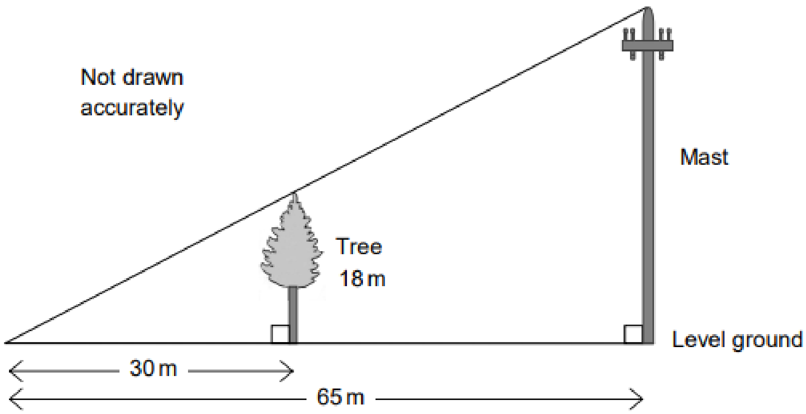
All questions are taken from AQA Level 2 Further Maths papers that are free to find online and are questions that could be found on a non-calculator GCSE Maths exam.

This resource is designed for students revising GCSE maths and not Level 2 Further Maths. The paper numbers are given for refrence only.

Practice Paper Set 1 Paper 1

3 The diagram shows a tree of height 18 metres and a mast on level ground.



The mast is about to fall over, pivoting about its base.

Could it hit the tree?

Show clearly how you decide.

....

....

....

....

....

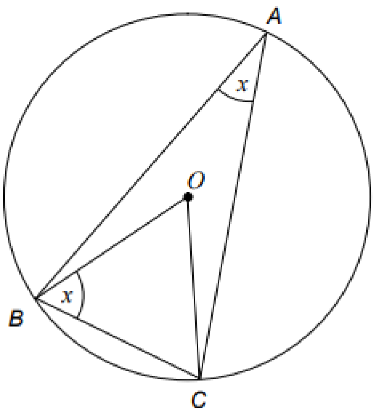
....

....

(4 marks)

8 A, B and C are points on a circle, centre O.

Angle BAC = angle $OBC = x$.



Not drawn accurately

Prove that angle $BOC = 90°$

....

....

....

....

....

....

(4 marks)

10 Simplify fully $\dfrac{3x^2 - x - 14}{9x^2 - 4} \div \dfrac{x + 2}{3x^2 + 2x}$

....

....

....

....

....

....

....

....

....

Answer .... *(5 marks)*

12 Make x the subject of $\frac{12}{y} = \frac{4}{x} - \frac{1}{3}$

....

....

....

....

....

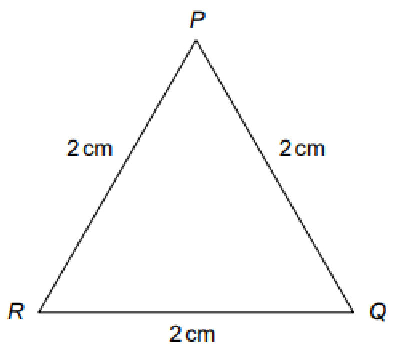
....

....

....

Answer .... *(5 marks)*

15 (a) Use the equilateral triangle *PQR* to show that $\cos 60° = \frac{1}{2}$



Not drawn accurately

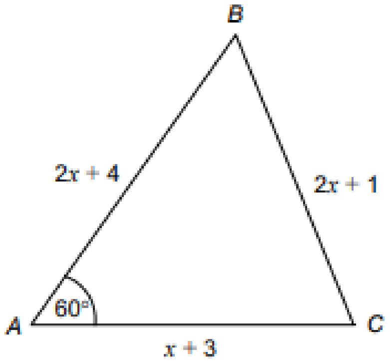
....

....

....

