A Magic Square of Squares

		I

2		
		1

2		
		1
	3	

2		
		1
4	3	

2		
	5	1
4	3	

2		6
	5	1
4	3	

15

2		6
	5	1
4	3	

15 Magic number!

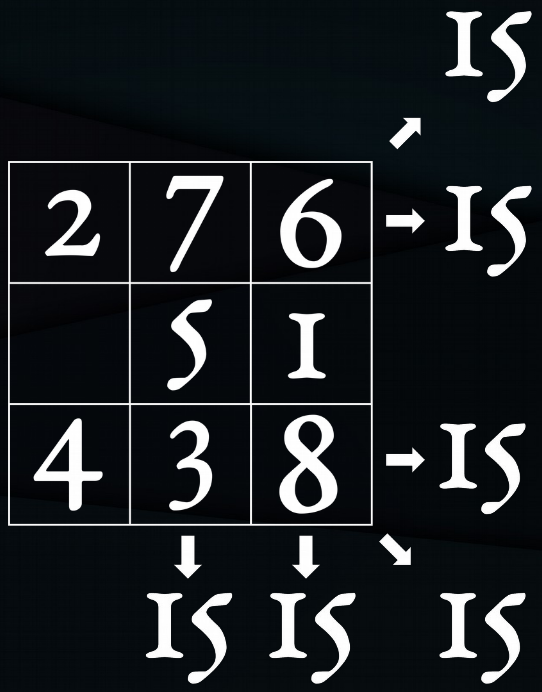
2		6
	5	1
4	3	

2	7	6
	5	1
4	3	

15

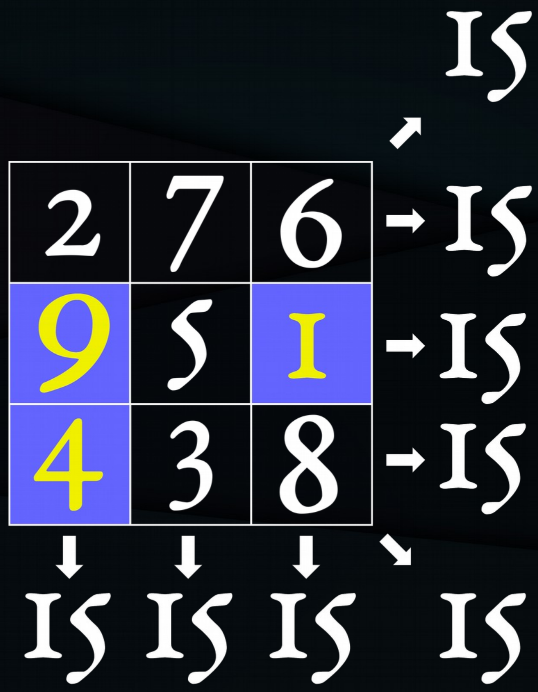
15

15


15
2 7 6 15
5 1
4 3 8 15
15 15 15

			15
2	7	6	15
9	5	1	15
4	3	8	15
15	15	15	15

A 3 x 3 magic square of unique integers with 3 perfect squares.



- Finding a magic square of squares is hard, maybe impossible.
    - but one with seven squares was found.

Found by
Dr Andrew Bremner
in 1997.

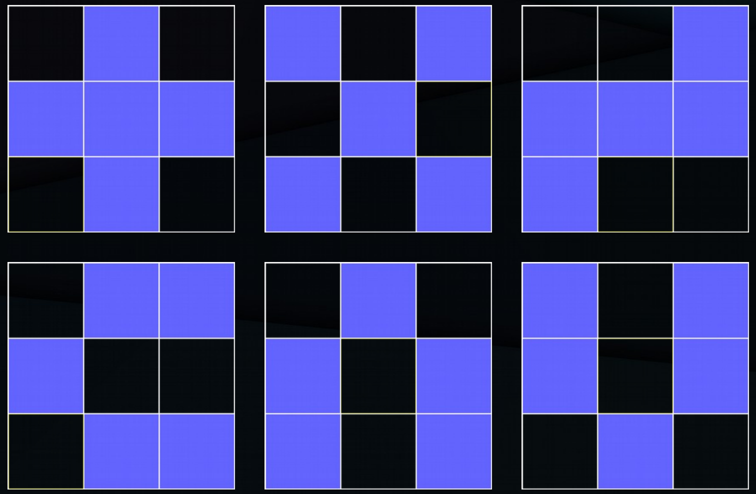
222121	23^2	565^2
527^2	425^2	289^2
205^2	360721	373^2

