The Principles of Linguistic Philosophy F Waismann



THE PRINCIPLES OF LINGUISTIC PHILOSOPHY

By the same author *

THE PHILOSOPHY OF MATHEMATICS

By the same editor

AN INTRODUCTION TO THE LOGIC OF SCIENCES MATTER AND METHOD

THE PRINCIPLES OF LINGUISTIC PHILOSOPHY

Вy

F. WAISMANN

LATE READER IN THE PHILOSOPHY OF SCIENCE UNIVERSITY OF OXFORD

EDITED BY

R. HARRÉ

FELLOW OF LINACRE HOUSE LECTURER IN THE PHILOSOPHY OF SCIENCE UNIVERSITY OF OXFORD

 $\begin{array}{c} MACMILLAN\\ London \cdot Melbourne \cdot Toronto \end{array}$

ST MARTIN'S PRESS New York 1965 Copyright © the literary executors of F. Waismann, and R. Harré, 1965

MACMILLAN AND COMPANY LIMITED Little Essex Street London WC 2 also Bombay Calcutta Madras Melbourne

THE MACMILLAN COMPANY OF CANADA LIMITED 70 Bond Street Toronto 2

ST MARTIN'S PRESS INC 175 Fifth Avenue New York 10010 NY

PRINTED IN GREAT BRITAIN

CONTENTS

INTRODUCTION

page xi

PART I

The Transition from the Classical to the Linguistic View of Philosophy

CHAPTER	
I THE NATURE OF A PHILOSOPHICAL PROBLEM	3
I The influence of logic upon philosophy	3
2 The nature of a philosophical problem	5
3 What is lack of clarity?	9
II EXAMPLES OF PHILOSOPHICAL PROBLEMS AND THEI	R
SOLUTIONS	15
1 The problem of the trustworthiness of memory	-5
2 Can two persons have the same experiences?	22
The timeless nature of truth	27
4 Can a justification be given for the rules of grammar?	34
5 Augustine's problem	40
III IS THERE <i>a priori</i> KNOWLEDGE?	4.4
The nature of geometrical propositions	44
2 The meaning of equations	44 50
2 Can a surface be both red and green?	50
4 Are there synthetic <i>a priori</i> judgements?	67
IN GRAMMATICAL MODELS	60
The vagueness of language	60
2 Language games	- - - - - -
3 The metaphysical aura around certain words	81
Appendix to Part 1	87
I Achilles and the tortoise	87
2 The antinomies	88

v

CONTENTS

PART II

Elements of a Philosophical Grammar

v I		93
I	Purpose of the investigation	93
2	The learning of language	94
3	Types of words	96
4	Categories	103
5	Ostensive definition	104
6	Must there be ostensive definition in every language?	107
7	The concept of a sign	108
VI	THE CAUSAL INTERPRETATION OF LANGUAGE	111
I	Language considered as a mechanism of signs	111
2	Criticism of this view	114
3	Further criticism	116
4	Causal explanation and explanation of meaning	118
5	Reason and cause	119
6	Language as a calculus	122
7	The definition as the reason for, and the cause of, the usage	
0	of a word	125
8	Sign and symptom	127
VII	WHAT IS A RULE?	129
I	Rules in a game	129
2	Rules in language	131
3	The normative aspect	132
4	Extension of the concept of a rule	135
5	Attempts to define the concept of a rule	137
6	A characterization of rules	140
7	Explanations	144
8	Criteria for the validity of a rule	146
9	Are there any final and conclusive explanations?	149
10	On the borderline between rules and assertions of fact	150
VIII	MEANING	153
Ι.	Substantive and substance	153
2	Meanings of the word 'meaning'	155
3	Meaning as use	156
4	Meaning and mental image	158
5	An objection	162

	CONTENTS	vii
IX	DIFFERENT TYPES OF EXPLANATION	163
I	Examples of explanation	163
2	Analogies in language	176
3	Families of concepts	179
4	Examples of families of concepts	183
5	Different meanings of one word	187
6	Essential and inessential rules	190
7	On the meaning of signals	19 2
x	Names	194
I	Names in general	194
2	Names of physical objects	197
3	The object in the flux of becoming	2 04
4	. Is 'this' a proper name?	205
5	Is there an ideal language for describing phenomena?	207
6	Proper names of people	212
7	The criterion of identity in the case of persons	213
8	Understanding the ostensive definition	216
XI	NAMES OF SPECIES	221
I	General names	221
2	Names of tones	225
3	Concept and object	227
4	. Colour names	228
5	The learning of colour words	231
6	The simile of the 'body of meaning'	234
7	Other ways of explaining a 'colour name'	237
XII	PROBLEMS OF COMMUNICATION	240
1	The problem stated	240
2	Is only structure communicable?	243
3	Removable and irremovable barriers to understanding	248
4	How can we describe our subjective experiences to one another?	257
5	Physical language	260
6	What sort of experience is presupposed by mutual under-	
	standing?	204
XII	I STRUCTURAL DESCRIPTION	269
1	The concept of description	209
2	Structural description	270
3	Does the ostensive definition lead us out of language?	277

viii	CONTENTS	
XIV	WHAT IS A PROPOSITION?	280
	a. Proposition and Experience	
I	Sentence and intonation	281
2	Understanding a sentence	281
3	Can a sentence be defined as an expression of a fact?	282
4	Psychological criteria for a proposition	283
	b. The Proposition as Part of a Calculus	
5	Language games with the words 'true' and 'false'	285
6	Confirmation of our criterion by examples	288
7	Towards a definition of the words 'true' and 'false'	290
8	Can a proposition be defined ostensively?	291
9	Are there different kinds of truth?	292
10	Lying	294
II	Having an idea in mind	295
12	Extension of the concept of a proposition	298
13	Types of propositions	298
	c. Supposals	
14	Is there a specific experience of judging?	300
15	Supposals	300
16	The assertion sign	302
хv	THE THEORY OF THE COMMON STRUCTURE	304
I	The problem stated	304
2	Wittgenstein's theory	307
3	Criticism of this theory	311
4	Continuation	317
5	Our own account	320
XVI	MEANING AND VERIFICATION	323
I	Language and symbols	323
2	When do we understand the meaning of a sentence?	324
3	Are unverifiable statements meaningless?	325
4	'The meaning of a statement is the method of its verification'	329
5	The description of verification	331
6	'Meaningful' and 'meaningless'	333
7	The meaning as shadow of reality	335
XVII	Some remarks on the concepts 'to be able', 'to	
	KNOW', 'TO UNDERSTAND'	338
Ι	The idea of possibility	338

342

2 To be able

•••

3	Toknow	345
4	To understand	346
5	Understanding a word	349
6	Is understanding a process in time?	351
7	Substantive and transitive parts of the stream of thought	352
8	Is there such a thing as blindness for concepts?	255
9	Different ways of understanding a word or a sentence	355
10	Is the entire future use of a word contained in how we now understand it?	258
II	Understanding as an experience	260
12	Are there degrees of understanding?	261
13	Can understanding approach reality?	262
- 5	ean anaoistantaing approach reality.	304
XVI	II COMBINATIONS OF PROPOSITIONS	364
I	Two ways of joining propositions	364
2	Tendencies to formalization	367
3	How far is the conclusion of an inference contained in the premisses?	369
4	Do the logical relations between propositions show them-	271
5	The propositional calculus	3/1
5	Can we give reasons justifying logic?	3/4
7	Tautology and contradiction	3/4
/	Tattology and contradiction	373
XIX	THE LOGICAL CALCULUS	377
Ι	Is logic a theory?	377
2	Does altering the axioms of the logical calculus give rise to a	
	non-Aristotelian logic?	380
3	Logic and ordinary thought	382
4	Generality	383
xx	TOWARDS A LOGIC OF OUESTIONS	287
т	What does 'to look for' mean?	287
2	Search in mathematics	201
2	Sums and mathematical problems	306
ວ ∡	Is search for the impossible possible?	208
4 ~	Discovery	320
5 6	Towards a grammar of questions	400
U	romation a Graninitation quotionite	τ°Э

Index

ix

INTRODUCTION

LINGUISTIC philosophy is not a set of doctrines, but a critical technique. The origins of the movement are to be found in reactions rather than in insights. G. E. Moore reacted against the absurdities of traditional metaphysics and its paradoxical conclusions like the denial of the reality of ordinary things; Wittgenstein against the formalism of Russell and his own early attachment to formal logic as an analytical tool; G. Ryle and J. L. Austin against both ancient and modern metaphysical myths, like the belief in mental substance and in sense data. Each of these philosophers brought his own contribution to the method. Moore's way of dealing with philosophy involved paying great attention to the exact way statements were expressed, and so bringing out verbal ambiguities and mistakes arising from the way theories were presented. Wittgenstein, once he had seen that the use of essentially mathematical models of discourse frequently put aside problems without solving them, devised the method of extensive description of the way words are used as a therapy for the equivocations which seemingly led to philosophical theories about such topics as the relation of mind and body and the true characterization of knowledge. Ryle and Austin seem to have arrived independently at the idea that close attention to the language in which philosophers have expressed themselves shows a haze of muddle and confusion, and, because of the much greater complexity of the language used to state the facts of, say, our mental life, by comparison with the language used by philosophers in theorizing about it, philosophers are particularly prone to verbal fallacies, varieties of equivocation.

