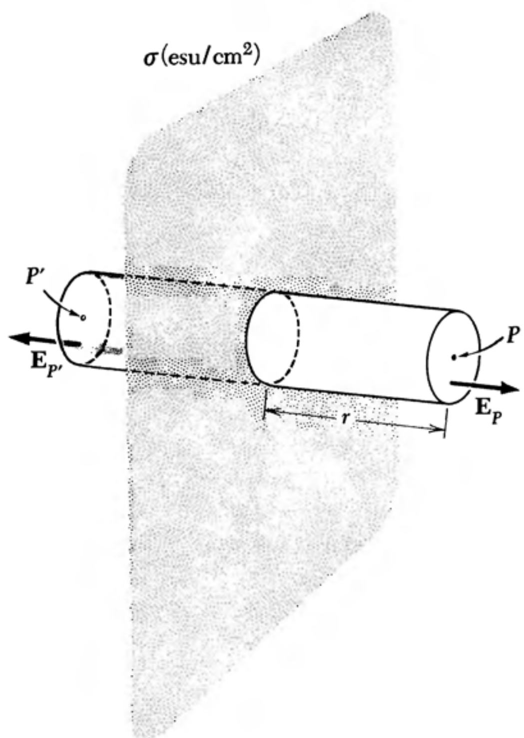


FIGURE 1.23
Using Gauss' law to find the field of an infinite flat sheet of charge.

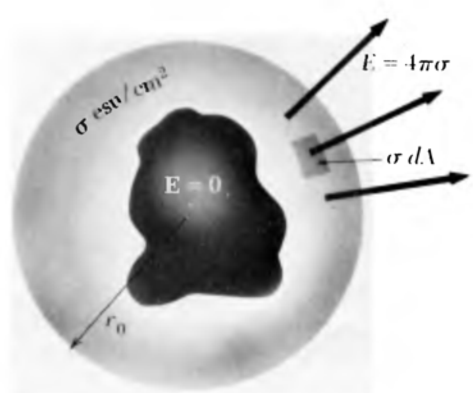
from the sheet, which just offsets the $1/r^2$ decrease in the field from any given element of charge.

THE FORCE ON A LAYER OF CHARGE

1.14 The sphere in Fig. 1.24 has a charge distributed over its surface with the uniform density σ, in esu/cm^2. Inside the sphere, as we have already learned, the electric field of such a charge distribution is zero. Outside the sphere the field is Q/r^2, where Q is the total charge on the sphere, equal to $4\pi r_0^2\sigma$. Just outside the surface of the sphere the field strength is $4\pi\sigma$. Compare this with Eq. 28 and Fig. 1.23. In both cases Gauss' law is obeyed: The *change* in E, from one side of the layer to the other, is equal to $4\pi\sigma$.

What is the electrical force experienced by the charges that make up this distribution? The question may seem puzzling at first because the field **E** arises from these very charges. What we must think about is the force on some small element of charge dq, such as a small patch of area dA with charge $dq = \sigma\, dA$. Consider, separately, the force on dq due to all the other charges in the distribution,

FIGURE 1.24
A spherical surface with uniform charge density σ.



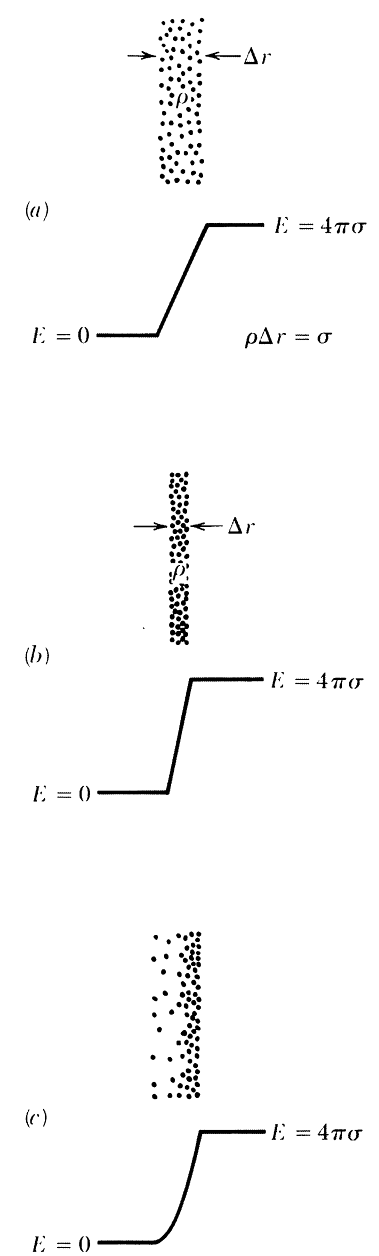


FIGURE 1.25
The net change in field at a charge layer depends only on the total charge per unit area.

and the force on the patch due to the charges within the patch itself. This latter force is surely zero. Coulomb repulsion between charges within the patch is just another example of Newton's third law; the patch as a whole cannot push on itself. That simplifies our problem, for it allows us to use the entire electric field **E**, *including* the field due to all charges in the patch, in calculating the force $d\mathbf{F}$ on the patch of charge dq:

$$d\mathbf{F} = \mathbf{E}\, dq = \mathbf{E}\, \sigma\, dA \tag{29}$$

But what E shall we use, the field $E = 4\pi\sigma$ outside the sphere or the field $E = 0$ inside? The correct answer, as we shall prove in a moment, is the *average* of the two fields.

$$dF = \tfrac{1}{2}(4\pi\sigma + 0)\, \sigma\, dA = 2\pi\sigma^2\, dA \tag{30}$$

To justify this we shall consider a more general case, and one that will introduce a more realistic picture of a layer of surface charge. Real charge layers do not have zero thickness. Figure 1.25 shows some ways in which charge might be distributed through the thickness of a layer. In each example the value of σ, the total charge per unit area of layer, is the same. These might be cross sections through a small portion of the spherical surface in Fig. 1.24 on a scale such that the curvature is not noticeable. To make it more general, however, we have let the field on the left be E_1 (rather than 0, as it was inside the sphere), with E_2 the field strength on the right. The condition imposed by Gauss's law, for given σ, is in each case

$$E_2 - E_1 = 4\pi\sigma \tag{31}$$

Now let us look carefully within the layer where the field is changing continuously from E_1 to E_2 and there is a volume charge density $\rho(x)$ extending from $x = 0$ to $x = x_0$, the thickness of the layer (Fig. 1.26). Consider a much thinner slab, of thickness $dx \ll x_0$, which contains per unit area an amount of charge $\rho\, dx$. The force on it is

$$dF = E\rho\, dx \tag{32}$$

Thus the total force per unit area of our charge layer is

$$F = \int_0^{x_0} E\rho\, dx \tag{33}$$

But Gauss's law tells us that dE, the change in E through the thin slab, is just $4\pi\rho\, dx$. Hence $\rho\, dx$ in Eq. 33 can be replaced by $dE/4\pi$, and the integral becomes

$$F = \frac{1}{4\pi}\int_{E_1}^{E_2} E\, dE = \frac{1}{8\pi}(E_2^2 - E_1^2) \tag{34}$$