Experiment 2: Bernoulli's Equation

Key concept: Validity of Bernoulli Equation

Refer to: Roberson, Crowe & Elger, 7th ed., Chapter 5, pp. 158-180

I. Introduction

This lab exercise tests the validity of the **Bernoulli equation**, one of the most widely used, and misused, equations in the analysis of fluid flow. The Bernoulli equation is derived from application of Newton's Second Law to a differential fluid element aligned with a streamline. It relates the pressure, elevation, and velocity between any two points on a streamline in an inviscid (ideal), constant density fluid flowing at steady state.

Consider a fluid element aligned *along a streamline*. A streamline is a line drawn in the flow field in such a manner that the velocity vector at each and every point on the streamline is tangent to the streamline at any instant.

In Figure 8, s is the coordinate along the streamline, Δh is the vertical height of the fluid particle, $p(s, t)$ is the pressure, $V(s, t)$ is the velocity of the fluid, W is the weight of the fluid particle, ΔA is the cross-sectional area of the fluid particle (normal to the paper and the streamline coordinate), and θ is the angle the fluid particle makes with the horizontal.

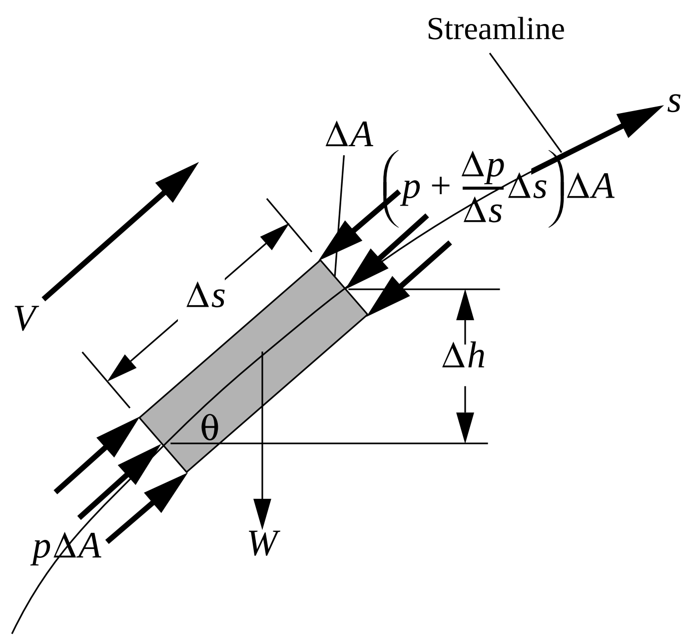

Figure 8. Differential fluid element oriented along a streamline

First, assume *the fluid is inviscid* (i.e., has no viscosity). How would this derivation differ if the fluid was viscous? Application of Newton's Second Law along a streamline yields:

$$\sum F_s = ma_s \tag{1}$$

where F_s are the external forces acting along the streamline, m is the mass of the fluid particle and a_s is the acceleration of the fluid particle along the streamline. Substitution of quantities from Figure 8 yields:

$$p\Delta A - \left(p + \frac{\Delta p}{\Delta s}\Delta s\right)\Delta A - \gamma\Delta s\Delta A\sin\theta = \rho\Delta s\Delta A a_s \tag{2}$$

where γ and ρ are the specific weight and density of the fluid, respectively. Furthermore, Figure 8 shows that $\sin\theta = \Delta h/\Delta s$. Algebraic manipulation yields:

$$-\frac{\Delta p}{\Delta s} - \gamma\frac{\Delta h}{\Delta s} = \rho a_s \tag{3}$$

In the limit, as Δs approaches zero:

$$-\frac{\partial p}{\partial s} - \gamma\frac{\partial h}{\partial s} = \rho a_s \tag{4}$$

Given that velocity is a function of time and position along the streamline ($V(t, s)$), acceleration along the streamline is (by chain rule):

$$a_s = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \tag{5}$$

For *steady flow*, (5) reduces to:

$$a_s = V\frac{\partial V}{\partial s} \tag{6}$$

Introduction of (6) into (4) yields:

$$-\frac{\partial p}{\partial s} - \gamma\frac{\partial h}{\partial s} = \rho V\frac{\partial V}{\partial s} \tag{7}$$

Use of the product rule, $V\frac{\partial V}{\partial s} = \frac{1}{2}\frac{\partial V^2}{\partial s}$, yields:

$$-\frac{\partial p}{\partial s} - \gamma\frac{\partial h}{\partial s} = \frac{\rho}{2}\frac{\partial V^2}{\partial s} \tag{8}$$

If *density is constant*, the differentials may be gathered to yield:

$$\frac{\partial}{\partial s}\left(p + \gamma h + \frac{\rho V^2}{2}\right) = 0 \tag{9}$$

Integration over s yields:

$$p + \gamma h + \frac{\rho V^2}{2} = \text{Constant along a streamline} \tag{10}$$

Finally, division by γ yields:

$$\frac{p}{\gamma} + h + \frac{V^2}{2g} = \text{Constant} = H \qquad \text{(Bernoulli's equation)} \tag{11}$$

where H is total head. The first term on the left hand side of Bernoulli's equation is the pressure head, the second is the elevation head and the third is the velocity head; pressure head plus elevation head is called static head while velocity head is also referred to as dynamic head (static head plus dynamic head is total head). Therefore, Bernoulli's equation relates pressure, elevation and velocity between any two points **along a streamline** in a flow field that is **inviscid**, **steady**, and **constant density**. Each term in the equation has the dimension of length.

Consider the application of Bernoulli's equation to a pitot-static probe (Figure 9).