In his presentation of the linguistic method in philosophy Waismann synthesized these three main strands. He makes use of the technique of analysis, the setting out of the various meanings an equivocal statement might have. He shows how formal analysis is not enough, and how only an extensive description of the way the key words are used can preserve us from falling into equivocation, led on by misleading verbal analogies. Waismann was not a great originator, but he was receptive to great new ideas, and he had an unrivalled capacity for systematic and painstaking development of such ideas. In this textbook, the first draft of which was written thirty years ago, and in his later development of the original text, he sought to expound the technique in a clear, systematic and, above all, useful way. Some philosophical problems, he believed, could be solved by the methods of linguistic philosophy, and on all a clearer light could be made to shine by its use as a preliminary to any theorizing.

The form in which he wrote the book preserves its origin, as a reaction against formalism, against the idea that mathematical rigour is the only kind of clarity that dispels metaphysical fog. But though owing much to Wittgenstein, in whose struggle against formalism Waismann was at one time an intimate ally, the book is not just a systematic version of Wittgenstein's *Philosophical Investigations*. The final form owes a good deal to Waismann's later Oxford days. He was the only man, except Wittgenstein, who lived as a participant through both main phases of modern philosophy, for most who started with mathematical techniques never moved on to more extensive linguistic analysis, and the later co-inventors of the linguistic technique had never been mathematical model builders.

The original version of this book was written and prepared for publication before the Second World War, but was withdrawn by Waismann on the eve of publication. Thereafter he worked over and over the galleys, adding to and developing the material, and compiling hundreds of sheets of inserts. All the material was found among his papers when he died, and most of it has been edited into a continuous exposition, as far as possible using only what seems to have been the latest of the modifications and additions which he had written. Permission for the publication of this book has been granted by Waismann's literary executors.

I am particularly grateful to Mrs. I. Mash, who typed a very ugly pile of paper into an elegant script.

R. HARRÉ (Editor)

Part I

THE TRANSITION FROM THE CLASSICAL TO THE LINGUISTIC VIEW OF PHILOSOPHY

CHAPTER I

THE NATURE OF A PHILOSOPHICAL PROBLEM

1. THE INFLUENCE OF LOGIC UPON PHILOSOPHY. It is characteristic of philosophy that every great turning-point in its history is greeted by many as its rebirth. Great thinkers of all times have refused to accept the ideas of their predecessors as unquestionable truths; they have preferred to try to reach the foundations of knowledge by their own labours. Descartes, Locke and Kant each felt himself to be a turning-point, the beginning of a new philosophy; they were not mistaken in so thinking, for each made a step which we can never go back upon.

The philosophy of the last eighty years contains almost more contradictions and diverging opinions than ever before.

Some philosophers, discouraged by the collapse of the great metaphysical systems of the nineteenth century, believe that they have found a solution in a return to Kant; others try to construct a picture of the universe from the results of science, drawing from scientific knowledge conclusions about our position in the universe and the meaning of life. A third group rejects the sovereignty of science, believing that only intuition can help us to understand the essence of being. Yet others, tired of these conflicting opinions, say that philosophy can provide no objective knowledge; it is to be understood merely as the expression of personality, of psychological type. This leaves to the philosopher only the task of cataloguing the works of his predecessors, without hoping or wishing to find *the* solution to the problems that exercised them.

This chorus of conflicting opinions, each claiming to be the sole possessor of the truth, is undoubtedly a sign of serious crisis and has been widely recognized as such. We have mentioned it here only in order to emphasize that at the present day a new trend has come into philosophy. What this trend is will be made clear in the course of this book; at the moment we will give merely a preliminary outline of its character.

Previous philosophers have almost always directed their attention to the answers given in reply to philosophical questions. Their disputes were all concerned with these answers, their truth or falsity, their proof or refutation. The new point of view differs from all the others in that, from the start, it ignores the answers and directs all its attention towards the questions. It is well known that we often think that we understand precisely what is meant by a question, whereas further examination shows us that we have deceived ourselves in thinking this and have been led astray by superficial linguistic analogies. The great mistake of philosophers up to now, which has led to so many misunderstandings, is that they have produced answers before seeing clearly the nature of the questions they have been asking. They seem to have been quite unaware of the possibility that the form of the question itself might conceal an error. This has meant that they have never reached the roots of the intellectual discomfort which they have felt; they have been satisfied by pseudo-solutions which, though they dazzled the mind for a little while, could not stand the test of time.

The change, when it came, was from quite a new direction. Without many being aware of it, logic had developed in the hands of mathematicians into an instrument which far surpassed the logic of the Schools in delicacy and expressive power. Though originally made for the purpose of analysing mathematical inferences, logic did not remain restricted to that field. Frege made a remark in his Begriffsschrift which today has the air of a prophecy. Logic, he said, will perhaps one day be of use to philosophy in the task of 'breaking the tyranny of words over thought, by bringing to light the confusions which are almost unavoidable in the use of language'. What Frege predicted has been realized by Russell, Moore and Wittgenstein. They used the recently discovered methods and ideas to illuminate the structure of language, and through their work it became apparent how much the logical structure of thought is concealed and misrepresented in the verbal form in which it is expressed. These investigations led to a much clearer understanding of the character of philosophical questions. Such problems disturb us only if we do not see how language functions; if we think we are discussing questions of fact, when we are merely being misled by peculiarities of linguistic form. The danger is that there are innumerable ways of being misled by the analogies, metaphors and similes of language, and even if we are constantly on the watch we are continually being caught in a linguistic snare.

We come, then, to an entirely new solution of philosophical

questions, one very different from what had been expected. It had been expected that such questions should be answered either by 'yes' or by 'no'. Instead of this the analytic method leads us to the view that the questions themselves rest upon misunderstandings; it frees us from them by making the meaning of our words and the way they are combined in language so clear that we no longer feel driven to ask the questions.

2. THE NATURE OF A PHILOSOPHICAL PROBLEM. Everyone understands what is meant, for instance, by the unsolved problem of the origin of life, or by saying that there are still many problems about prehistoric times remaining to be solved. Solving these problems consists in gaining new information, which is expressed and communicated in sentences. We are so used to this sort of problem that it never occurs to us that the problems of philosophy might be quite different. Yet in fact 'problem' and 'solution' are often used in quite different ways. If we talk of the problem of the transmission of force by radio the solution we want is a technical discovery, a piece of practical activity, which is connected with our knowledge of physics only by being suggested and prepared for by the latter. No one would hold that solving a social question consists in acquiring some theoretical ideas about it. The problem of depicting movement in modern painting introduces yet another sense of 'problem'. If we consider these examples, one after the other, it becomes clear that very different sorts of things are collected under the name 'problem'.

The first thing that strikes us is that philosophy does not deal with one homogeneous sort of object in the way that, for instance, history or astronomy does. In fact, from any question, if we follow it far enough back, we can reach a typically philosophical question. For example, from the question whether a judge has decided justly in a particular case — that is to say, has given a judgement in accordance with the law — we can pass to the question whether the laws themselves are just; from the question how a particular natural phenomenon is to be explained we can pass to what is meant by explanation and whether all that explanations achieve is merely to push back the inexplicable a step further; from the question whether a particular story is true we can pass to the problem of the nature of truth. In these cases we can perceive dimly that the direction of the question has changed, that we move, as it were, in a new dimension. It seems then that what gives a problem its peculiarly philosophical air lies less in its subject-matter than in the way it is put; in what direction we are forced to move when we try to solve it.

At this point we might be tempted to say that philosophical questions are those which are the most general. In daily life we are interested, for example, in the purpose of a particular action, but the philosopher examines the nature of purpose in general, the concept of purpose. The scientist looks for explanations of facts, but according to this view the meaning of explanation is itself a problem for the philosopher. The legal philosopher examines the essence of justice, the philosopher of languages the essence of language and so on. The most general concepts of science — space and time, chance and law, life and consciousness, meaning and purpose — can only be illuminated philosophically.

Though there is doubtless some truth in this opinion, it is not an adequate account of the matter. In the first place, there are philosophical questions which, far from being general, are particular. For instance: How does it come about that, although the image on the retina is the wrong way up, we see things the right way up? and, how does it come about that, in a mirror, things are still the right way up although left and right are the wrong way round? What is even more significant is that there are questions of a very general nature in science and mathematics which are not philosophical. Certain present-day investigations in mathematics (e.g. the theory of sets, the theory of abstract groups, modern algebra) can hardly be surpassed in generality. Yet there is a quite unphilosophical way of dealing with these questions. So generality alone cannot be the differentiating mark of philosophical questions.

From Plato to Schopenhauer all philosophers were agreed that wonder is the source of philosophizing. This wonder is not directed towards extraordinary and rare happenings but towards just those things which are familiar to everyone. The philosopher might be described as the type of man who is given to wonder about the things of everyday knowledge. Consider, for example, the amazement with which Augustine contemplated the fact of memory. It was not exceptional achievements of memory which amazed Augustine, but the fact that there is such a thing as memory at all. His dilemma can be put in the following way: a sense impression, a smell or a taste, lasts for a moment and then disappears. It is now here, now gone. Yet in the halls of memory copies of the vivid impressions of the moment are stored up. From these I draw each one out as I wish. But the copy is not transitory like the sensation it mirrors; it has a continued existence. It is the same as, yet different from, the sensation. The past is preserved as it was, yet it is intrinsically different. How can such a thing come about?

Here the fact of memory itself becomes a problem. The philosopher, as he ponders over a question, has the appearance of a man who is disturbed and irritated by something. He seems to be struggling to do something which is beyond his strength. The words in which such a problem is expressed do not reveal its true meaning. If we try to penetrate into the disturbed background from which the question arises and takes shape, we come upon something else which it is difficult to express in words. We might perhaps call it alarm on coming up against the inconceivable. We can try, by various examples, to get ourselves nearer to this queer state of mind. If while walking along we saw the place which we had just left suddenly appearing again in front of us, we should feel a sort of dizziness, we should say in bewilderment 'But how can this be?' Similarly the philosopher says to himself 'Certainly there is such a thing as memory, but how is it possible?'

