 63)
	10.5 or $10\frac{1}{2}$ or $\frac{21}{2}$	A1	
	Additional Guidance		
	M1 is for a full area calculation (= 63)		

12(a)	Valid common denominator with at least one numerator correct	M1	eg $\frac{7x}{9x^2}$ and $\frac{a}{9x^2}$ or $\frac{7x+a}{9x^2}$ or $\frac{b}{9x\times 3x^2}$ and $\frac{2\times 9x}{9x\times 3x^2}$ numerators and denominators may be seen as products a can be numerical or algebraic b can be numerical or algebraic
	Valid common denominator with both numerators correct	M1dep	$\frac{7x}{9x^2}$ and $\frac{6}{9x^2}$ or $\frac{7\times 3x^2}{9x\times 3x^2}$ and $\frac{2\times 9x}{9x\times 3x^2}$ numerators and denominators may be seen as products
	$\frac{7x+6}{9x^2}$ or $\frac{7x+6}{(3x)^2}$ with no further work	A1	
	Additional Guidance		
	$\frac{21x^2+18x}{27x^3}$ or $\frac{21x+18}{27x^2}$ or $\frac{7x^2+6x}{9x^3}$		M2A0
	$\frac{7x^{-1}+6x^{-2}}{9}$		M2A0
	$7x + 6 / 9x^2$		M2A0

12(b)	Changes division to multiplication and inverts to $\frac{3x+12}{x^2}$	M1	may be implied
	$(3x + 12 =) 3(x + 4)$	M1	may be implied
	Correct expression written as a single fraction or a product must have factor $(x + 4)$ in a numerator and denominator $x + 4$ or correct expression written as a single fraction or a product must have denominator x^3 or x^2 or x or 1	A1	may be implied by final A1 eg $\frac{3x(x+2)(x+4)}{x+4}$ or $\frac{(3x^2+6x)(x+4)}{x+4}$ or $\frac{x}{x+4} \times \frac{x+2}{1} \times 3(x + 4)$ or $\frac{x}{x+4} \times 3(x + 2)(x + 4)$ or $\frac{3x^4(x+2)}{x^3}$ or $x^4 \times \frac{x+2}{x} \times \frac{3}{x^2}$ or $\frac{(x+2)}{x^3} \times 3x^4$ or $\frac{3x^3(x+2)}{x^2}$ or $\frac{3x^2(x+2)}{x}$ or $\frac{3x(x+2)}{1}$ or $x \times (x + 2) \times 3$ or $3x(x + 2)$
	$3x^2 + 6x$	A1	SC2 $\frac{x(x+2)(3x+12)}{x+4}$
	Additional Guidance		
	The list of examples in the first A1 is not exhaustive		
	$3x^2 + 6x$ with no incorrect working		4 marks

14(a)	$6pq^2r(2q - 3r + 4)$	B2	B1 correct factorised expression with a common factor involving at least two variables eg $pq(12q^2r - 18qr^2 + 24qr)$ or $2q^2r(6pq - 9pr + 12p)$ or common factor $6pq^2r$ with two out of the three terms in the bracket correct eg $6pq^2r(2q - 3r + 4p)$
	Additional Guidance		

14(b)	Correct factorised expression with a common factor	M1	eg $(y+3)[6(y+3)^4+4(y+3)^3]$ or $2[3(y+3)^5+2(y+3)^4]$ or $2(y+3)^2[3(y+3)^3+2(y+3)^2]$
	$2(y+3)^4[3(y+3)+2]$ or $2(y+3)^4(3y+9+2)$ or $(y+3)^4[6(y+3)+4]$ or $(y+3)^4(6y+18+4)$ or $(y+3)^4(6y+22)$	A1	
	$2(y+3)^4(3y+11)$	A1	
	Additional Guidance		

14(c)	$3(4+5x)(4-5x)$ or $3(-4-5x)(5x-4)$ or $-3(4+5x)(5x-4)$ or $-3(-4-5x)(4-5x)$	B2	B1 Partial factorisation eg $3(16-25x^2)$ or $-3(25x^2-16)$ or $(12+15x)(4-5x)$ or $(12-15x)(4+5x)$
	Additional Guidance		
	Brackets in either order for B2 or B1		
	$-(75x^2-48)$		B0
	$(-5x+4)$ is equivalent to $(4-5x)$ etc		
	Incorrect notation eg $(4+5x)3(4-5x)$		B1