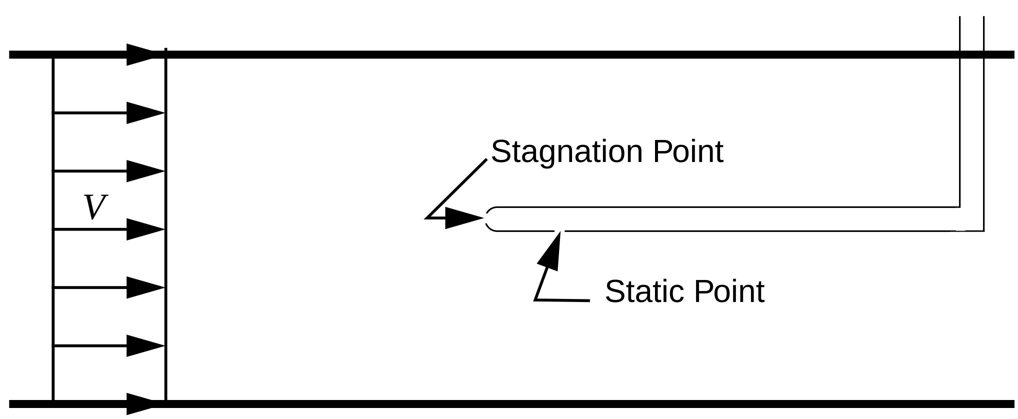

Figure 9. Pitot-static probe

Bernoulli's equation may be written between any two points, 1 and 2, on a streamline:

$$\frac{p_1}{\gamma} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + \frac{V_2^2}{2g} \quad (12)$$

Let Point 1 be the stagnation point in Figure 9 and Point 2 be the static point. In a Pitot tube, the air velocity at the stagnation point is zero while the air velocity at the static point is constant. The elevation of the two points is the same. If the pressure at the stagnation and static points is measured, the velocity at the static point can be determined from equation (12) [Note: validate this yourself using the above definitions and equation (12)]:

$$V_2 = \sqrt{2\frac{p_1 - p_2}{\rho}} \quad (13)$$

Indeed, Bernoulli's equation is a very useful relation. However, it must be used with great care! Remember that if the four assumptions required in the derivation of Bernoulli's equation are not met, then Bernoulli's equation is not valid. For example, if the fluid has significant viscosity, Bernoulli's equation can not be used to analyze fluid motion. In this case, one must turn to the general energy equation, which applies when any, or all, of the Bernoulli assumptions are not satisfied.

In this experiment the fluid, air, is assumed to have constant density. This assumption is valid for flow of gas at low velocity. Low velocity is defined when the Mach number (see "Dimensionless Fluid Parameters" on page 8) is less than $1/3$. Furthermore, air has a small enough viscosity that it may be assumed to be inviscid for the purposes of this experiment. Be sure to consider the relative validity of these assumptions in analysis of your errors.

II. Objective

Validate Bernoulli's assumptions and equation by determining if the summation of the terms in the Bernoulli equation at several locations along a streamline is a constant, H.

III. Anticipated Results

At a minimum, you should be able to anticipate:

1) Will Bernoulli's equation apply?
2) What effect will the obvious violations of Bernoulli's assumptions have (and how much)?

IV. Apparatus

1) Miniature wind tunnel device with variable flow control and uniform air supply.
2) Converging - diverging test section (Venturi).
3) Multiple tube inclined manometer.

4) Pitot-Static probe.
5) Measuring scale (mm), thermometer.

V. Procedure

1) Measure and record the ambient air temperature.
2) Measure the dimensions of the test section.
3) Ensure that the manometer is leveled correctly. The fluid level should be equal in each tube. Level the manometer if required.
4) Adjust the flow rate to low and start the fan. Move the Pitot-Static tube vertically so that the manometer reads a high reading (low pressure). Then, carefully adjust the flow rate to high, but DO NOT ALLOW the manometer to overflow at any time. If the manometer is going to overflow, reduce the flow rate until it is safe to continue. Ensure that the high pressure (low readings) are within the scale of the manometer.
5) Position the Pitot-static probe vertically so that at least two readings can be taken in each test section (converging, constant, and diverging). Record the manometer readings [millibars] for the stagnation point, static point, and air box pressures. At each test location, measure the cross-sectional area of the test section and the distance from the top of the test section (use the static point).
6) Adjust to a low flow rate and repeat step 5.

VI. Data Control

Data control consists of plotting $\frac{p}{\gamma} + \frac{V^2}{2g}$ for the static point versus the distance from the entrance of the Venturi meter. A flat line indicates good observed data.

VII. Results

1) Plot the stagnation pressure, dynamic pressure, static pressure, and total pressure versus the distance from the entrance of the Venturi. On a separate plot, plot static head, dynamic head, and total head versus the distance from the entrance of the Venturi. Hint: Pressure and head are related. Any pressure can be converted to an equivalent head (and vice-versa) by the relationship $h\gamma = p$, where h is the head and p is the pressure.
2) Repeat for the second air flow. Make sure there is a good legend to clarify the data.
3) Calculate the fluid velocity, V and the flowrate, Q for each reading. Compare to expected results. Hint: conservation of mass yields $Q = VA$, where A is the cross-sectional area at which the velocity is acting.
4) Is the flowrate constant? Is the Mach number in an acceptable range? Is the total head constant?

VIII. Suggested Data Sheet Headings ([ ] indicate the units of measurement)

Flow	Distance	Pressure [ ]			Width
[ ]	[ ]	Stag Point	Static	Airbox	[ ]

Length of Venturi section ______________ [ ]

Air temperature ______________ [ ]