Everyone who has brooded over a philosophical question is familiar with the experience that his mind seems to become blurred, that everything, even the apparently most certain and self-evident, takes on a new and puzzling air. Plato experienced this astonishment when general ideas suddenly became a problem for him. He started from the question: What is meant by a general term? What, for example, does the word 'horse' mean? Does it mean a single particular horse? Clearly not, for the word can be used to refer to this or that or any horse. Then does it mean all horses? No, for even if there were no horses, the word would still have a meaning. But if it means neither a single horse nor all horses what does it mean?

The idealist knows the shock of realizing for the first time that the world is merely an idea in his mind, that he 'knows neither a sun nor an earth but only an eye that sees a sun, a hand that feels an earth' (Schopenhauer). Does anything, then, exist besides individual consciousness? Kant must have felt this sort of thing when the existence of mathematics suddenly became a riddle to him. How is it possible, he thought, that geometry, which is independent of all experience, should agree so miraculously with reality? Can mind, without the assistance of experience, fathom the properties of real things merely by cogitation?

A peculiar mental unrest arises from considering such questions as these. It seems as if we had previously passed carelessly over difficulties which we now notice. We ask ourselves in horror 'But how can such a thing be?' We ask this question only if the facts astonish us, if something about them seems to us out of place, incredible, even absurd.

What we can do to overcome this sensation of unanswerable difficulties we shall see presently. We shall see also why our philosophical disturbance cannot be quieted by winning fresh knowledge. We shall be forced to the opinion that philosophy is not a temple of knowledge, that in it there are neither suppositions nor affirmations, that it is something fundamentally different from these, namely the clarification of thought. Wittgenstein was the first to reach this opinion and we can use his words as a motto for this book:

'Philosophy is not one of the natural sciences.... The object of philosophy is the logical clarification of thoughts. Philosophy is not a theory but an activity.... The result of philosophy is not a number of "philosophical propositions", but to make propositions clear. Philosophy should make clear and delimit sharply the thoughts which otherwise are, as it were, opaque and blurred.'¹

As we have said, the sense of these words will gradually become more comprehensible to us. Even now we can mention certain general facts which point towards our view of philosophy. First of all, we have often experienced the difficulty of explaining to a practical man the *meaning* of a philosophical question. This cannot be because he lacks some necessary technical knowledge, for many philosophical questions can be put in the simplest words of everyday language. What he lacks is the capacity to share the bewilderment which the philosopher feels at the question. Another important sign is the prevalence of misunderstanding in a philosophical argument, the danger of the parties speaking at cross purposes. Everyone who has to speak to an audience on a philosophical subject feels at a certain loss; he feels that arranging ideas in a logical sequence is not quite the right way to communicate his views to his listeners. All he can do is to conduct them through the ideas in the way he went through them himself, so that they feel for themselves the mental discomfort which he felt, so that the

¹ Tractatus Logico-Philosophicus, 4. 111, 4. 112 (London, 1922, new trans., 1961).

doubts rise before them as they rose before him and they make the same attempts at solution which he has made. Then step by step they may become clearer until they see the affair with his eyes. A philosophical opinion cannot be communicated like a dead formula. The best part of philosophy consists in training the intellect, not in communicating ready-made truths. Outstanding thinkers have always felt this to be so. Thus Kant, for example, said that he did not wish to teach philosophy but philosophizing.

Another sign that our view is correct can be found in the history of philosophy. For though the sceptic makes out that philosophy is merely a disheartening swaving between one system and another. this is not the case. If we look into the matter carefully, we find, under the contradictions on the surface, a continuous development, a gradual progress of views. This change is not so much in the assertions made as in the attitude adopted to the questions, a change in what are looked upon as problems and what are regarded as unfruitful questions. For example, what Hume, in his famous critique of the concept of causality, showed most plainly was that we perceive only the succession of events and not an inner connection joining cause and effect together. The tangible gain in this idea lay not in any philosophical axiom giving rise to further propositions, but in the clarification of the meaning of causal sentences; not in adding to the number of our propositions, but in cutting it down; in getting rid of the pseudo-truths and fancied knowledge which accompany the idea of an inner connection in causality.

3. WHAT IS LACK OF CLARITY? What do we mean by saying that someone is unclear about something? Do we mean that he is in a certain peculiar mental state? Is becoming clear the beginning of a new mental state, as it were a ray of light in the mind?

In order to understand the meaning of the words 'clear' and 'unclear' we shall start by looking at typical examples of their use. When do we say that someone is unclear about something? Is it when he lacks certain knowledge? Can his confusion be removed by extending his field of knowledge? We should not say that Newton was unclear about Optics although he knew far less about it than is known by present-day physicists. But we should say that he was unclear about the differential calculus, that, for example, he had nebulous notions of 'infinitely small' quantities. This is shown by the fact that it is very easy, from the point of view adopted by Newton, to ask questions which make us feel perplexed. We will not, however, discuss Newton's confusion in detail, but will turn instead to a simpler example of lack of clarity. Suppose we consider the infinite series I - I + I - I + I - ... The question which leads us at once to a difficulty is this: 'What is the sum of the series?' We might first of all say 'It is zero, since the numbers can be paired so that each pair gives zero and the sum of zeros is zero'. We could, on the other hand, say 'The sum is I'; for we can begin with the first number and then pair off the successive numbers in this way:

$$I - (I - I) - (I - I) \dots$$

Euler, applying the formula for obtaining the sum of a geometrical progression, suggested that the sum is $\frac{1}{2}$ and gave the following justification of his procedure: If you end the series after an even number of terms the sum is zero, and after an odd number it is I. Since the series has an infinite number of terms and infinity is neither even nor odd, the sum is neither zero nor I but their mean.

Here we have a problem which differs from a mathematical problem in a typical way. This difference may be revealed by the curious discomfort which arguments like Euler's produce in us. It is evident that our question cannot be answered by doing further calculations. The calculations lie before us, but they contradict each other, and we must now try to see how this comes about. We are not dealing with a calculation to be performed but with the *meaning* of a calculation.

The first step in resolving the conflict lies in turning from the question 'What is the sum of this series?' to the question 'What does it mean to say that a number is the sum of an infinite series?' As soon as we have made this transition, as soon as we no longer ask whether what is disputed is true or false, but ask what it *means*, we have passed from the domain of calculation to that of clarification, from mathematics to philosophy. Here we have a characteristic example of that change in direction of attention of which we have so often spoken.

The next step consists in seeing that the word 'sum', first of all explained only for a finite number of terms, has as yet no meaning when applied to an infinite series. We must first *define* what we wish to understand by the sum of an infinite series. If we give a definition, such as the usual one, which defines the sum as the limit to which the partial sums taken successively tend, the problem disappears. It then becomes evident that the series under consideration has no sum at all.¹

In what, therefore, does the solution of the problem consist? It consists in explaining exactly how we operate with a finite and with an infinite series, in stating precisely and completely the rules for manipulating each of these types of series and in comparing these rules. This comparison then shows indeed how far a finite series is *similar* to an infinite one.

Suppose it is now asked how this confusion can have arisen or why the correct way of escape was not obvious from the start? The answer is that the lack of clarity has its roots deep in the forms of expression of our language. For with both finite and infinite we use the expressions 'sum', 'series', 'addition'; with both we use the plus sign and other mathematical symbols. These facts conceal the fundamental difference in logic between finite and infinite series, so that we do not suspect that in going from one to the other we pass to quite another region. We are amazed to learn that an infinite series obeys quite different laws from those obeyed by a finite series, that, for example, it may happen that its sum can be altered simply by changing the order of its terms. We are in the habit of reading into the infinite series the properties of the finite.

The difficulty in such a problem is to avoid applying the wrong system of concepts. It is always language which leads us into the fallacy of misapplied concepts and which as *a matter of course* uses the same words with different meanings. The effect of this is the effect of a conjuring trick; the change occurs so innocently that it escapes attention. We apply the word 'sum' quite as a matter of course, both to finite and to infinite forms. All the traps are camouflaged by our use of words.

Perhaps the most famous example of clarification is Einstein's analysis of simultaneity. At the end of the last century difficulties of unknown origin arose in classical physics and manifested themselves in a variety of ways. The situation can perhaps best be described by saying that in the classical view it was quite uncertain whether two events taking place at two very widely separate places (for example, one on the Earth and the other on Sirius) were simultaneous or not. In this view, the answer to the question depends upon the state of motion of the bodies relative to the

¹ But one can apply a number to the series as a sum, if one uses another definition of 'sum'.

ether. Light takes about eight years to travel from Sirius to Earth, so that an event which is observed to be taking place now on Sirius actually happened eight years ago - but only if the Earth-Sirius system is at rest in the ether. If the system is moving in the direction from Sirius to the Earth, the light takes longer than eight years, because the Earth is flying away from it. But if the system is moving in the opposite direction, the light takes less than eight years. In order, therefore, to assign a definite place in time to an event on Sirius, the size and direction of motion of the system must be known. But experience (the Michelson-Morley experiment) has shown that such a motion cannot be ascertained. In order to explain this fact, Lorentz invented a remarkable hypothesis according to which the measurements of our apparatus become shorter owing to their motion, and at the same time clocks (and all other natural processes), go slower in such a way that the effect of the motion is exactly compensated. In other words, Lorentz says 'Motion through the ether and the contraction of the measuring instruments are both real processes. If only one part of these processes took place we could very well measure the motion; but in fact, the processes are so adjusted to each other that the effect of the whole is exactly zero'; that the motion, though real, escapes notice; that it becomes impossible to determine whether an event on the Earth is simultaneous with one on Sirius or not.