(2 marks)

15 (b) In triangle ABC, angle $BAC = 60°$



Not drawn accurately

Use the cosine rule to show that $x = 4 + 2\sqrt{7}$

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(6 marks)

3	(tan x) $= \frac{18}{30} = \frac{m}{65}$	M1	oe eg, $\frac{65}{30} = \frac{m}{18}$
	$m = \frac{18}{30} \times 65$	M1	
	39	A1	
	(65 – 30 =) 35 and their 39 and Yes	B1 ft	

8	Angle *BOC* $= 2x$ Angle at centre = 2 × angle at circumference	M1	
	Angle *BCO* $= x$ Isosceles triangle	M1	Isosceles triangle
	$x + x + 2x = 180$ Angle sum of triangle = 180	M1	
	$2x = 90$	A1	

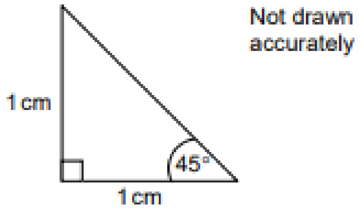
10	$\frac{(3x-7)(x+2)}{(3x-2)(3x+2)}$	B2	B1 For numerator B1 For denominator
	$(3x^2 + 2x =) \quad x(3x+2)$	B1	
	(their fraction) × $\frac{x(3x+2)}{x+2}$	M1	
	$\frac{x(3x-7)}{3x-2}$ or $\frac{3x^2-7x}{3x-2}$	A1	

12	Multiplies throughout by x or y or 3 or xy or $3x$ or $3y$ or $3xy$	M1	
	$36x = 12y - xy$	A1	
	Collects terms in x on one side from their equation eg, $36x + xy = 12y$	M1	
	Factorises to $x(\ldots\ldots..)$ eg, $x(36 + y) = 12y$	M1	
	$x = \dfrac{12y}{36 + y}$	A1	oe

15(a)	Shows 60° angle and a right-angled triangle (with right angle marked) and side 1 (cm) marked	B2	B1 Any 2 of the 3 criteria shown
Alt 15(a)	$2^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \cos 60$	M1	oe
	$4 = 8 \cos 60$	A1	
15(b)	$(2x + 1)^2 = (2x + 4)^2 + (x + 3)^2 - 2(2x + 4)(x + 3)\frac{1}{2}$	M1	
	$4x^2 + 2x + 2x + 1 = 4x^2 + 8x + 8x + 16 + x^2 + 3x + 3x + 9 - (2x^2 + 4x + 6x + 12)$	M1	Any of the 4 term expansions or all four with ≤ 3 errors
	$4x^2 + 2x + 2x + 1 = 4x^2 + 8x + 8x + 16 + x^2 + 3x + 3x + 9 - (2x^2 + 4x + 6x + 12)$	A1	All correct
	$x^2 - 8x = 12$ or $x^2 - 8x - 12 = 0$	A1	oe Must be simplified to 3 terms
	$(x - \frac{\text{their } 8}{2})^2 = \text{their } 12 + (\frac{\text{their } 8}{2})^2$ or $\dfrac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times -12}}{2 \times 1}$	M1	oe Substitutes $x = 4 + 2\sqrt{7}$ in their equation
	$x = 4 + \sqrt{28}$ Must reject the other solution	A1	Shows substitution satisfies the correct equation.

Practice Paper Set 2 Paper 1

11 (a) Here is a right-angled triangle.



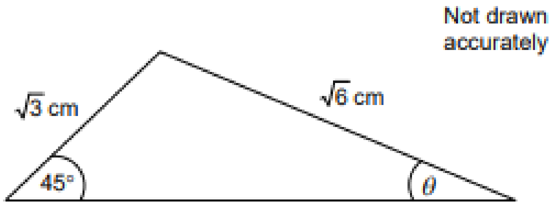
Show clearly that $\sin 45° = \frac{1}{\sqrt{2}}$

....

....

(1 mark)

11 (b) Here is a triangle.



Work out the value of θ.

....

....

....

....

....

....

....

....

....

....

Answer .... degrees *(5 marks)*

13 Work out the value of x if $\frac{\sqrt{x} \times \sqrt{8}}{\sqrt{3}} = 4\sqrt{5}$

....

....

....

....

....

....

....

....

Answer x = ... *(4 marks)*

16 n is a positive integer.

Prove that $(n + 2)^2 + (n + 1)^2 - 1$ is **always** a multiple of 4.

....

....

....

....

....

....

....

....

....

....