- The search has been for another Seven, but no other has been found.
- There are plenty of Sixes.
- And we make Sixes by making Fives.

Making Fives

- Use parametric formulas on pairs of *co-primes*.
- *M,N,P,R*

Dr Lee Morgenstern



Overlapping progressions of 3 perfect squares

Making Sixes

- Fives are ubiquitous, Sixes are rare, and the enigma is if there is only one Seven.
- Make Fives and hope another perfect square shakes out.
- Can we increase the likelihood of finding more Sixes?

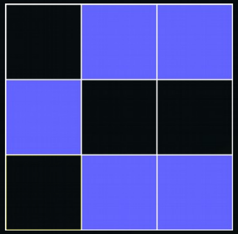
 (and thereby having more chances at finding a Seven.)

The Plan

- Use pairs of primes for *M,N,P,R*.
- Try every pair of primes up to the first 1000 primes.
- Record which pairs generate Sixes.

e.g. $M=5$ $N=43$ $P=2$ $R=5$

(2254^2 is the overlapping number)



4269576676	92414^2	1874^2
2254^2	64354^2	8537210116
8538778756	1394^2	65366^2

There is structure!

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This square just happened to shake out.

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8538778756	1394^2	65366^2

Magic number / 3 !

The Plan

- Generate squares from the first 1000 primes.

  throw away the Fives and Sevens (if any.)

- Determine which *M,N,P,R* Mod 10, finds the most Sixes.

  check which modulo finds more.

- Also determine which one finds the least, and also pick a middle one.

  best, middle & worst candidates.

The Plan

- Count which *M,N,P,R* mod 10 make the most/least progressions of 3 perfect squares. We dislike multiples!
- For example, maybe our best candidate is 1,7,7,1

  generate the Fives again, but this time with the first 1000 primes that are 1 mod 10 for *M*, 7 mod 10 for *N*, and so on.

- Our *M,N,P,R* values will be larger.

  maybe we'll find a bigger Six.