Surveying this argument today it is perfectly clear that a way out of this dilemma could only be found by turning away from the world of facts to a consideration of concepts. The decisive step consists in passing from the question 'Are the two events simultaneous?' to the question 'What exactly does it mean to say that they are simultaneous?' The answer to this is that initially it does not mean anything; for the word 'simultaneous' has only a clear meaning when it is applied to events at more or less the same place. If it is used to refer to events in quite different places, we require a statement of what it is to mean in this new context. This step was taken by Einstein. He neither discovered hitherto unknown facts, nor did he suggest a hypothesis which explains better the known facts; rather he cleared away from the concept of simultaneity the confusion which had surrounded it. He simply drew attention to the fact that the word 'simultaneous' must be redefined if it is to be used to apply to events in quite different portions of space. The realization that it is here a matter of our having to determine the

use of a word at once made the difficulties of classical physics disappear. For these were precisely due to the fact that one regarded what is only a matter of convention as if it were a problem of physics, that one tried to ascertain whether certain events were simultaneous instead of defining the word 'simultaneous'.

Here again we see how confusion, in the guise of a problem, comes about. The use of the one word 'simultaneous' misleads us about the logical difference of the various concepts which it denotes very much as the word 'sum' did in the previous example. We find exactly the same situation at two other points in the history of scientific thought; in the argument of Zeno against the possibility of motion, and in the antinomies of the theory of sets. In order not to introduce too many interruptions of our train of thought, these examples have been placed in an Appendix (p. 87).

In both the examples we have just examined (and also in those in the Appendix) a confusion arose, and in all cases it was got rid of by the same method, viz. by going back to and dissecting the meaning of the words and signs involved. All the tantalizing problems disappear once we are clear how the expressions 'series', 'sum', 'addition' and 'simultaneous' are to be used.

Let us put this result in a slightly different way and so make its real significance clearer. We spoke, in the examples, of *the analysis* of the meaning. In what does this consist? In nothing more than giving the rules for the use of a certain sign, whether a word or a mathematical symbol — for example, the rules for manipulating infinite series, the definition of simultaneity for events at different places, the logical rules for the use of the term 'all'. Now these rules of calculations, definitions, conventions, etc., form only a small part of the body of rules governing the usage of our language. These rules are called the *grammar* of the language.

If it is permissible to extend the word 'grammar' to apply to all rules for the use of signs, then we can say that the clarification of meaning is a process in grammar. Confusion was removed by the establishment of rules; consequently the confusion *was* a confusion about the rules. It is in this sense of the word that we shall in future be concerned with grammar.

We must certainly be clear what we mean by this. By grammar we mean everything about language which can be fixed before language is applied. We contrast the grammar with the actual application of language. The relation between grammar and

language is similar to that between deciding upon the metre as the unit of length and carrying out a measurement, or indeed between the adjustment of a telescope and an observation made through it. Grammar is, as it were, the installation and adjustment of a system of signs, in preparation for their use. If we take the word 'grammar' in this wide sense, it includes not only the rules it usually includes -rules of accidence and syntax - but also, for example, the definition of simultaneity, the rules governing mathematical calculation, the rules of logical inference, and ostensive definitions of the sort 'this colour is yellow' (pointing at a yellow patch), for this pointing too is part of the preparation of language. In short, grammar includes all the enormous number of conventions which, though nowhere expressly formulated, are presupposed in the understanding of everyday language. It is precisely the formulating and bringing to consciousness of these tacit conventions, the discovery of this complicated network of rules which constitutes the philosophical clarification of our ideas.

If this account is correct, it must be possible to confirm it from history. Whenever science has come to a crisis, a turning-point, where it ceased to go in its old direction, and where the way out could only be found by an examination of fundamental concepts, this was immediately felt to be a *philosophical* achievement. The solution of the antinomies of the infinite and Einstein's analysis of simultaneity are the most famous examples of this. If laying bare, the structure of concepts, the analysis of language, the clarification of meaning is the peculiar task of the philosopher, then we must say that the philosophic attitude is an essential part of all scientific thought. ••••••

Pages 15 ... 92 deleted

••••••

CHAPTER V

INTRODUCTION

I. PURPOSE OF THE INVESTIGATION. Human beings possess the faculty of thought and of communicating their thought to one another. The word 'language' will be used here as a term applying to everything that serves the end of expression and communication. It is of set purpose that we extend this concept to cover not only verbal languages but also gesture language, picture language, the 'language' of maps, the formal 'languages' of mathematics and logic, besides signal systems, and much else. It follows from the convention that all communication takes place by means of language.

This explanation will, of course, only be understood by a person who already knows the meaning of the words 'communicate', 'expression', 'meaning', etc. This might lead us to think that the first task of a logical examination of language should consist in an effort to fix the meaning of these words unambiguously; we must therefore decide how we are going to approach our subject-matter.

Ought we to begin by giving a set of precise definitions? In considering such a question it is important to keep in mind the purpose which the definitions are to serve. In a strictly deductive science definitions provide starting-points from which the deduction can proceed. Any inaccuracy in the definitions would result in some uncertainty in the deductions from these definitions. But it is not our intention to construct a theory of language comparable to formal logic, a theory which develops through concepts which it has itself created. Our subject is rather language as it is, with all its irregularities and moods; our purpose is to trace out the contours of language, exploring the casual inflections and deviations of linguistic usage so as to throw into relief the points from which spring our philosophical qualms. The fact that we aim at something quite different from the construction of a theory of language or the discovery of general principles takes away the main reason for seeking strict definitions.

Of course many of the concepts with which we shall have to deal

hereafter might themselves be so clear and perspicuous as to yield readily to definition. In fact, though, closer examination reveals a peculiar phenomenon, which we shall understand better hereafter but to which we must devote a few words here. It is characteristic of many of the terms which are particularly important in our study, that they are used in a fluctuating and irregular manner to express various particular meanings. Examples of such terms are 'meaning', 'understanding', 'expression', 'thought', and also 'sign' and 'language'. To force the meanings of these words into an exact formula is to do violence to them.

Accordingly we prefer to start with meanings with which the reader is familiar and which he finds given in the ordinary usage of language. This leaves it open to us to become more familiar with the logical nature of these concepts later, if our purposes require this. That is, we take for granted in what follows that we already understand in some degree the meanings of the words 'language', 'sign', 'communicate', 'express', 'understand', 'meaning', and of many other words, and we shall use them from this point onwards without any special hesitation.

The ability to use language is not present from birth, it has to be acquired. This leads to the question: *how is language learned?* What processes are involved in such learning?

2. THE LEARNING OF LANGUAGE. St Augustine says that a child learns the use of language in the following way: grown-ups point to things, direct the child's attention towards them, and at the same time pronounce words. In this way the child learns the *names* of things.

Certainly something corresponding closely to this description occurs during the learning of language. But is it the whole of the learning of language? The person who thus describes what takes place may have cases in mind in which the child is shown, say, a man, an animal, a piece of sugar, and at the same time the words 'man', 'dog', 'sugar' are pronounced to him; or he may be thinking of such a case as that in which one points to someone and says the words 'That is Jack'. But what about such words as 'yes' and 'no', 'can', and 'may', 'true' and 'false', 'how' and 'why'? Obviously these need to be explained or taught in a different manner.

We will call explanations of the use of words of the kind just given ostensive definitions. We can then say 'A child does not learn all the words of our language by means of ostensive definition'. It is an important point that we can imagine a primitive language, a system of communication much less extensive than our language, in which all the words are taught by ostensive definitions, a language for which what St Augustine says about the learning of language holds true exactly. Here we shall follow some suggestions of Wittgenstein.¹

Examples: (i) Suppose that A is erecting a building and that B has to hand him stones. There are different kinds of them, cubes, columns, slabs. B is so trained that when 'Cube!' is called, he hands over a cube, and so on.

What has just been described will be called a *simplified language* game. (Such games are actually played in the course of the learning of language.) B learns the game through being shown repeatedly how to play it until he imitates his teacher. In such a case the teaching of the game is essentially a training.

We shall see that the great variety of ways in which words are used is equalled by the variety of ways in which their uses are learned. Let us compare other examples with the one just given.

(ii) Suppose there are building-stones of different colours in the game. *B* is trained, on being given orders like 'White cube!', 'Red cube!' to hand over a cube of the corresponding colour. Part of the teaching of this game will be something that can be called the 'explanation' of the colour words. In the cases of 'white' and 'red', the explanation might consist in pointing to something white and saying the word 'white', and pointing to something red and saying the word 'red', and repeating this by pointing successively to different white things and saying the word 'white' each time and similarly in the case 'red'. This is an extension of game (i).

(iii) We can extend the game farther still and introduce the word 'here' and 'there'. When the order 'Cube here! slab there!' is given, B has to set down the building-stones at the places indicated.

This game serves as an instance of the penetration of gestures into verbal language, not only in the sense that the gestures play a part in the first definition of the word, but also in the sense that they are constantly *applied* in the actual *use* of language. In the order 'A slab there!' the gesture of pointing is part of the expression of the command, it is essential to its sense, i.e. the words of the command without the gesture would be incomplete. But ought we not to say that the gesture 'defines' the meaning of 'here' and

¹ Philosophical Investigations, § 2, §§ 8-10.