(6 marks)

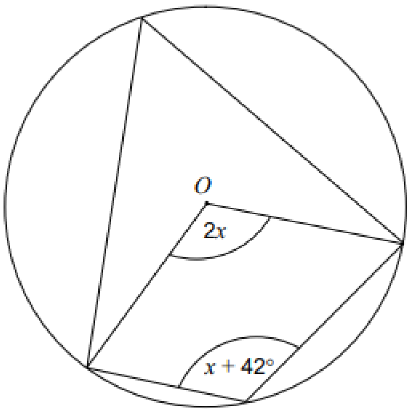
11(a)	$\sqrt{(1^2 + 1^2)} = \sqrt{2}$	B1	
11(b)	$\frac{\sin\theta}{\sqrt{3}} = \frac{\sin 45°}{\sqrt{6}}$	M1	or $\frac{\sqrt{6}}{\sin 45°} = \frac{\sqrt{3}}{\sin\theta}$
	$\sin\theta = \frac{\sqrt{3}}{\sqrt{6}} \times \frac{1}{\sqrt{2}}$	M1	
	$\sin\theta = \frac{\sqrt{3}}{\sqrt{12}}$ or $\frac{\sqrt{3} \times \sqrt{12}}{12}$ or $\sqrt{\frac{1}{4}}$	M1	
	$\sin\theta = \frac{1}{2}$	A1	
	$\theta = 30°$	A1	

13	$\frac{\sqrt{(8x)}}{\sqrt{3}} = 4\sqrt{5}$	M1	
	$\sqrt{(8x)} = 4\sqrt{15}$	M1	
	$8x = 16 \times 15$	M1	
	$(x =)\ 30$	A1	
Alt 1 **13**	$\frac{\sqrt{x} \times 2\sqrt{2}}{\sqrt{3}} = 4\sqrt{5}$	M1	
	$\sqrt{x}\sqrt{2} = 2\sqrt{15}$	M1	$x = \frac{15 \times 4}{2}$
	$\sqrt{(2x)} = \sqrt{60}$	M1	
	$(x =)\ 30$	A1	
Alt 2 **13**	$\sqrt{x} = \frac{4\sqrt{5}\sqrt{3}}{\sqrt{8}}$	M1	
	$\sqrt{x} = \frac{4\sqrt{15}}{\sqrt{8}}$	M1	
	$x = \frac{16 \times 15}{8}$	M1	
	$(x =)\ 30$	A1	
Alt 3 **13**	$\sqrt{(8x)} = 4\sqrt{5}\sqrt{3}$	M1	
	$\sqrt{(8x)} = \sqrt{(240)}$	M1	
	$x = \frac{240}{8}$	M1	
	$(x =)\ 30$	A1	
Alt 4 **13**	$\sqrt{\frac{8x}{3}} = 4\sqrt{5}$	M1	
	$\sqrt{\frac{8x}{3}} = \sqrt{80}$	M1	
	$x = \frac{3 \times 80}{8}$	M1	
	$(x =)\ 30$	A1	

16	$n^2 + 4n + 4$	M1	
	$n^2 + 2n + 1$	M1	
	$2n^2 + 6n + 4$	A1	
	$2(n^2 + 3n + 2)$	A1	
	$2(n + 1)(n + 2)$	M1	Explaining that $2n^2 + 6n + 4$ or $2(n^2 + 3n + 2)$ is divisible by 2 scores this mark
	$(n + 1)$ and $(n + 2)$ are consecutive numbers so one of them is even. So two factors of 2 … hence divisible by 4	A1	

Practice Paper Set 3 Paper 1

4 O is the centre of this circle.



Not drawn accurately

Work out the value of x.

....

....

....

....

x = ... degrees *(3 marks)*

8 $x^{\frac{1}{2}} = 6$ and $y^{-3} = 64$

Work out the value of $\frac{x}{y}$

....

....

....

....

....

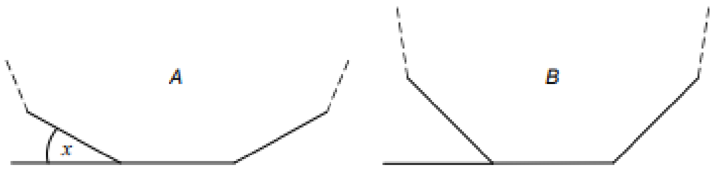
....

Answer .... *(4 marks)*

9 *A* and *B* are regular polygons.

An exterior angle of *A* is x.

Not drawn accurately



Here is some information about them.

	A : *B*
Ratio of exterior angles	1 : 3
Ratio of interior angles	7 : 6

9 (a) Write down an expression in x for an exterior angle of polygon *B*.

Answer .... *(1 mark)*

9 (b) Prove that polygon *A* has 30 sides.

....

....

....

....

....

....

....

....

(5 marks)

15 $2x^2 - 4x + 5 \equiv a(x + b)^2 + c$

Work out the values of a, b and c.