16	**Alternative method 1**		
	$2(2-5x)+3(3x-1)$ or $4-10x$ or $9x-3$	M1	
	$4-10x+9x-3=1-x$	M1dep	
	$(1-x)^2=1-2x+x^2$	A1	must see working for M2
	$2-5x+3x-1+x^2=1-2x+x^2$	B1	
	Alternative method 2		
	$4(2-5x)^2+6(2-5x)(3x-1)$ $+6(2-5x)(3x-1)+9(3x-1)^2$	M1	oe allow $+12(2-5x)(3x-1)$ for $+6(2-5x)(3x-1)+6(2-5x)(3x-1)$
	$4(4-10x-10x+25x^2)$ $+6(6x-2-15x^2+5x)$ $+6(6x-2-15x^2+5x)$ $+9(9x^2-3x-3x+1)$ $=16-40x-40x+100x^2+36x-12$ $-90x^2+30x+36x-12-90x^2$ $+30x+81x^2-27x-27x+9$	M1dep	oe must see expansions must see working for 1st M1 allow $+12(6x-2-15x^2+5x)$ for $+6(6x-2-15x^2+5x)$ $+6(6x-2-15x^2+5x)$
	$1-2x+x^2$	A1	must see working for M2
	$2-5x+3x-1+x^2=1-2x+x^2$	B1	

16	**Alternative method 3**		
	$2(2-5x)+3(3x-1)$ or $4-10x$ or $9x-3$	M1	oe
	$(4-10x+9x-3)^2$ $=16-40x+36x-12-40x+100x^2$ $-90x^2+30x+36x-90x^2+81x^2$ $-27x-12+30x-27x+9$	M1dep	oe must see expansions
	$1-2x+x^2$	A1	must see working for M2
	$2-5x+3x-1+x^2=1-2x+x^2$	B1	

17(a)	$(-5)^2+2^2=29$	B1	oe involving use of –5 and 2 eg $(-5-0)^2+(2-0)^2=29$ or $(0--5)^2+(0-2)^2=29$ or $\sqrt{(-5)^2+2^2}=\sqrt{29}$ or $29-(-5)^2=2^2$ or $29-2^2=(-5)^2$ or $\sqrt{29-(-5)^2}=2$ or $\sqrt{29-2^2}=-5$

17(b)	**Alternative method 1** Using gradients		
	(gradient OP =) $\frac{2-0}{-5-0}$ or $-\frac{2}{5}$ or –0.4	M1	oe may be implied eg $y = -\frac{2}{5}x$ or gradient of tangent = $\frac{5}{2}$ (with gradient OP not seen)
	(gradient tangent =) $\frac{-1}{\text{their } -\frac{2}{5}}$ or $\frac{5}{2}$ or 2.5	M1	oe correct or ft their $-\frac{2}{5}$
	$y - 2 = \text{their } \frac{5}{2}(x - -5)$ or $0 - 2 = \text{their } \frac{5}{2}(x - -5)$ or $2 = \text{their } \frac{5}{2} \times -5 + c$	M1dep	oe dep on 2nd M1 equation of their tangent with or without substitution of y = 0 implied by $y = \frac{5}{2}x + \frac{29}{2}$ oe or $0 = \frac{5}{2}x + \frac{29}{2}$ oe
	$-\frac{29}{5}$ or –5.8	A1	oe allow $\left(-\frac{29}{5}, 0\right)$ SC2 answer –10 (grad tangent = $\frac{2}{5}$) SC2 answer $-\frac{21}{5}$ or –4.2 oe (grad tangent = $-\frac{5}{2}$)