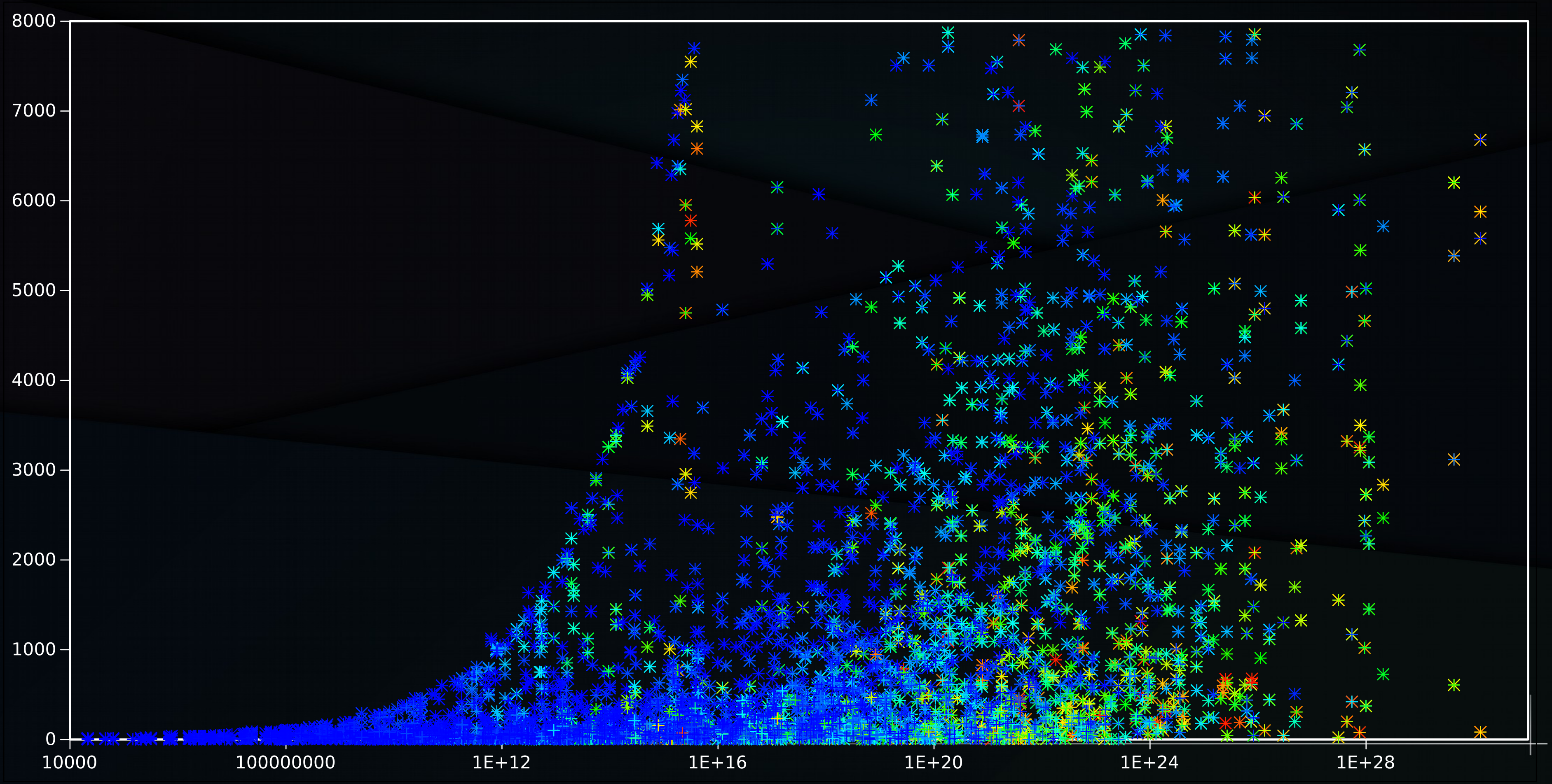
The QUESTION

- Can we use this strategy as a predictor of finding more Sixes?
- Hypothesis: We will generate more Sixes with our best candidate than with our worst.
- The Nagging Doubt: Sixes get rarer as *M,N,P,R* gets higher, so maybe we can't tell a) that it's random, or b) there's some other structure that lower values don't shed light on.

Results of Run #1

- We found 1534 Sixes.
- With modulo 10…
  - The best candidate is: 1, 3, 7, 9
  - The middle candidate is: 1, 3, 1, 7
  - The worst candidate is: 9, 9, 7, 1
- The best modulo is actually 6 with 5, 1 , 5, 1.