....

....

....

....

....

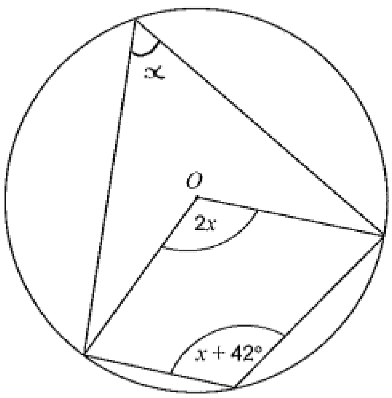
....

....

....

a = ..., b = ..., c = ... *(4 marks)*

4 O is the centre of this circle.



Not drawn accurately

Work out the value of x.

Angle at circumference = 1/2 angle at centre = x

x + x + 42 = 180° (cyclic quadrilateral)

→ 2x + 42 = 180

→ 2x = 138

x = 69

x = 69 degrees *(3 marks)*

8 $x^{\frac{1}{2}} = 6$ and $y^{-3} = 64$

Work out the value of $\frac{x}{y}$

$x^{1/2} = 6$

$\Rightarrow x = 36$

$y^{-3} = 64$

$\Rightarrow \frac{1}{y^3} = 64$

$\Rightarrow y^3 = \frac{1}{64}$

$\Rightarrow y = 1/4$

$\therefore \frac{x}{y} = \frac{36}{1/4}$

$= 36 \times 4 = 144$

Answer ....144............ *(4 marks)*

9a)

$3x$

9b)

$$\frac{180 - 3x}{180 - x} = \frac{6}{7}$$

$$1080 - 6x = 1260 - 21x$$

$$15x = 180$$

$$x = 12$$

$$\frac{360}{12} = 30$$

15 $2x^2 - 4x + 5 \equiv a(x+b)^2 + c$ complete the square!

Work out the values of a, b and c.

$2[x^2 - 2x + 2.5]$

$2[(x-1)^2 + 1.5]$

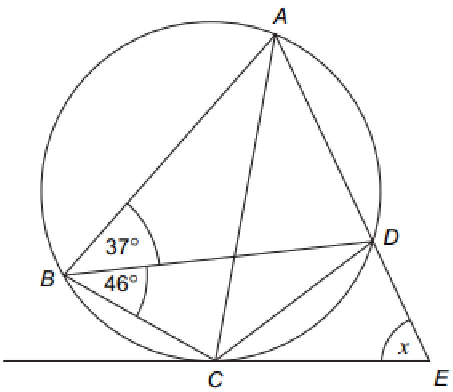
$\rightarrow 2(x-1)^2 + 3$

$a =$ 2, $b =$ −1, $c =$ 3 *(4 marks)*

June 2012 Paper 1

7 The diagram shows a cyclic quadrilateral *ABCD*.

ADE is a straight line.
CE is a tangent to the circle.



Not drawn accurately

Work out the size of angle x.

....

....

....

....

....

$x =$ .... degrees *(3 marks)*

9 Write this ratio in its simplest form

$$\sqrt{12} : \sqrt{48} : \sqrt{300}$$

....

....

....

....

....

Answer...................... : .... : .... *(3 marks)*

10 The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1 .

Prove algebraically that **all** the terms in the new sequence are multiples of 5 .

....

....

....

....

....

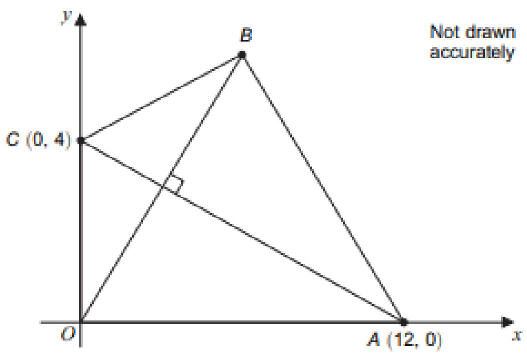
....

....

....

(4 marks)

11 *OABC* is a kite.



11 (a) Work out the equation of *AC*.

....

....

....

Answer... *(2 marks)*

11 (b) Work out the coordinates of *B*.

....

....

....

....

....

....

....