	Alternative method 2 Using similar triangles (see diagram in Additional Guidance)		
	$\frac{a}{2} = \frac{2}{5}$	M1	oe equation any letter
	$a = \frac{2}{5} \times 2$ or $a = \frac{4}{5}$	M1dep	
	–5 – their $\frac{4}{5}$	M1dep	dep on M2
	$-\frac{29}{5}$ or –5.8	A1	oe allow $\left(-\frac{29}{5}, 0\right)$ SC2 answer –10 (grad tangent = $\frac{2}{5}$) SC2 answer $-\frac{21}{5}$ or –4.2 oe (grad tangent = $-\frac{5}{2}$)

23	$(a^2 + 3) \times k$ or $ka^2 + 3k$	M1	oe eg $b = ka^2 + 3k$ may be seen on diagram
	$(ak)^2 + 3$ or $a^2k^2 + 3$	M1	oe eg $b = a^2k^2 + 3$ may be seen on diagram
	$ka^2 + 3k = a^2k^2 + 3$	M1dep	oe equates and expands brackets correctly dep on M2 may include $-b$ on each side
	$a^2(k - k^2) = 3 - 3k$ or $ka^2(1 - k) = 3 - 3k$ or $ka^2 - a^2k^2 = 3(1 - k)$ or $a^2(k - k^2) = 3(1 - k)$ or $ka^2(1 - k) = 3(1 - k)$ or $a^2(k^2 - k) = 3k - 3$ or $ka^2(k - 1) = 3k - 3$ or $k^2a^2 - ka^2 = 3(k - 1)$ or $a^2(k^2 - k) = 3(k - 1)$ or $ka^2(k - 1) = 3(k - 1)$	M1dep	oe eg $(a^2 =) \frac{3 - 3k}{k - k^2}$ or $(a =) (\pm)\sqrt{\frac{3 - 3k}{k - k^2}}$ or $(a^2 =) \frac{3k - 3}{k^2 - k}$ or $(a =) (\pm)\sqrt{\frac{3k - 3}{k^2 - k}}$ collects terms in a^2 and factorises correctly on at least one side must use a^2 as a factor if awarding mark for factorising $ka^2 - a^2k^2$ dep on M3
	$(a^2 =) \frac{3(1-k)}{k(1-k)}$ or $(a^2 =) \frac{3}{k}$ or $(a =) (\pm)\sqrt{\frac{3(1-k)}{k(1-k)}}$	M1dep	oe eg $(a^2 =) \frac{3(k-1)}{k(k-1)}$ correct fraction with numerator and denominator factorised correctly dep on M4
	$(a =) \sqrt{\frac{3}{k}}$ or $(a =) \left(\frac{3}{k}\right)^{\frac{1}{2}}$	A1	oe eg $(a =) \frac{\sqrt{3}}{\sqrt{k}}$ or $(a =) \left(\frac{k}{3}\right)^{-\frac{1}{2}}$ $(a =) \pm\sqrt{\frac{3}{k}}$ M5A0 $\quad$ $(a =) -\sqrt{\frac{3}{k}}$ M5A0

	Alternative method 1 Powers of 3		
	$(3^2)^{0.5p}$ or $(3^3)^{2p-1}$ or $3^{2\times0.5p+4}$	M1	oe powers of 3 eg 3^p or 3^{6p-3} or 3^{p+4} brackets not needed if intention clear eg $3^{2^{0.5p}}$
	$(3^2)^{0.5p}$ and 3^4 and $(3^3)^{2p-1}$ or $3^{2\times0.5p+4}$ and $(3^3)^{2p-1}$	M1dep	oe powers of 3 eg 3^p and 3^4 and 3^{6p-3} or 3^{p+4} and 3^{6p-3}
	$2\times0.5p+4=3(2p-1)$ or $p+4=6p-3$	M1dep	oe equation dep on M2
24	1.4 or $\frac{7}{5}$	A1	oe
	Alternative method 2 Powers of 9		
	$9^{0.5p+2}$ or $(9^{1.5})^{2p-1}$	M1	oe power of 9 eg $9^{3p-1.5}$ brackets not needed if intention clear eg $9^{1.5^{2p-1}}$
	9^2 and $(9^{1.5})^{2p-1}$ or $9^{0.5p+2}$ and $(9^{1.5})^{2p-1}$	M1dep	oe powers of 9 eg 9^2 and $9^{3p-1.5}$ or $9^{0.5p+2}$ and $9^{3p-1.5}$
	$0.5p+2=1.5(2p-1)$ or $0.5p+2=3p-1.5$	M1dep	oe equation dep on M2
	1.4 or $\frac{7}{5}$	A1	oe