Distribution of Sixes in Run #1



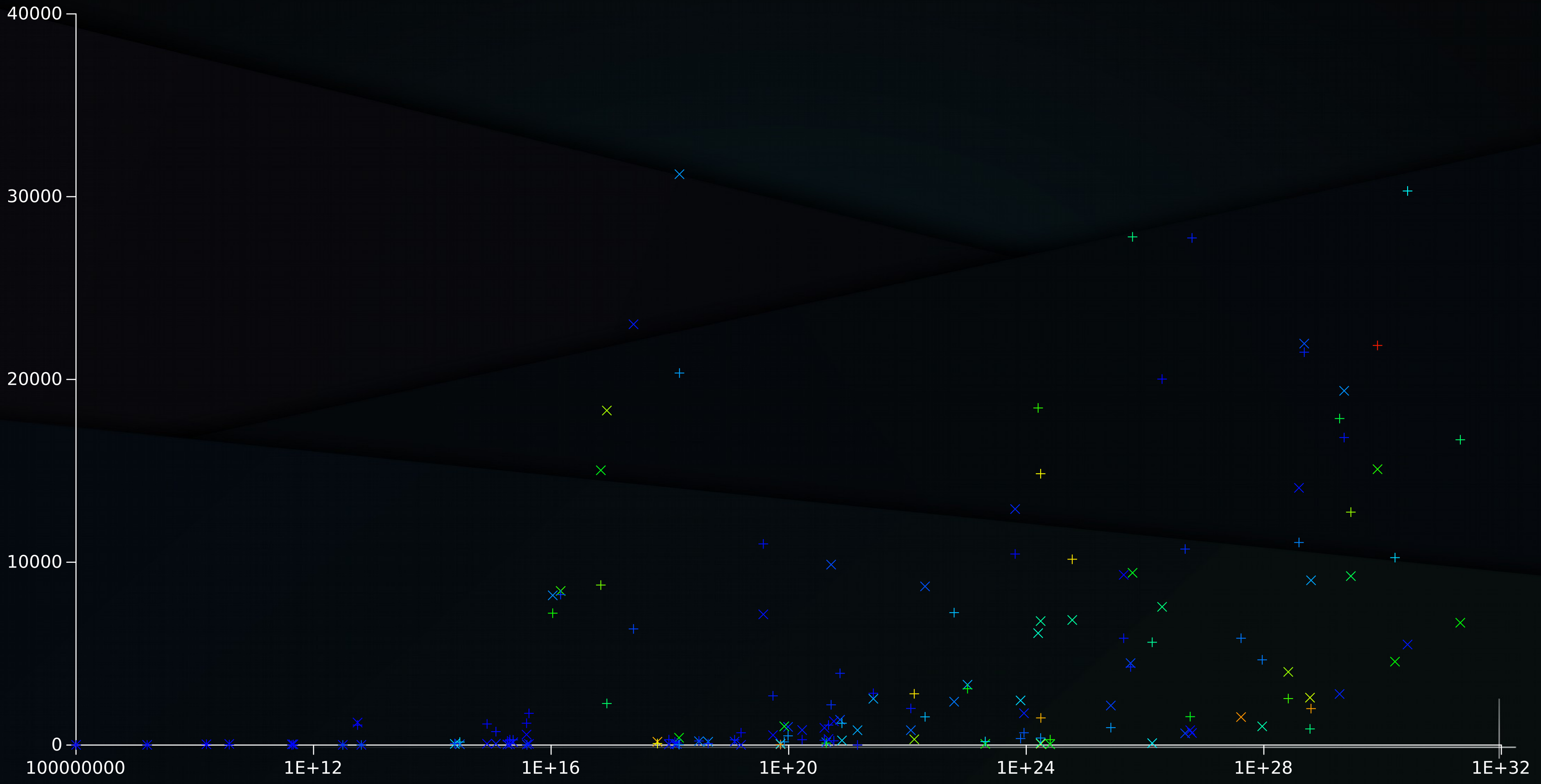
Results of Run #2

	Sixes	3sq prog.
Best (1,3,7,9)	98	210
Faux-Best (1,7,7,1)	77	147
Middle (1,3,1,7)	120	236
Worst (9,9,7,1)	19	25