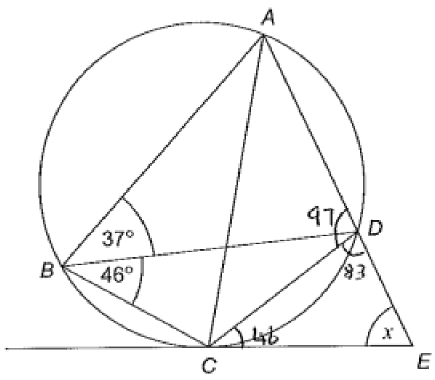
....

....

Answer (........................ , ...) *(6 marks)*

7 The diagram shows a cyclic quadrilateral *ABCD*.

ADE is a straight line.
CE is a tangent to the circle.



Not drawn accurately

Work out the size of angle x.

ADC = (180 - (37+46)) = 97° (opposite angles in cyclic quad add to 180°)

∴ CDE = 180 - 97 = 83°

ECD = 46° (alternate segment theorem)

∴ x = 180 - 83 - 46

$x =$ 51° degrees *(3 marks)*

9 Write this ratio in its simplest form

$$\sqrt{12} : \sqrt{48} : \sqrt{300}$$

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

$\sqrt{300} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

→ $2\sqrt{3} : 4\sqrt{3} : 10\sqrt{3}$ ($\div \sqrt{3}$)

2 : 4 : 10 ($\div 2$)

Answer 1 : 2 : 5 *(3 marks)*

10 The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that **all** the terms in the new sequence are multiples of 5.

nth term = $(5n-3)^2 + 1$

$= (5n-3)(5n-3) + 1$

$= 25n^2 - 15n - 15n + 9 + 1$

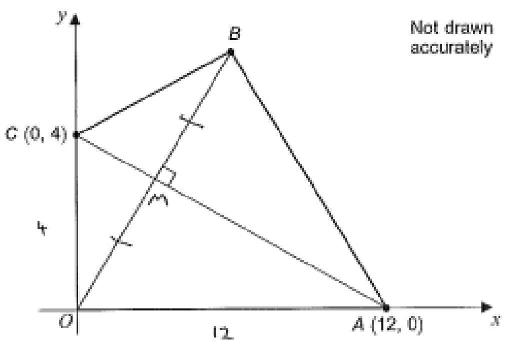
$= 25n^2 - 30n + 10$

$= 5(5n^2 - 6n + 2)$

Anything multiplied by 5 must be a multiple of 5

(4 marks)

11 *OABC* is a kite.



11 (a) Work out the equation of *AC*.

gradient = $-4/12 = -1/3$

y-intercept = (0, 4)

Answer $y = -\frac{1}{3}x + 4$ *(2 marks)*

11 (b) Work out the coordinates of *B*.

OB is perpendicular to AC

∴ gradient = 3

y-intercept = (0, 0) → Equation of OB = $y = 3x$

M = crossing point of OB & AC

At M: $3x = -\frac{1}{3}x + 4$

$+\frac{1}{3}x$ → $3\frac{1}{3}x = 4$

→ $\frac{10}{3}x = 4$

$x = \frac{12}{10} = \frac{6}{5} = 1.2$

$y = 3x → 3(\frac{6}{5}) = \frac{18}{5} = 3.6$

B must be 2 × M

→ $x = 2 \times 1.2 = 2.4$

$y = 2 \times 3.6 = 7.2$

Answer (2.4 , 7.2) *(6 marks)*

June 2018 Paper 1

4 $P = 4x$ and $Q = 7x$

P **increases** by 25%

Q **decreases** by 40%

Now, *P* is 28 greater than *Q*.

Work out the value of x.

[4 marks]

Answer ______________________

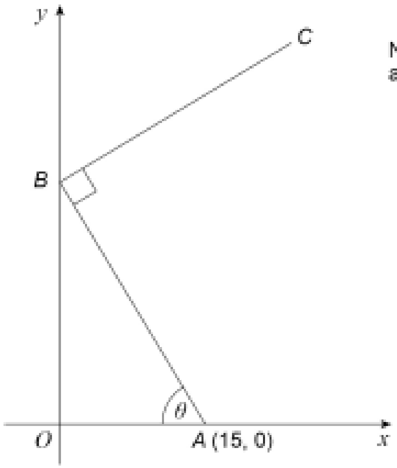
10 Rearrange $\frac{1}{xy} = 4 - \frac{3}{y}$ to make x the subject.

[3 marks]

12 In the diagram,

A is the point (15, 0) and *B* lies on the y-axis.

Angle $ABC = 90°$ and $\tan\theta = \frac{5}{3}$



Not drawn accurately

Work out the equation of the line *BC*.