24	**Alternative method 3** Powers of 27		
	$\left(27^{\frac{2}{3}}\right)^{0.5p}$	M1	oe power of 27 eg $27^{\frac{2}{3}\times 0.5p}$ or $27^{\frac{1}{3}p}$ brackets not needed if intention clear eg $27^{\frac{2}{3}0.5p}$
	$\left(27^{\frac{2}{3}}\right)^{0.5p}$ and $27^{\frac{4}{3}}$	M1dep	oe powers of 27 eg $27^{\frac{2}{3}\times 0.5p}$ and $27^{\frac{4}{3}}$ or $27^{\frac{1}{3}p}$ and $27^{\frac{4}{3}}$ M2 $27^{\frac{2}{3}\times 0.5p+\frac{4}{3}}$ or $27^{\frac{1}{3}p+\frac{4}{3}}$
	$\frac{2}{3}\times 0.5p+\frac{4}{3}=2p-1$ or $\frac{1}{3}p+\frac{4}{3}=2p-1$	M1dep	oe equation dep on M2
	1.4 or $\frac{7}{5}$	A1	oe

24	**Alternative method 4** Powers of 81		
	$(81^{0.5})^{0.5p}$ or $(81^{0.75})^{2p-1}$ or $81^{0.5\times 0.5p+1}$	M1	oe powers of 81 eg $81^{0.25p}$ or $81^{1.5p-0.75}$ or $81^{0.25p+1}$ brackets not needed if intention clear eg $81^{0.5^{0.5p}}$
	$(81^{0.5})^{0.5p}$ and $(81^{0.75})^{2p-1}$ or $81^{0.5\times 0.5p+1}$ and $(81^{0.75})^{2p-1}$	M1dep	oe powers of 81 eg $81^{0.25p}$ and $81^{1.5p-0.75}$ or $81^{0.25p+1}$ and $81^{1.5p-0.75}$
	$0.5\times 0.5p+1=0.75(2p-1)$ or $0.25p+1=1.5p-0.75$	M1dep	oe equation dep on M2
	1.4 or $\frac{7}{5}$	A1	oe

June 2019 Paper 2

3 $-2 < a < 0$ and $-1 < b < 1$

Tick the correct box for each statement.

[4 marks]

	Always true	**Sometimes true**	**Never true**
$a^2 < 0$	☐	☐	☐
$-1 < b^3 < 1$	☐	☐	☐
$\frac{b}{a} < 0$	☐	☐	☐
$a - b > 0$	☐	☐	☐

8 (a) A linear sequence has first term $7 + 12\sqrt{5}$

The term-to-term rule is

add $9 - 2\sqrt{5}$

One term of the sequence is an integer.

Work out the value of this integer.

[2 marks]

Answer _____

8 (b) The nth term of a different sequence is $\dfrac{3n^2 - 1}{n^2 + 1}$

Work out the sum of the first three terms.

[2 marks]

9 Factorise fully $(p + 6)^{11} - (p + 6)^{10}$

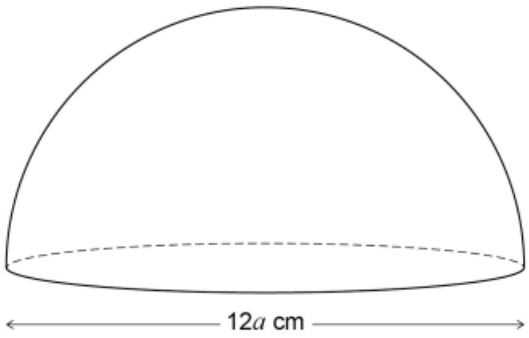
[2 marks]

Answer _____

12 The diagram shows a solid hemisphere.

The diameter is $12a$ cm

The volume is 486π cm^3



Work out the value of a.