'there' all over again each time it is used? That is, that it is used as an ostensive definition each time? What would be the significance of saying this? To explain the meaning of a word surely means to fix its future use, to prepare it for use. Now we would not say that the gesture that goes with the order 'A slab there!' prepares 'there' for future use; whereas pointing to a cube in explaining the use of 'cube' *does* serve to determine the future use of 'cube'. If 'explanation of the meaning' is limited in its application to what can be done once and for all, then the pointing in game (iii) is not an explanation of the meaning of the word 'there'.

(iv) B is trained on being given an order like 'Five cubes!' to repeat the numerals from 1 to 5 and to hand over a cube as he says each one. This extension of our primitive language changes it so that, at this stage, the description of it that we gave initially does not apply. With the introduction of the numerals learned by heart we are taking a step in a new direction.

We shall leave ostensive definition for the time being and come to a new sort of explanation of the meaning of a word.

3. TYPES OF WORDS. Compare the orders 'Two cubes here!' and 'Three columns there!' It is to be noted that there is a closer similarity between the use of the words 'cube' and 'column' than between that of the words 'cube' and 'two', and likewise a closer similarity between the use of the words 'two' and 'three' than between that of the words 'two' and 'here'. Let us now group words according to similarities of usage, that is to say, in types of words.

Do 'two' and 'three' belong to different types of words? To say that they do would be to go against our feelings of language, yet it is not correct to say that they are used in *exactly* the same way; the truth of the matter is that we put words *differing only slightly* in their use into the same group. But before we go on, ought we not to define the expression 'differing only slightly'?

We shall put 'red', 'yellow', 'green', 'blue' together in one class, but shall we put 'black' and 'white' in the same class? And if we do add 'black' and 'white' to the others, there is still the question whether we ought to add 'light' and 'dark' too. Whether we should do this or not is not clearly settled one way or the other; we are free to choose whichever of the alternatives we prefer, and that this is the case is a hint that the idea of a type of words is a fluid one.

But can logic tolerate such fluid concepts? Must it not every-

where insist on precision and clarity? Let us see whether a more exact classification of words can be found.

It might be suggested as a way of making the classification exact that we should put in a single class all the words that can be substituted for one another in a sentence without a misuse of language resulting. Thus if we wished to decide if the expression 'table' and 'surface of the table' are of the same class, we could try substituting the one for the other in a sentence or in sentences. Now we can say 'Put the table in the corner!', but we cannot say 'Put the surface of the table in the corner!' This is evidence that these expressions belong to different types of words. Does this fully solve the problem of the classification of words? Let us carry the examination farther. In general, the words 'white', 'red', 'green', 'black' can be substituted for one another, and so, on the suggested basis, they could be of the same class. But what if I have to describe the colour of a signal-light when it flashes? In this case can I state 'black'? The numerals are even more similar in their use than the colour words; it looks at first as though they were interchangeable in all cases; but in fact they are not. We can say of a playground that it is divided into two parts, three parts, four parts, but not that it is divided into one part. The suggested criterion in this case would, then, separate the number 'one' from the other numerals. It would be easy to multiply examples of this kind.

So we see that, in practice, the suggestion advanced if acted upon does not fulfil its promise. The reason why this is so may perhaps be made clear by a comparison. Suppose we have to arrange various tools according to the similarity of their uses. How shall we set about performing this task? The auger and the awl belong together, they are closely related. But what about hammer and nail? The one hits, the other is hit. Better to put the hammer and the axe together. Or ought we to put the axe with the drivingchisel? Do the shears go with the knife or with the pincers? The fact of the matter is that the tools have different sorts of similarity, and so can be grouped in more than one way. There will be arbitrary features even in the most carefully chosen classification. The case is quite similar with words which are 'the tools of thought'. With words also, all that is possible is a sort of classification according to typical use, without attempting to attain precision in every detail. Anyone to whom these remarks appear too vague should consider that precision consists in describing things as they

are, and in not drawing precise lines of demarcation where in fact there are none. If we constructed an artificial language we could naturally define precisely the use of the words in it. But if we wish to describe actual language, it is essential to admit that its structure is very irregular, a maze of lines running this way and that; and then the demand for absolute precision will be felt as an unnatural one.

School grammar provides us with a sketch of a variety of the constituents of language by arranging words in certain categories such as nouns, adjectives, verbs and so on. This classification is by no means worthless, but it is too rough and primitive for our purposes; if we were content with it, we should have to say that words with such fundamentally different uses as 'moon' and 'justice', or 'walk' and 'exist' belong to the same category. This we must refuse to do if the classification is to reflect finer details of the architecture of concepts, it must employ a far larger number of classes. We shall have to speak, not of nouns, adjectives and verbs, but of colour words, numerals, shape words, sound words, texture words and so on. Shall we also speak of metal words, poison words, fish words?

To clarify this point let us contrast the concepts 'metal', 'poison', 'fish' with the concepts 'colour', 'number', 'shape' and see what the comparison reveals. But first: how do we attain to comprehension of such words? The meaning of 'poison' or 'metal' is explained by adducing different properties characteristic of a poison or of a metal. On the other hand, what about explaining the meaning of 'number' to someone, how should we go about this? Perhaps we would say '1, 2, 3, 4, 5, and so on, are numbers'. That is, we give a series of instances, and add 'and so on'. Experience shows that this is the way a child in fact learns the use of the word 'number', that is, in general a child is able to carry out an order like 'Tell me a number' correctly. On the other hand, we do not give a child an explicit definition of the concept 'number'; we do not, for instance, say to him 'A number is a relation between two magnitudes' (Newton's definition); in fact, we would be puzzled ourselves if we were asked to give such a definition. This last fact is significant, for it shows that no such definition is naturally familiar to us.

The meaning of 'colour' is explained in a similar way. We say 'Red is a colour, yellow is a colour, green is a colour, and so on' but we do not give a formal definition. Compare the propositions:

'Phosphorus is a poison.'

'The whale is not a fish.'

with the propositions:

'Red is a colour.'

'5 is a number.'

What does this contrast reveal? The first two propositions are experiential. The effects of phosphorus must have been observed before it can be known that it is a poison. But what of the sentence 'Red is a colour'? Is this an experimental proposition? What experimental fact does it express? Can we imagine observations that would falsify it? Unquestionably, no. Perhaps the reader will say that it just lies in the nature of red that it is a colour. Challenged to explain the meaning of this phrase he may perhaps answer that a certain necessity inheres in the statement 'Red is a colour', that we cannot imagine it not being true. It is natural to ask on what this necessity rests.

By way of answering this question let us consider in what circumstances or for what purpose we should actually utter such a sentence. There are different cases in which it might be used:

(i) I wish to explain the idea of colour to somebody and say 'Red is a colour, yellow is a colour, and so on'. In this case the expression 'Red is a colour' is part of the explanation of the meaning of the word 'colour'.

(ii) A child understands the word 'colour' but does not know the meaning of 'beige'. If I say to him 'Beige is a colour', I thereby give him a hint how he is to use the word. I am letting him know that the use of the word is similar to the use of the word 'yellow', 'green', 'brown', etc., which he already knows; that is, I indicate, as it were, the section of linguistic territory to which the word belongs. (What is lacking still, of course, is an indication *which* colour it is, i.e. the ostensive definition.)

(iii) If the words 'red' and 'colour' are already understood, the purpose of the sentence 'Red is a colour' cannot be either of those described. But there is another case in which it might be used. Suppose that someone is told to paint a wall some colour and he paints it red. He is taken to task for this and says 'I was to paint the wall some colour or other, and red is a colour'. In so saying he would be giving a justification of what he had done. Here the sentence 'Red is a colour' reminds the other of a point about how 'colour' and 'red' are used; *hence* its obvious truth. We can now answer the question with which we began, as to why the names of colours, but not those of poisons, make up a special class of words. It can be told from the usage of language that red is a colour, but in this sense it cannot be told from the usage of language that phosphorus is a poison. Since in its contexts in sentences the word 'phosphorus' has a function quite analogous to that of words like 'sulphur', 'iron', 'coal', etc., there is no good reason for grouping the names of poisons into a special class. Thus in order to avoid misunderstandings we may say that 'Phosphorus is a poison' asserts a property of phosphorus, whereas 'Red is a colour' does not assert a property of red.

Let us consider yet further the sentence 'Red is a colour'. It is in different ways analogous to a variety of sentences - for example, to 'Vanilla is a flavour'. If we have this analogy in mind we are tempted to say that 'Red is a colour' is about red, states a quality of red. This view leads to the difficulties already mentioned; for example, because it seems to make no sense to deny such a statement or to describe circumstances in which it would be false, people have concluded that it expresses a necessary truth. In order to avoid the dangers of such a view as this it is advisable to turn our attention to various other analogies. For example, 'Red is a colour' is like 'Red is a property' or 'Walking is an activity'. Yet we should hardly be so ready to say that these statements are about 'red' or 'walking'. 'Red is a property' means 'the word "red" is used as a property word',¹ it says something about the part played by the word 'red' in our language the appearance it has of saving something about red is due only to a superficial analogy between its form and that, for example, of 'Iron is a metal'. The same is true of 'Walking is an activity'. We could instead say 'Walking is called an activity', or ' "Walking" is an activity word'.