[4 marks]

Answer ______________________

14 Work out the value of $\left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)^2$

You **must** show your working.

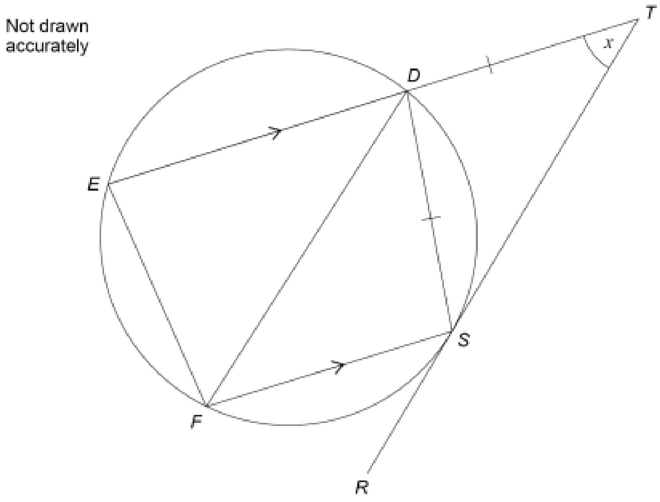
[3 marks]

18 D, E, F and S are points on a circle.

RST is a tangent.

The straight line EDT is parallel to FS.

$DS = DT$



Prove that FD is parallel to RST.

Use angle DTS as x to help you.

[5 marks]

	Alternative method 1		
	$1.25 \times 4x$ or $5x$	M1	oe
	$0.6 \times 7x$ or $4.2x$	M1	oe
	their $5x$ − their $4.2x = 28$ or $0.8x = 28$	M1dep	oe eg their $5x$ = their $4.2x + 28$ dep upon at least one of previous M marks earned
	$x = 35$	A1	
	Alternative method 2		
	two numbers in the ratio 4 : 7	M1	
4	correct increase by 25% and decrease by 40% calculations **and** comparison with 28	M1dep	If difference is not 28, then first numbers must be clearly rejected
	second trial with correct calculations and comparison	M1dep	correct first trial means 2nd and 3rd M marks scored automatically
	$x = 35$	A1	

	A correct first step using algebra	M1	Here are some of the possible alternatives $\frac{1}{x} = y\left(4 - \frac{3}{y}\right)$ multiplying through by y $1 = xy\left(4 - \frac{3}{y}\right)$ multiplying through by xy $1 = 4xy - \frac{3xy}{y}$ multiplying through by xy $y = 4xy^2 - 3xy$ multiplying through by xy^2 $\frac{1}{xy} = \frac{4y - 3}{y}$ making the RHS an algebraic fraction $\frac{1 + 3x}{xy} = 4$ rearranging **and** making the LHS an algebraic fraction
10	Further correct algebra which leads to an equation that is one step from the final answer.	M1dep	Following two of the above alternatives ... $y = 4xy^2 - 3xy$ $y = x(4y^2 - 3y)$ M1dep gained $\frac{1 + 3x}{xy} = 4$ $1 + 3x = 4xy$ $1 = 4xy - 3x$ $1 = x(4y - 3)$ M1dep gained
	A correct final answer in **any** form	A1	$x = \frac{1}{4y - 3}$ $x = \frac{-1}{3 - 4y}$ $x = \frac{y}{4y^2 - 3y}$ $x = \frac{-y}{3y - 4y^2}$ $x = \frac{1}{y\left(4 - \frac{3}{y}\right)}$ $x = \frac{-1}{y\left(\frac{3}{y} - 4\right)}$ $x = \frac{1}{\left(4 - \frac{3}{y}\right)} \div y$

12	$\frac{5}{3} \times 15$ or 25 seen as the length of *OB* or the coordinates of *B*	M1	
	gradient *AB* = $\frac{0 - \text{their } 25}{15 - 0}$ or $-\frac{5}{3}$	M1	oe
	gradient *BC* = $-1 \div$ (their $-\frac{5}{3}$) or $\frac{3}{5}$	M1	oe
	$y = \frac{3}{5}x + 25$	A1	oe eg $y = \frac{15}{25}x + 25$ or $5y = 3x + 125$
	Additional Guidance		
	We must see y = ... for A1 (or any other correct equation) Look for this in their working if it isn't written on the answer line. A sign error in their gradient *AB*, after a correct expression, can be recovered. eg gradient *AB* = $\frac{0 - 25}{15 - 0} = \frac{25}{15} = \frac{5}{3}$ gradient *BC* = $\frac{3}{5}$ (positive gradient because they can see it from the diagram) equation *BC* is $y = \frac{3}{5}x + 25$ ... this scores 4 marks similarly, recovery can be from ... gradient *AB* = $\frac{25}{15} = \frac{5}{3}$ ... without seeing $\frac{0 - 25}{15 - 0}$ ... and can still lead to 4 marks		