[3 marks]

Answer _____

15 Rearrange $m = \frac{2p+1}{p} + \frac{p+5}{3p}$ to make p the subject.

[4 marks]

Answer _____

16 The curve $y = 2\sqrt{x-a} + 5$ passes through the point (1, 8)

Work out the value of a.

[3 marks]

Answer _____

17 Show that $(x + 1)(x + 3)(x + 4) - x(x^2 + 7x + 11)$

can be written in the form $(x + a)(x + b)$ where a and b are positive integers.

[5 marks]

18 Solve $4(x - 5)^2 = k^2$ where k is a constant.

Give your answers in their simplest form in terms of k.

[3 marks]

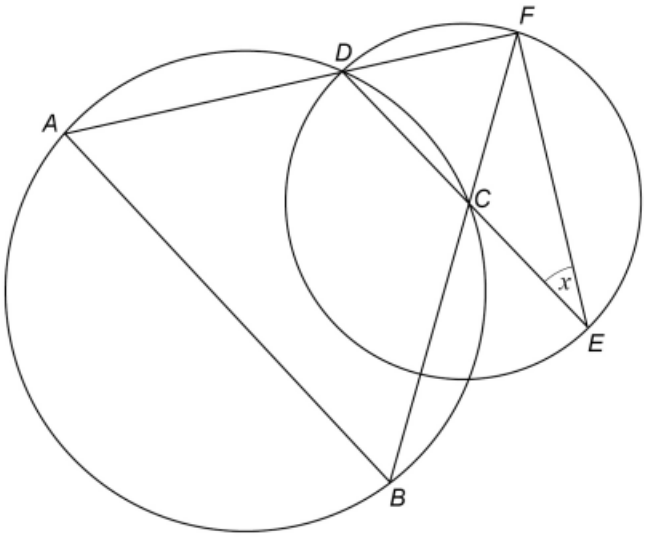
Answer _____

20 *A*, *B*, *C* and *D* are points on a circle.

D, *E* and *F* are points on a different circle, centre *C*.

DCE, *ADF* and *BCF* are straight lines.

angle *DEF* = x



Not drawn accurately

20 (a) Prove that angle *BAD* = $2x$

[3 marks]

20 (b) In the case when *AB* is parallel to *DE*, work out the size of angle x.

[2 marks]

Answer _____ degrees

3	$a^2 < 0$ ☐ ☐ ✓ $-1 < b^3 < 1$ ✓ ☐ ☐ $\frac{b}{a} < 0$ ☐ ✓ ☐ $a - b > 0$ ☐ ✓ ☐	B4	B1 for each correct row
	Additional Guidance		
	Two boxes ticked in a row with other 3 rows fully correct		B3

8(a)	$7 + 12\sqrt{5} + 6(9 - 2\sqrt{5})$ or $12\sqrt{5} + 6(-2\sqrt{5}) = 0$ or $12\sqrt{5} \div 2\sqrt{5} = 6$ or states that need to add 6 lots of $(9 - 2\sqrt{5})$ or 7th term	M1	oe eg $7 + 6 \times 9$ or $7 + 54$ or $6 \times -2 = -12$ allow $7 + 12\sqrt{5} + (n - 1)(9 - 2\sqrt{5})$ with $n = 7$ allow $7 + 12\sqrt{5} + n(9 - 2\sqrt{5})$ with $n = 6$
	61	A1	

8(b)	$\frac{29}{5}$ or $5\frac{4}{5}$ or 5.8	B2	oe eg $5\frac{8}{10}$ B1 any two of 1, $\frac{11}{5}$, $\frac{26}{10}$ oe values
	Additional Guidance		
	Terms must be evaluated for B1 unless correct answer seen eg1 $\frac{3-1}{1+1} + \frac{12-1}{4+1} + \frac{27-1}{9+1}$		B0