There are other considerations which lead to the same point of view. What statements exclude 'Red is a colour'? What opinion do we wish to oppose or to correct when we assert such a proposition? 'Red is a colour' is on a level with 'Red is not a thing', 'Red is not an activity', 'Red is not a shape', etc., and remarks of this sort are not made without cause, i.e. unless someone is in doubt as to the use of these words: if, for example, he has on being shown a piece of red paper, mistaken the ostensive definition of 'red' for an

¹ 'Red is a property' could also be used along with other examples to explain the expression 'property'.

explanation of the word 'paper'. We could then try to correct his misunderstanding by saying 'Red is not a thing'. And similar misunderstandings may be imagined to evoke the other negations. We may now be tempted to say 'Red is a colour' means 'The word "red" is a colour word'. But this would not be altogether

We may now be tempted to say 'Red is a colour' means 'The word "red" is a colour word'. But this would not be altogether true. For in the aforementioned example would it not be curious if the person who had been told to paint the wall some colour, instead of saying 'I had to paint the wall any colour, and red is a colour', were to say 'I had to paint the wall any colour, and "red" is a colour word'?

We might therefore put the question as follows: is 'The word "red" is a colour word' a *translation* of 'Red is a colour'? If so, then the one must always be substitutable by the other. But this is not the case. Let us, however, remember that the expression 'Red is a colour' can be used for various purposes (the three examples on page 99), and that sometimes, particularly in philosophical discussion where one looks upon the expression divorced from its usage, it is not clear what is meant by it. If someone is perplexed as to what it means, or inclined to suppose it embodies a piece of *a priori* knowledge, we can direct his attention to the large number of cases which bear greater or less resemblance to the one in question, and which are more obviously about language. Eventually when we ask such a person 'Could you bring yourself, instead of saying "Red is a colour", to say "Red is called a colour" or "The word 'red' is a colour word"?' — it is evident that we are not asserting that these expressions mean the same, but are suggesting to him a new way of speaking. Naturally we cannot force anyone to adopt this new usage; but if anyone adopts it, and notices that he is thereby freed from tormenting confusions, that it assists him in finding his bearings, we shall have influenced him in a way we wished.

Perhaps we were going too far in our criticism. The sentence 'Red is a colour' naturally suggests various expressions in our language which are formed in analogous ways. One series of analogues is, for instance: 'Red is a quality', 'Red is a predicate', 'The word "red" is a predicate', "Red" is a colour-word'. But there is another analogy with such sentences as 'Vanilla is a flavour', 'Quinine is a febrifuge'. Up to now we have been pursuing the first analogy, throwing into relief those points which assist us in passing from the original expression to the others. Let us now ask whether it is not possible to follow up the second, i.e. to interpret the sentence 'Red is a colour' in such a way that it describes a fact of experience. This is quite possible. Let us start from the remark that white and black are commonly contrasted with colours in the strict and proper sense (cf. 'illustrations in colour' and 'illustrations in black and white'). There are however, certain situations in which we are inclined to treat white as a genuine colour as, for instance, in the case of the white in the tricolour. This fact may be expressed by saying that, in certain special contexts, white makes the impression of a genuine colour. Supposing a psychologist were to inquire into the conditions necessary for the occurrence of such a phenomenon, then he could summarize his conclusions by saying: white produces the effect of a colour only in such and such circumstances; red, on the other hand, is always felt to be a colour; or more briefly: red is a colour. Even if this phrase appears not quite natural, it might be adopted as an intelligible abbreviation: and we can even recognize the naturalness of such a sentence as 'White is not a colour in the way in which red is'. In making such a statement he would be describing a certain fact of experience — for it is conceivable that, even in those circumstances, white might not have been felt as a colour.

The conclusion is that we may interpret the words 'Red is a colour' in such a way that they describe a fact of experience, although there are only rare occasions in which such an interpretation would be useful. The fact that we can interpret the sentence in two different ways helps us to understand how the philosophical trouble arises. It is important, in this connection, to notice that, in every suggested interpretation of the sentence, we are following the lead of certain analogies, even if these analogies are not expressed or formulated, but only felt in a dim, half-conscious way. These analogies, blurred and nebulous, form, as it were, a background, which acts upon us almost like a kind of magnetic field, pulling us in two opposite directions: on the one hand, these are analogies like 'Red is a quality', 'Red is a predicate' that force themselves upon our minds and suggest strongly one sort of interpretation. On the other hand there are analogies like 'Vanilla is a flavour', 'Quinine is a febrifuge'; and the pursuit of this second series of analogies makes it appear as if 'Red is a colour' ascribes a property to red; this leads us to the problem whether such a statement is necessarily true. The whole problem is but the result of two conflicting forces, trying to assimilate the expression to two different patterns.

It is best not to make any general remarks, but to examine actual cases of the use of 'Red is a colour'. We will then see that among these various uses there is none in which this sentence could be used to make an *a priori* statement about red. Compare with 'Red is a colour', the sentences '5 is a number', 'Frank is a Christian name', 'Tuesday is a weekday', 'The bishop is a chessman', 'Negation is a logical concept'. They all admit of a formulation having reference, not to reality or to 'ideal objects', but merely to the use of words. And it is only in this guise that their sense is clearly revealed. What ought we to say in reply to the question whether zero is really a number? Types of words might be ranged under headings such as 'colour', 'shape', 'number'. A proposition like 'Red is a colour' draws attention to the fact that 'red' is included in a certain one of these classes.

But is not colour the determinable of which red is a determinate, a more generalized concept including red? No doubt; but colour is a more generalized concept including red in a different sense from that in which metal is a more generalized concept including iron. It is usually stated in the textbooks of logic that we pass from more particular concepts to more general ones by a process of abstraction. We are said to reach the concept 'metal' from the concept 'iron' by suppressing certain attributes while retaining certain others. But we cannot pass from the concept 'red' to the concept 'colour' in this way (nor from 'This shade of red' - pointing - to the concept 'red'). In the case of what is usually called abstraction the type of words remains the same throughout the process; generalization of the sort illustrated in the examples just given leads from one type of words to another. Thus the relations between concepts to which the term 'relations of superordination' can be applied are of very different sorts, and the word 'generalization' has correspondingly different senses.

4. CATEGORIES. Just as we group words into 'types of words' according to similarities of their use, so we might group a number of types of words whose use is closely related into wider classes of words. Shape, colour, hardness, etc., can be brought together under the general term 'quality'. Accordingly, quality constitutes something like a second-order class of words. (Sometimes 'red' and 'sweet' are adduced as members of this class, sometimes 'colour' and 'taste' are given instead.) In the case of these second-order classes the vagueness of this concept-formation by abstrac-

tion, rooted as it is in analogy, shows itself still more plainly. We are much less able to define exactly the limits of the idea of quality than those of the idea of colour. It seems clear, to start with, what a quality is: namely, whatever is expressed by a quality word. Thinking of this, one has in mind chiefly a few typical quality words like 'red' and 'sweet'. But if we ask whether 'impossible' is a quality of a round square, or if 'true' and 'false' are qualities of sentences we begin to waver. We can say about this only that these words have a use that is very unlike the use of 'red' and 'sweet'. The question whether they are *really* qualities is more or less on a level with the question whether zero *really* is a number.

The concept 'relation' is similarly vague and blurred, and so is the concept 'object'. Both 'love' *and* 'divisibility' are adduced as examples of a relation. We say 'The table was littered with small objects of a special kind'; but we also say 'For these tribes the goddess is an object of veneration', and we speak of the object of an investigation. These examples throw a light on the extensive and fluctuating uses of these words.

The classification of words according to school grammar has much more far-reaching effects than might at first be thought; most attempts to construct philosophical systems of categories have been influenced by this classification. Things have been made to correspond to nouns; properties to correspond to adjectives, processes or activities to correspond to verbs, and so on. With this model in mind, concepts like 'state', 'event', 'relation', 'number', 'structure', 'reality', 'possibility', 'necessity' have been elevated to this status of *categories*, made into eternal, inescapable forms of thought. What can be said about these highest concepts is that they are also the most obscure. Think of the innumerable controversies that each of them has generated in the history of philosophy. Our intention is to replace them by a more exact scheme, corresponding more closely to the realities of language.

5. OSTENSIVE DEFINITION. Before we follow the trail of investigation any further, let us return to the idea of ostensive definition and try to gain a clear view of it. What different sorts of cases does the term cover?

(i) someone points to an apple or pear and says 'Apple' or 'Pear', or points to a person and says 'That is John'.

- (ii) I span a length with two fingers and say 'That is an inch'.
- (iii) It is possible to speak of an ostensive definition also in the

case of tone. While a tone is sounding, or standing forth against a background of others, I may call attention to it with a gesture; for example, I can point in the direction it comes from, or motion for silence and say 'Listen, a tone is sounding', or 'That is high C'.

(iv) Is there also an ostensive definition in the case of the numerals? For the initial terms of the series of integers the answer is 'Yes'. I point to a group of things whose number can be seen at a glance, or ring it round with a motion of my hand, and say 'Two nuts' or 'Three apples', etc.; and this is a serviceable way of making the meaning of these words clear. With larger numbers, of course, this method of explanation breaks down, and this calls attention to the distinction that could be described as that between 'visual number' and 'inductive number'. That the first four numerals are declined in Greek seems to indicate that a difference was felt between the numbers that can be taken in at a glance and the others.

(v) How is negation explained? A child is told 'No, no more sugar' and the sugar-lump is taken away; or someone says 'Don't make such a noise!' and puts a hand over the child's mouth. The meaning of negation could also be explained by drawing a figure and saying 'Look, this point is in the circle, that one is not'.

