Hypothesis Testing

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Suppose we have a_board that depends on the roll of one die and attaches special importance to rolling a 6. In a particular game, the die is rolled 235 times, and 6 comes up 51 times. If the die is fair, we would expect 6 to come up 235 x 1/6 = 39.17 times.

Binomial test - Wikipedia

In statistics, the binomial test is an exact test of the statistical significance of deviations from a theoretically expected distribution of observations into two categories using sample data. The binomial test is useful to test hypotheses about the probability () of success: where is a user-defined value between 0 and 1.

W https://en.wikipedia.org/wiki/Binomial_test

Theory

Hypothesis

$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

We then set the level of significance α

Distribution

Using a Normal approximation to the Binomial we know that the random variable of the population is Normally distributed :

$X \sim N(\mu, \sigma)$

Where

$\mu = np = n_s$

$\sigma = \sqrt{np(1-p)}$

Therefore, under the null hypothesis, the standardized random variable has a Standard Normal distribution :

$H_0 : Z = \frac{\mu - \mu_0}{\sigma} \sim N(0,1)$

$H_0 : Z = \frac{\hat{x} - \mu_0}{s} \sim N(0,1)$

Since

$lim_{n \to \infty} \hat{x} = \mu$

$lim_{n \to \infty} s = \sigma$

Confidence intervals demonstration

Under the null hypothesis, the standardized variable should be contained between Standard Normal critical values :

$H_0 : -Z_{1-\alpha/2} \leq \frac{\mu-\mu_0}{\sigma} \leq Z_{1-\alpha/2}$

$H_0 : -Z_{1-\alpha/2}\sigma \leq \mu - \mu_0 \leq Z_{1-\alpha/2}\sigma$

$H_0 : -\mu - Z_{1-\alpha/2}\sigma \leq -\mu_0 \leq Z_{1-\alpha/2}\sigma - \mu$

$H_0 : \mu + Z_{1-\alpha/2}\sigma \geq \mu_0 \geq -Z_{1-\alpha/2}\sigma + \mu$

We expect the null average to fall within the confidence interval.

$H_0 : \mu - Z_{1-\alpha/2}\sigma \leq \mu_0 \leq \mu + Z_{1-\alpha/2}\sigma$

Example solved

Using the standardized Z-score

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Under the null hypothesis, the die is fair :

$$H_0 : \mu = 39$$

$$H_1 : \mu \neq 39$$

We set the level of significance $\alpha = 0.05$

$$Z_{1-\alpha/2} = 1.96$$

Recall that

$$H_0 : Z = \frac{\hat{x} - \mu_0}{s} \sim N(0,1)$$

Since

$$lim_{n \to \infty} \hat{x} = \mu$$

$$lim_{n \to \infty} s = \sigma$$

Where

$$\hat{x} = n\hat{p} = \hat{n_s}$$

$$\hat{x} = 51$$

$$s = \sqrt{n\hat{p}(1-\hat{p})}$$

$$s = \sqrt{235 \times 0.217(1 - 0.217)}$$

$$s = 6.319$$

Result

$$Z = \frac{51-39}{6.319} = 1.90$$

> The standardized variable result we observe is lower than the critical value of 1.96. We fail to reject the null hypothesis

Using the confidence intervals

📜 Recall that

...

$$H_0 : \mu - Z_{1-\alpha/2}\sigma \leq \mu_0 \leq \mu + Z_{1-\alpha/2}\sigma$$

We observe that

$$H_0 : 51 - 1.96 \times 6.319 \leq \mu_0 \leq 51 + 1.96 \times 6.319$$

$$H_0 : 38.614 \leq \mu_0 \leq 63.385$$

Result

$$38.614 \leq 39 \leq 63.385$$

> The null hypothesis is within the confidence interval. We fail to reject the null hypothesis

Reference

Test statistic - Wikipedia

A test statistic is a statistic (a quantity derived from the sample) used in statistical hypothesis testing. A hypothesis test is typically specified in terms of a test statistic, considered as a numerical summary of a data-set that reduces the data to one value that can be used to perform the hypothesis test.

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