8(c)	**Alternative method 1**		
	(Second differences =) 4 or $2n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$-3 - 2 \quad 3 - 8 \quad (13 - 18 \quad 27 - 32)$ or $-5 \quad -5 \quad (-5 \quad -5)$	M1dep	subtracts $2n^2$ from the given terms
	$2n^2 - 5$	A1	oe eg $2n^2 + 0n - 5$ does not need terms collected
	Alternative method 2		
	(Second differences =) 4 or $2n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$3a + b = 3 - -3$ and substitutes $a = 2$ or $b = 0$	M1dep	oe
	$2n^2 - 5$	A1	oe eg $2n^2 + 0n - 5$ does not need terms collected
	Alternative method 3		
	Any three of $a + b + c = -3$ $4a + 2b + c = 3$ $9a + 3b + c = 13$ $16a + 4b + c = 27$	M1	
	$3a + b = 3 - -3$ and $5a + b = 13 - 3$ or $a = 2$ and $b = 0$	M1dep	oe obtains two correct equations in same two variables from their equations
	$2n^2 - 5$	A1	oe eg $2n^2 + 0n - 5$ does not need terms collected
8(c) cont	**Alternative method 4**		
	(Second differences =) 4 or $2n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$2 \times 1^2 + b \times 1 - 5 = -3$ or $2 + b - 5 = -3$ or $b = 0$	M1dep	$2n^2 + bn - 5 = -3$ with $n = 1$ substituted oe eg $2 \times 2^2 + b \times 2 - 5 = 3$
	$2n^2 - 5$	A1	oe eg $2n^2 + 0n - 5$ does not need terms collected

9	$(p+6)^{10}(p+5)$ or $(p+5)(p+6)^{10}$	B2	B1 $(p+6)^{10}(p+6-1)$ or states $x = p+6$ and $x^{10}(x-1)$ (any letter for x other than p) or correct partial factorisation eg $(p+6)[(p+6)^{10}-(p+6)^{9}]$ or $(p+6)^{2}[(p+6)^{9}-(p+6)^{8}]$
	Additional Guidance		
	Any shape of bracket may be used		
	$(p+6)^{10}((p+6)-1)$		B1
	Missing brackets must be recovered eg $p+5\,(p+6)^{10}$ not recovered and B1 response not seen		B0

12	**Alternative method 1**		
	$\frac{1}{2} \times \frac{4}{3} \times \pi \times (6a)^3$ or $\frac{2}{3} \times \pi \times 216a^3$ or $144\pi a^3$	M1	oe eg $\frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{12a}{2}\right)^3$ or $\frac{2}{3} \times \pi \times (6a)^3$
	$a^3 = \frac{486\pi}{144\pi}$ or $a^3 = \frac{27}{8}$ or $a^3 = 486 \div \left(\frac{2}{3} \times 6^3\right)$ or $a^3 = 3.375$ or $\sqrt[3]{3.375}$	A1	oe equation of form $a^3 =$ or calculation allow $(6a)^3 = 729$ or $6a = 9$
	$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	SC1 answer 0.75 oe or answer 1.19… or answer 4.95…

12 cont	**Alternative method 2**		
	$r^3 = \frac{486\pi}{\frac{2}{3}\pi}$ or $r^3 = 729$ or $\sqrt[3]{729}$ or 9	M1	oe equation of form $r^3 =$ or calculation
	$6a = \sqrt[3]{\frac{486\pi}{\frac{2}{3}\pi}}$ or $6a = 9$ or $9 \div 6$	A1	oe equation or calculation allow $(6a)^3 = 729$
	$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	SC1 answer 0.75 oe or answer 1.19… or answer 4.95…

15	**Alternative method 1**		
	$3mp = 3(2p + 1) + p + 5$ or $(m =) \frac{3(2p+1)}{3p} + \frac{p+5}{3p}$ or $(m =) \frac{6p+3+p+5}{3p}$	M1	oe fractions eliminated or common denominator eg $(m =) \frac{3p(2p+1)}{3p^2} + \frac{p(p+5)}{3p^2}$ or $(m =) \frac{6p^2+3p+p^2+5p}{3p^2}$
	$3mp = 6p + 3 + p + 5$ or $3mp = 7p + 8$	M1dep	oe brackets expanded and fractions eliminated eg $3mp^2 = 7p^2 + 8p$ implies M2
	$3mp - 7p = 8$ or $\frac{8}{3m-7}$ or $\frac{-8}{7-3m}$	M1dep	oe terms collected eg $p(3m - 7) = 8$ or $7p - 3mp = -8$ implies M3
	$p = \frac{8}{3m-7}$ or $p = \frac{-8}{7-3m}$	A1	oe eg $\frac{8}{3m-7} = p$