(vi) We say 'Give me the apple *and* the pear' and accompany the words with a comprehensive gesture; or we say 'Will you have the apple *or* the pear?' and present the other person with the choice in a gesture. Do these gestures explain the meaning of 'and' and 'or'? This is not easy to decide. At any rate they help the other person to understand these words; and in so far as they do this no objection need be raised to calling them part of the explanation of the meaning.

In each of these cases attitudes and gestures enter into the explanation of the meaning of a word. But the reader will observe that the parts played by the gestures in the different cases are very different. We cannot point to a tone in the way we can point to a body; and to a logical constant, such as 'not' and 'and', we cannot point at all. Does this mean that we should give up talking of an ostensive definition in these cases? But where exactly are we to draw the line? Are we to say 'Up to here it is an ostensive definition, beyond here it is not?' Surely, it is purely arbitrary to draw a line anywhere. Our purpose in juxtaposing these widely different cases of ostensive definition is not to show that we do not yet know where the boundary line runs, but on the contrary to show that no exact boundary line is to be found. We just produce the examples and leave it open to each to decide for himself what he will or will not call ostensive definition.

A further question arises: can an ostensive definition be misunderstood? For instance, if I point to a red piece of paper of elliptical shape and say 'That is elliptical', can I be sure that the person to whom I speak the words understands what I mean? If all that I do and say is to point and say the words, he may misunderstand me. He could take 'elliptical' to mean any one of the following: elliptical, red, brightly coloured, paper, smooth, thin — and perhaps many other things also. But if I say 'The colour is red, the shape is elliptical', the mention of the type words prevents misunderstanding.

We now begin to see the value of these names of word types. The learning of language is achieved by stages. The first stage consists in learning the use of expressions like 'cube', 'column', 'white', 'red', '1', '2', '3'. At the second stage the meaning of second-order concepts like 'shape', 'colour', 'number' are explained with the help of the words already learned. At the third stage these secondorder concepts are used to shorten the process of ostensive definition and to prevent it being misunderstood.

One might here speak of a stratification of concepts. But the order of the strata is not quite unambiguously determined. A person can first be taught the words 'oak', 'maple', 'fir', and taught the word 'tree' *afterwards*; or this order might be reversed. But can the meaning of 'number' be explained before the particular numbers are taught? Why not? What about the intermediate case of 'colour' and particular colours?

Compare the definitions 'Red', 'That is red', and 'This colour is red', where the person giving the definition points to a sample of the colour in each case. Are these all definitions in the same sense? Perhaps there will be an inclination to deny that the first is a definition at all (at least in certain circumstances), and to call it instead an early stage in a training. The second is what might be called an articulate definition, but is liable to be misunderstood in various ways. It is only the third that is given to someone familiar with language. Accordingly, we can distinguish between primitive and fully developed forms of ostensive definition — without indeed being always able to specify the point exactly where the 'definition' begins and 'training' ceases. And this constitutes a further uncertainty which we find in our concept.

6. MUST THERE BE OSTENSIVE DEFINITION IN EVERY LAN-GUAGE? What part does ostensive definition play in the examples we have given? It may seem that the act of ostensive definition is the link between language and reality; there may be an inclination to say that there must be a system of ostensive definitions in *every* language. So let us compare some examples of other languages with the language of sounds we use.

(i) Imagine a language made up entirely of written signs, a sort of ideography for dumb people. Suppose that the people who use this language, as well as being dumb, employ very few gestures in communicating with one another. What form might the teaching of what would correspond to the building-stone game take in this language? Perhaps A might train B as his helper in the following way. He hands B a scrap of paper on which a sign for 'cube' is written. This is to correspond to the order to hand him a cube. B has been given a table that co-ordinates the ideograms with pictures of the corresponding building-stones, and B already knows how to use such a table in learning the use of signs. B compares the signs given with the signs in his list till he finds the right one, runs his eye from it to the picture opposite, and takes a stone of the corresponding shape. It may be objected that this is not how the process would actually develop; rather would the connection between the sign and what it signifies be furnished by association. But this is immaterial, and in many cases not even true. For instance, a chemist's apprentice is given a piece of paper with the name of a substance written on it, and he may seek for it by going from jar to jar till he finds one with the same name written on it. And someone in a strange town may look for a street by comparing the address he has written down with the sign-plates marking the names of the streets.

Is there anything corresponding to ostensive definition in the game just described? In ostensive definition the gesture serves, as it were, to establish a connection between a word and an object. What establishes the connection between the signs and the building-stones in this example? Or — it comes to the same thing for our purposes — what establishes the connection between the signs and the *pictures of* the building-stones? The answer is: their relative spatial position. If the signs were on one sheet of

paper and the pictures on another, we would not know which sign corresponded to which picture. Thus in this example the juxtaposition of signs and pictures takes the place of the gesture in ostensive definition — we could use the term 'ostensive definition' of this case also, if we wished — though, of course, its sense would be quite different.

(ii) What about the case of a picture language, say a language of hieroglyphics? In this case very much can be expressed without ostensive definition. But could everything be so expressed? A row of pictures can tell us a story; but by itself it cannot tell us any history. It is not clear from a mere picture of a little figure with a crown on its head that this represents King Kambyses. It is only if the picture is inserted in a text, or if the name of the king is written beneath it that we understand this. (What connects a name with a person in this sort of case, if it is not an ostensive definition, will be seen later on.) It might be said that the likeness between the picture and the person it represents shows for whom the picture stands; and in many cases this is correct, but what if the likeness is a very bad one? We should then have to say either that it is not a portrait at all; or that it is a portrait of whoever it looks like; and to say either of these would be contrary to the sense in which the word 'portrait' is used.

(iii) But the non-essential character of ostensive definition becomes manifest when we turn to an entirely different kind of language: a language of gestures. If in such a language an order to eat an apple is given by first pointing to an apple and then making a gesture of eating, is there anything corresponding to ostensive definition in this? Obviously not; the gesture here does not serve to *explain* a meaning; it is not a preparation of an expression, but a *part* of an expression. It might be asked if in talking of a gesture language we have not really let ourselves be carried outside the territory of language proper. Are gestures perhaps *reality* in contrast to language? No, for a measure of understanding is possible, communication can be carried on and orders issued in a gesture language, though ostensive definition in the proper sense is lacking. 'Ostensive definition', as we have explained the term, is characteristic of a particular sort of language, not every language.

7. THE CONCEPT OF A SIGN. In the previous sections we have spoken a good deal about signs, but we did not define precisely what a sign is. This will in the eyes of many appear as a serious defect. Ought we not sharply to delimit our theme? We shall disappoint those who have such hopes, in that we shall not try to lay down an exact definition of the concept 'sign'. We intentionally draw no sharp boundaries; we say 'Words, spoken or written, gestures, drawings, signals such as the waving of a flag or the flashing of a light, are signs, and so also are many other things more or less related to these', and by adding this last clause we refuse to tie ourselves down. It must not be thought that we make a virtue out of necessity; that we are simply not in a position to give a clear definition. It would, of course, be easy to fix the concept rigidly by saying, for instance, 'Only things of such and such a form are signs', and enumerating four or five kinds. Yet how can we be sure that this enumeration satisfies us, and that we will not later find ourselves wanting to include other things under the name 'sign'? Have we, for example, borne in mind that the pitch of a voice, the breaking of a twig, and innumerable other things, can function as signs, as means of communication?

Yet should we not aim at finding what is common to these examples and bringing it into a clear formula? But what have they in common? Is it that they are used according to fixed rules? But the pieces used in a game are used in accordance with rules, and we do not call them signs. If it is said that there must be a definite intention behind the sign, for instance, an intention to communicate, we must remember that this also holds only in certain cases; we often speak without expressly intending to communicate anything, as in the case of the soliloquy. If it is said that it is a matter of choice how we determine the limits of the concept 'sign', that I can arbitrarily decide that only such and such structures are to be called signs, we can object that such a delimitation would be quite unnatural, a mere dead letter by which no one would seriously be bound. If we abide by the use of language we can say only that the concept is not exactly delimited. It is then best to give a few typical examples and leave it open to the person to whom we are talking to decide how far he allows the concept to extend.

If this laxity, this lack of precision, seems to be incompatible with logic, we may consider the following points. Many of our commonest concepts rest upon dimly felt analogies, and it is to these that a considerable part of the expressiveness of language is due. If our concepts were really as rigid as the textbooks of logic make us believe, we should soon come to the boundaries of communication, and would have to be continually inventing new words. It is therefore useful for concepts to be left to some extent open; language is thereby flexible and able to adapt itself to describing new situations as they arise.

Perhaps the following example will illuminate what has been said. Let us suppose that someone waves a handkerchief in the air. This can be regarded as making a sign; or else we can say that he waves it simply because the fancy takes him. What is the difference between the two? There are a number of differences. If someone wishes to make a sign he places himself where he can be seen by as many people as possible. Moreover, it can be seen from his expression and bearing that he wishes to communicate something. In short, if we call something a sign, we include it among a large number of familiar phenomena. It is in the context of these phenomena that the process of making signs is manifested.

We cannot conceive of a 'sign' unless we also think of the complex machinery of which it is a part. The complete interdependence between signs and our own life can be seen if we attempt to divorce the two. Let us envisage, for the sake of example, two imaginary spherical creatures; now, what exactly does it convey to us if we say that these theoretically constructed creatures 'make signs'? If the one creature emits a scream, and continues to scream until the other rolls towards it, then we could interpret this screaming as the expression of a wish or a craving. If, on the other hand, instead of a scream we were to witness the appearance of a red patch on one of the creatures which also continued until its companion rolled towards it, then we should hardly be justified in interpreting its conduct, and any resemblance to what we ourselves call signs would have disappeared. To attempt an interpretation under these circumstances would be as pointless as to refer to the expression of a shelf, or the attitude of a wall. Thus the difficulties with which we are confronted when we attempt to define a sign are analogous to those which we have to face in endeavouring to define life itself.