14	**Alternative method 1**		
	$3^{\frac{1}{2}} \times 3^{\frac{1}{2}} + 3^{\frac{1}{2}} \times 3^{\frac{3}{2}} + 3^{\frac{1}{2}} \times 3^{\frac{3}{2}} + 3^{\frac{3}{2}} \times 3^{\frac{3}{2}}$ or $\sqrt{3}\sqrt{3} + \sqrt{3}\sqrt{27} + \sqrt{3}\sqrt{27} + \sqrt{27}\sqrt{27}$	M1	oe allow an error in one term
	3 or 9 or 27	M1dep	
	48	A1	
	Alternative method 2		
	$\sqrt{3}$ and $3\sqrt{3}$	M1	$3\sqrt{3}$ must come from correct working
	$(4\sqrt{3})^2$	M1dep	
	48	A1	
	Alternative method 3		
	$\left(3^{\frac{1}{2}}\right)^2 (1+3)^2$	M1	oe
	3×4^2	M1dep	oe
	48	A1	

	Angle *DST* = x	M1	' base angles of isosceles triangle *DST*' but we do **not** require a reason for this mark
	Angle *DFS* = x angle in alternate segment or Angle *RSF* = x corresponding	M1	either of these angles **with a correct reason** scores this mark no reason or an incorrect reason is M0
	Further evaluation of angles, **with correct reasons**, to arrive at a stage where ... either ... it is possible to use the converse of a theorem or ... which leads to the fact that *DTSF* is a parallelogram	M1dep	Here is a complete example ... angle *DST* = x angle *DSR* = 180 − x angles on a straight line angle *RSF* = x corresponding angle *FDS* = x *FDS* = *RSF*, angle in alternate segment
	A statement of the angles, or the values of the angles, that will complete the proof ... the angles must be clearly identified	M1dep	angle *DSR* + angle *FDS* = 180 − x + x = 180
18	A statement of the correct reason to accompany these angles, thus completing the proof	A1	*FD* is parallel to *RST* because these angles add to 180 ... using the (converse) of the co-interior angles theorem

June 2019 Paper 1

5 Solve $\sqrt[3]{\left(2\sqrt{x}-10\right)}=2$

[3 marks]

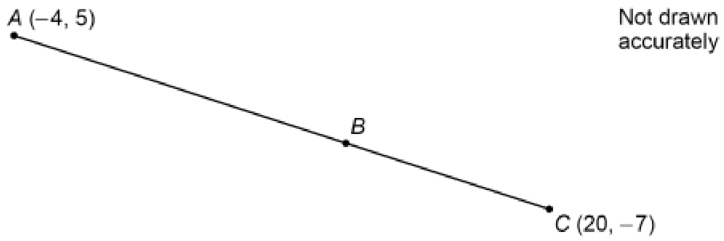
$x =$ ____________________

8 *ABC* is a straight line.

A is the point (−4, 5)

C is the point (20, −7)

AB : *BC* = 5 : 3



Work out the coordinates of *B*.

[4 marks]

11 A cone has base radius r cm, perpendicular height h cm and slant height l cm

The curved surface area is 60π cm^2

$l = 3r$

Work out the value of h.

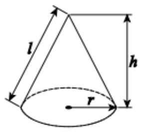
Give your answer in the form $a\sqrt{10}$ where a is an integer greater than 1

You **must** show your working.

[5 marks]

Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



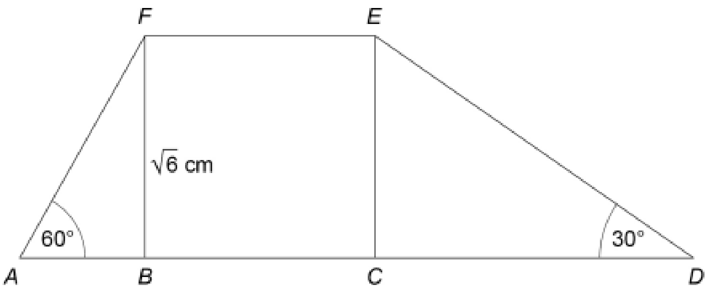
13 Prove that $(3x + 5)^2 - 5x(x + 10) \geqslant 0$ for all values of x.