15 cont	**Alternative method 2**		
	$(m =) \frac{3(2p+1)}{3p} + \frac{p+5}{3p}$ or $(m =) \frac{6p+3+p+5}{3p}$	M1	oe common denominator eg $(m =) \frac{3p(2p+1)}{3p^2} + \frac{p(p+5)}{3p^2}$ or $(m =) \frac{6p^2+3p+p^2+5p}{3p^2}$
	$m = \frac{7p+8}{3p}$ and $m = \frac{7}{3} + \frac{8}{3p}$ and $m - \frac{7}{3} = \frac{8}{3p}$	M1dep	simplifies numerator and isolates term in p eg $m = \frac{7p^2+8p}{3p^2}$ and $m = \frac{7}{3} + \frac{8}{3p}$ and $m - \frac{7}{3} = \frac{8}{3p}$ implies M2
	$\frac{3m-7}{3} = \frac{8}{3p}$	M1dep	converts $m - \frac{7}{3}$ to a single fraction implies M3
	$p = \frac{8}{3m-7}$ or $p = \frac{-8}{7-3m}$	A1	oe eg $\frac{8}{3m-7} = p$

16	$\frac{8-5}{2} = \sqrt{1-a}$ or $\frac{3}{2} = \sqrt{1-a}$ or $3^2 = 2^2(1-a)$ or $9 = 4(1-a)$	M1	
	$1-a = \left(\frac{3}{2}\right)^2$ or $1-a = \frac{9}{4}$ or $9 = 4-4a$ or $\frac{4-9}{4}$	M1dep	oe equation or calculation eg $1-a = \left(\frac{8-5}{2}\right)^2$ or $1-a = 2.25$ or $\frac{9-4}{-4}$ implies M2
	$-\frac{5}{4}$ or -1.25 or $-1\frac{1}{4}$	A1	

17	$x^2 + 3x + x + 3$ with three terms correct or $x^2 + 4x + k$ where k is a non-zero constant	M1	oe expansion attempt of one pair of brackets eg1 $x^2 + 4x + 3x + 12$ with three terms correct or $x^2 + 7x + k$ where k is a non-zero constant eg2 $x^2 + 4x + x + 4$ with three terms correct or $x^2 + 5x + k$ where k is a non-zero constant
	$x^3 + 3x^2 + x^2 + 3x$ or $x^3 + 4x^2 + 3x$ or $4x^2 + 12x + 4x + 12$ or $4x^2 + 16x + 12$	M1dep	attempt at a full expansion with correct multiplication of their 3 or 4 terms by one of the terms in the remaining bracket oe eg $x^3 + 4x^2 + 3x^2 + 12x$ or $x^3 + 7x^2 + 12x$ or $x^2 + 4x + 3x + 12$ or $x^2 + 7x + 12$ ($x^2 + 7x + 12$ must be from an attempt at a full expansion) or $x^3 + 4x^2 + x^2 + 4x$ or $x^3 + 5x^2 + 4x$ or $3x^2 + 12x + 3x + 12$ or $3x^2 + 15x + 12$
	$x^3 + 8x^2 + 19x + 12$	A1	fully correct expansion allow if terms not collected eg $x^3 + 3x^2 + x^2 + 3x + 4x^2 + 12x + 4x + 12$ or $x^3 + 4x^2 + 3x + 4x^2 + 16x + 12$
	$x^2 + 8x + 12$	A1ft	ft M2A0 full simplification of their $(x^3 + 8x^2 + 19x + 12) - x^3 - 7x^2 - 11x$ their $(x^3 + 8x^2 + 19x + 12)$ must be a cubic
	$x^2 + 8x + 12$ and $(x + 6)(x + 2)$ or $(x + 2)(x + 6)$	A1	oe product of brackets