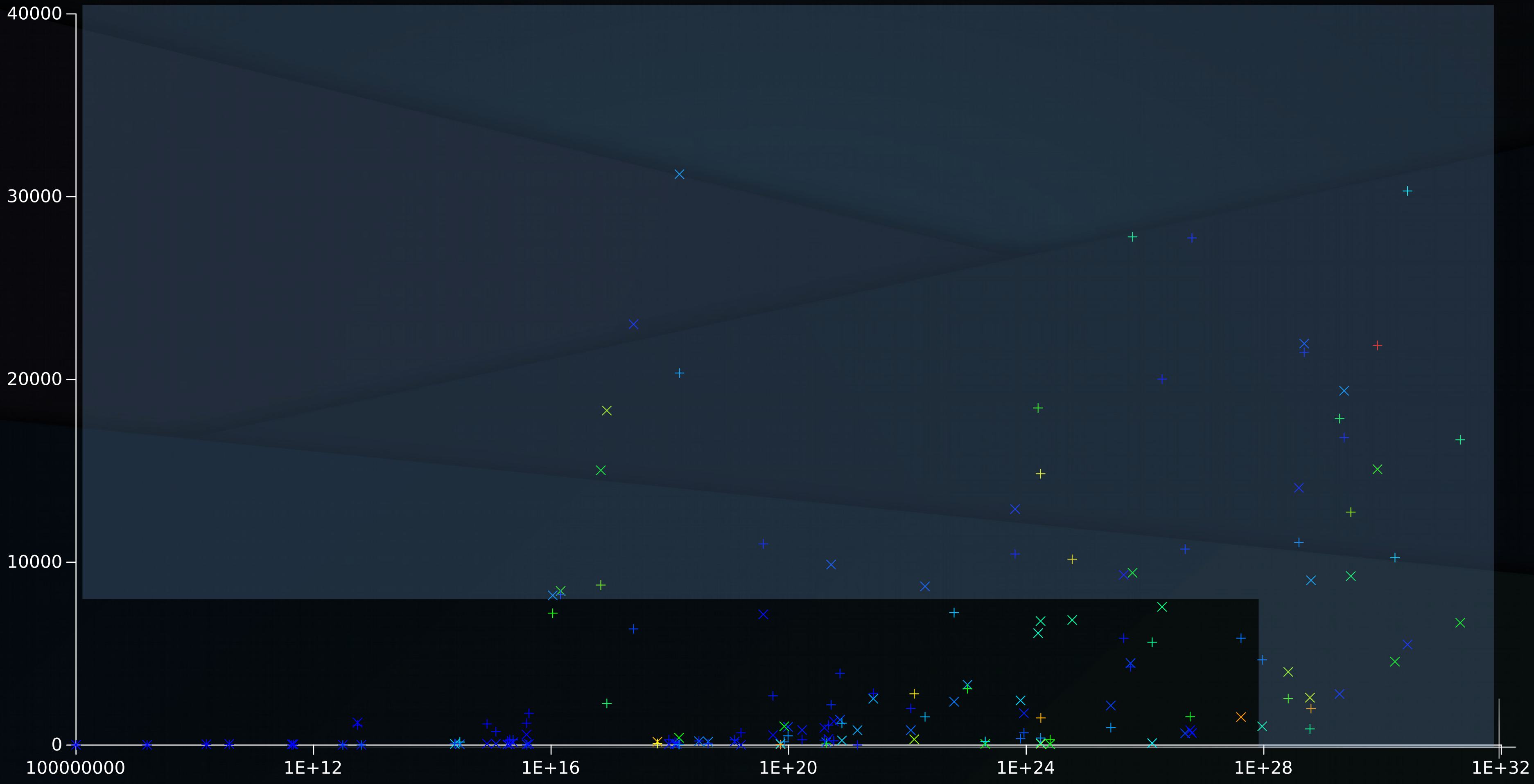
The ANSWER to the QUESTION

- Can we use this strategy as a predictor of finding more Sixes?
  - NO

Results of Run #2 (middle)



Results of Run #2 (middle, new)



What's Next?

- Check for candidates with M, N, P, R all > 100, > 1000.
- Look for candidates in 1, 3, 1, 7 with mod 100.
- Check for patterns in the *indices* of mod 10 primes.