••••••

Pages 111 ... 418 deleted

••••••

INDEX

'able', 342-5 'Achilles and the Tortoise', 87-88 affirmation, 298 'all', 383-5 analogies, linguistic, 60, 63 antinomies, 88-90 a priori, ch. III; synthetic a p., 67-68 Aristotle, 27 arithmetic, applications of, 55-57; a. games, 51; primitive a., 78-79; 'pure' and 'applied' a., 51-53, 55-57 assertion v. description, 33-34; a. sign, 302-3 Augustine, 40-43, 94-95, 153, 172, 174, 179, 412 beauty, 84-85 Becher, E., 17 n. Berkeley, G., 72 'block/slab game', 95 body, change of, 25 Boltzmann, L., 48, 77 Bolyai, 48 Bolzano, B., 31 Brouwer, L. E. J., 395-6 calculus, logical, ch. xix, 377-86; propositional c., 372-4; c. versus causal mechanism, 122, 124 Cantor, G., 398 Carnap, R., 197, 270, 275, 277, 407 categories, 104 causality, 9 cause v. reason, 119-21 Chomsky, N., 368 Chrysippos, 27

clarity, lack of, 9-14

'clear' and 'unclear', 9

'colour', 99-103, 164-5

colour words, 228-32, 237-9

Clifford, W.K., 48

classes, 88-89

commands, 114–15, 116–19, 134–5, 141 communication, 245; problem of c., 240-1; d. of structure, 241-3 concepts, 226-8; blindness for c., 355-7 consciousness, 18-20 continuity, 48 contradiction, 367, 375-6 Darwin, C., 81 deception and language, 75-76 definition: d. as cause of usage, 125; exhaustive d., 222-5; ostensive d., 94-96, 104-10, 195-6, 200, 206, 216-20, 225, 277-9, 291-2; purpose ofd., 03: d. as reason for usage, 125 Descartes, R., 3, 15, 405 description, 269-70; relational d., 272-7; structural d., 270-7 desires, 117 determinable and determinate, 103 dispositions, 343-5; d. and understanding, 359 doubt, 15, 17 Einstein, A., 11, 12, 48 empiricism, 58 Epimenides, 84 equations, 50-57 Euler, L., 10 'exact' and 'inexact', 208-12, 223 existential quantifier, 385-6 experience: of colours, 23-25; incommunicable e., 265-8; sameness of e., 24-26, 241-8; subjective e., 257-9 explanation, 149-50, 163-93 fact, 282-3 'false', 285-8, 290-1, 293-5, 298 'families' of concepts, 179-87 formalization, 368 Frege, G., 4, 46, 72, 164, 181, 183, 228, 234-6, 313, 373, 377, 401

games, 180, 182 Gauss, K. F., 48 general terms, 7 generality, 383-6 geometry, as grammar, 44, 48; g. and experience, 46-47, 48; natural g., 49; g. of visual field, 49-50 gestures, 105, 107, 108, 195 Goethe, 80-81, 178 grammar: 13-14 reasons for g., 40; rules of g., 34-40; 'school' g., 98; veiled rules of g., 58-59 Helmholtz, H., 48 Hjelmslev, 49 Hofmannsthal, H. von, 178 Hume, $D_{.,9}$

Idealism, 7 ideas, 295-8 identity, 200-4, 223, 227; i. of persons, 213-16 impossibility, 339-40 inference, 368-71 'is', 35-37

James, W., 19, 215, 352-4 judgement, 285, 301 judging, 300

Kant, I., 3, 7, 9, 67, 412 Kepler, J., 84, 412 Klein, F., 47 'know', 345-6 knowledge, 84, 151-2

language; analogies in l., 176-9; confusions in l., 4; conventions of l. 289; l. games, 71, 95; ideal l., 207-12; intertranslatability of languages, 252-3; learning of l., 94-96; physical l., 260-4; private l., 259; scope of l., 93; unlearnable languages, 252-6 Leibniz, G. W., 375 Letvis, C. I., 240 Lobatschewski, N., 48 Locke, J., 3, 72

logic, justification of, 374-5; non-Aristotelian 1., 380-2; ordinary thought and l., 382-3; as a theory, 377-80 Lorentz, H. A., 12 Lukasiewicz, J., 27, 34 lying, 204-5 Malinowsky, 193 Maxwell, J. C., 77 meaning, 153-62; analysis of m., 13; 'body of m.', 234-7; m. as an effect, 115-16, 119; explanation of m., 126, 144-5, 163-76; m. as image, 158-61, 352-5; meanings of m., 155-6; m. of names, 69-78; m. and notation, 372; m. as object, 312-13; picture theory of m., 307-10; 'real' m., 77; m. and rules, 65; m. as 'shadow of reality', 335-7; m. of signs, 116; m. as use, 156-8, 176, 213, 319; m. and verification, 329-31 meaninglessness, 38-39, 325-9, 333 meaningless question, 414-16 Meinong, A., 60, 197, 301-2 memory, 6–7, 15–22 metaphor, 177, 179 metaphysics, 81-86 Mill, James, 18 Mill, J. S., 58 mind of another, 23 Mitchelson-Morley experiment, 12 Moore, G. E., 4

names, 194-220; n. of colours, 228-32; generic n., 221-39; paradigm of n., 194; proper n., 212-13, 226-7; n. of species, 221-39; n. of tones, 225-7 negation, 105, 232-3, 298 Newton, I., 9 Nietzsche, F., 251 'nought', 83 numbers, 164-8, 185-6 numerals, definition of, 105 objects, 197, 204-5, 226, 228, 313-14;

objects, 197, 204-5, 226, 228, 313-14; naming of 0., 197-200 ostensive, *see* definition, ostensive. Peano, G., 373

- persons, 213-16, 262-3
- phenomenalism, 58, 60
- philosophy, crisis in, 3
- Plato, 6, 7, 84-85, 196
- possibility, 338-42; logical p., 339
- problem, as confusion, 13; mathematical p., 396-8; mathematical v. philosophical p., 10; philosophical p., 5-9, 78, 179; senses of p., 5
- property, 222
- propositional calculus, 372-4
- propositions, 280-303; combinations of p., 364-76; p. express thoughts?, 284, 295-8; types of p., 298-300
- qualities, 103-4; q. and predicates, 101-2
- questions and answers, 410-13; q. versus answers, 3, 4, 5; q. and assertion, 405-6; grammar of q., 405-17; identity of q., 409-10; insoluble q., 26; logic of q., 387-417; meaningless questions, 414-16; philosophical q., 6, 8, 9; q. and requests, 407

reason v. cause, 119-24

- 'red and green', 57-67; r. and g. as synonyms, 61-63, 64
- Reichenbach, H., 368
- 'reliable' and 'unreliable', 20-22
- 'represents', 311-12
- Richards, I. A., 193
- Riemann, G. F. B., 48
- rules, 13, 82-3, 218-19, 233; concept of r., 137-40, 141-4; essential and inessential r., 190-2; r. of games, 129-30; grammatical r., 34-40, 66, 68, 135-6; r. as guides, 132-3, 151; r. in a language, 131-2, 234-7; r. as statements, 60; validity of r., 147-8
- Russell, B., 4, 116-7, 164, 206, 269, 372-3, 377-83, 401

```
'same', 23, 201-4, 249-50
samples, 278
scepticism, 21
```

Schlick, M., 241-4, 264, 304-7, 311, Schopenhauer, A., 6, 7, 154 'search', 387-400; s. for the impossible, 398-400; s. in mathematics, 391-6; s. and questions, 405

sensations, 25

315

- sense-data, 72-75, 83
- sentences, 251-3
- series, sum of, 10-11, 13
- Sheffer, H. M., 402
- signals, 112, 128, 192-3
- signs, 108-10, 112, 113, 192; s. versus symptoms, 127-8
- simultaneity, 11
- Socrates, 84, 405
- species, 221, 225
- Spinoza, B., 179
- structure, 243, 244, 272, 314-16; s. and communication, 304-22

supposals, 300-3

- tautology, 367, 375-6
- 'thing', 204-5
- 'this', 205-6, 212
- time, 31-33, 172-6; measurement of t., 40-43; t. and truth, 27
- 'true', 285-8, 290-1, 293-5, 298; 'p is t. = p', 30
- truth, 27-34; t.-functions, 364-7; t.functional constants and ordinary language, 367-9; t. as a timeless quality, 28-29, 32
- 'trying', 183-4
- 'understand', 256, 346-8 understanding: degree of u., 361-2; u. as an experience, 360-1; u. the meaning, 325, 348; u. as a process, 351-2; u. a question, 413-14; u. and reality, 362-3; u. a word, 349-50, 357-8

vagueness, 69, 71 verification, 253-4, 323-4, 325, 331-3

Weininger, 162 Weyl, H., 225

 Whitehead, A. N., 273, 401
 different meanings of w., 187-90;

 wishes, 117, 141, 143, 289
 explanations of w., 144-6, 163; sen

 Wittgenstein, L., 4, 8, 31, 71, 95, 307 suous and mental w., 176-7; types

 20, 369, 372
 of w., 96-104

 wonder, as a source of philosophy,
 'world', 85

 6
 Zeno, 13

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD THE UNIVERSITY PRESS, GLASGOW

422