18	**Alternative method 1**		
	$x-5=\frac{k}{2}$ or $x-5=-\frac{k}{2}$ or $2(x-5)=k$ or $2x-10=k$ or $2(x-5)=-k$ or $2x-10=-k$	M1	oe linear equation eg $x-5=\sqrt{\frac{k^2}{4}}$ or $x=\frac{k}{2}+5$ or $\sqrt{4}(x-5)=\sqrt{k^2}$
	$x-5=\frac{k}{2}$ and $x-5=-\frac{k}{2}$ or $2(x-5)=k$ and $2(x-5)=-k$ or $2x-10=k$ and $2x-10=-k$	A1	oe eg $x-5=\pm\frac{k}{2}$ square root(s) must be processed implied by final A1
	$\frac{k}{2}+5$ and $-\frac{k}{2}+5$	A1	oe simplest form eg $\frac{10+k}{2}$ and $\frac{10-k}{2}$ or $\frac{k+10}{2}$ and $\frac{k-10}{-2}$ or $5\pm0.5k$
	Alternative method 2		
	$4x^2-40x+100-k^2\ (=0)$	M1	expands and collects terms
	$\frac{--40\pm\sqrt{(-40)^2-4\times4\times(100-k^2)}}{2\times4}$	A1	oe eg $\frac{40\pm\sqrt{16k^2}}{8}$ or $\frac{40\pm4k}{8}$ implied by final A1
	$\frac{k}{2}+5$ and $-\frac{k}{2}+5$	A1	oe simplest form eg $\frac{10+k}{2}$ and $\frac{10-k}{2}$ or $\frac{k+10}{2}$ and $\frac{k-10}{-2}$ or $5\pm0.5k$

20(a)	**Alternative method 1**		
	Full method leading to angle $BCD = 180 - 2x$	M1	eg angle $CFE = x$ and angle $FCE = 180 - 2x$ and angle $BCD = 180 - 2x$
	Full reasoning for their method	A1	eg (base angles of) isosceles (triangle are equal) and (sum of) angles in a triangle (is 180) and (vertically) opposite angles
	angle $BAD = 2x$ and (opposite angles of) cyclic quadrilateral (add to 180)	A1	must see M1
	Alternative method 2 Working out angle *DCF* using angle at centre		
	angle $DCF = 2x$	M1	
	angle at centre (is double angle at circumference)	A1	
	Full method leading to angle $BAD = 2x$ and full reasoning for their method	A1	must see M1 eg angle $BCD = 180 - 2x$ and angle $BAD = 2x$ and angles on a (straight) line (add to 180) and (opposite angles of) cyclic quadrilateral (add to 180)

20(a) cont	**Alternative method 3 Working out angle *DCF* not using angle at centre**		
	Full method leading to angle $DCF = 2x$	M1	eg angle $CFE = x$ and angle $DCF = 2x$
	Full reasoning for their method	A1	eg (base angles of) isosceles (triangle are equal) and exterior angle (of triangle is sum of interior opposite angles)
	Full method leading to angle $BAD = 2x$ and full reasoning for their method	A1	must see M1 eg angle $BCD = 180 - 2x$ and angle $BAD = 2x$ and angles on a (straight) line (add to 180) and (opposite angles of) cyclic quadrilateral (add to 180)

20(a) cont	**Alternative method 4**		
	Full method leading to angle $DFC = 90 - x$ and angle $ABC = 90 - x$	M1	eg angle $CFE = x$ and angle $DFE = 90$ and angle $DFC = 90 - x$ and angle $CDF = 90 - x$ and angle $ADC = 90 + x$ and angle $ABC = 90 - x$
	Full reasoning for their method	A1	eg (base angles of) isosceles (triangle are equal) and (angle in a) semicircle (is 90) and (sum of) angles in a triangle (is 180) and angles on a (straight) line (add to 180) and (opposite angles of) cyclic quadrilateral (add to 180)
	angle $BAD = 2x$ and (sum of) angles in a triangle (is 180)	A1	must see M1

20(b)	30	B2	B1 correct equation or calculation eg $90 + 2x + x = 180$ or $90 - x = 2x$ or $3x = 90$ or $6x = 180$ or $90 \div 3$
	Additional Guidance		
	Ignore any expressions for angles and any other calculated angles		
	Ignore any reasons		