[4 marks]

18 *ADEF* is a trapezium.

ABCD is a straight line.

BCEF is a square of side $\sqrt{6}$ cm



Not drawn accurately

18 (a) Show that $AB = \sqrt{2}$ cm

[1 mark]

18 (b) Show that $DE = 2\sqrt{6}$ cm

[1 mark]

18 (c) Work out the perimeter of the trapezium *ADEF*.

Give your answer in the form $t\sqrt{2} + w\sqrt{6}$ where t and w are integers.

You **must** show your working.

[3 marks]

Answer ______________________ cm

19 $f(x) = \dfrac{x-3}{2x}$

Solve $f(x+1) - f(2x) = 0.5$

You **must** show your working.

[6 marks]

5	$2\sqrt{x} - 10 = 2^3$ or $2\sqrt{x} - 10 = 8$ or $2\sqrt{x} = 18$	M1	
	$\sqrt{x} = \dfrac{2^3 + 10}{2}$ or $\sqrt{x} = \dfrac{8 + 10}{2}$ or $\sqrt{x} = 9$ or $4x = 18^2$ or $x = 9^2$	M1dep	
	$x = 81$	A1	± 81 scores A0
	Additional Guidance		

8	**Alternative method 1**		
	± (20 − −4) or ± (5 − −7) or ±24 or ±12 seen	M1	allow on diagram
	using $\frac{5}{8}$ or $\frac{3}{8}$ × ± their 24 or ±15 or ±9 or $\frac{5}{8}$ or $\frac{3}{8}$ × ± their 12 or ±7.5 or ±4.5	M1dep	oe
	(11, −2.5)	A2	A1 for each
	Alternative method 2		
	(x =) $\dfrac{3(-4) + 5(20)}{8}$ or (y =) $\dfrac{3(5) + 5(-7)}{8}$	M1	oe (condone 1 numerical error)
	(x =) $\dfrac{3(-4) + 5(20)}{8}$ and (y =) $\dfrac{3(5) + 5(-7)}{8}$	M1	oe
	(11, −2.5)	A2	A1 for each

11	**Alternative method 1**		
	$\pi \times r \times 3r = 60\pi$	M1	oe
	$r^2 = 20$ or $r = \sqrt{20}$ or $r = 2\sqrt{5}$	A1	oe
	$(l =)\ 3\sqrt{20}$ or $(l =)\ 6\sqrt{5}$ or $(l =)\sqrt{180}$ or $l^2 = 180$	A1	oe
	$(h^2 =)\ (3\sqrt{20})^2 - (\sqrt{20})^2$ or $(h^2 =)\ (6\sqrt{5})^2 - (2\sqrt{5})^2$ or $(h^2 =)\ (\sqrt{180})^2 - (\sqrt{20})^2$ or $(h^2 =)\ 160$	M1	oe using their l and r (this is independent so l and r can be anything) condone missing brackets
	$(h =)\ 4\sqrt{10}$	A1	
	Alternative method 2		
	$\pi \times \frac{l}{3} \times l = 60\pi$	M1	oe
	$l^2 = 180$ or $l = \sqrt{180}$ or $l = 3\sqrt{20}$ or $l = 6\sqrt{5}$	A1	oe
	$r^2 = 20$ or $(r =)\sqrt{20}$ or $(r =)\ 2\sqrt{5}$	A1	oe
	$(h^2 =)\ (3\sqrt{20})^2 - (\sqrt{20})^2$ or $(h^2 =)\ (6\sqrt{5})^2 - (2\sqrt{5})^2$ or $(h^2 =)\ (\sqrt{180})^2 - (\sqrt{20})^2$ or $(h^2 =)\ 160$	M1	oe using their l and r (this is independent so l and r can be anything) condone missing brackets
	$(h =)\ 4\sqrt{10}$	A1	

Q	Answer	Mark	Comments
13	**Alternative method 1**		
	$9x^2 + 15x + 15x + 25 - 5x^2 - 50x$ or $9x^2 + 30x + 25 - 5x^2 - 50x$ or $9x^2 + 15x + 15x + 25$ and $-5x^2 - 50x$ or $5x^2 + 50x$	M1	allow only one error in sign, omission or coefficient but not in more than one of these could be written as 2 separate expansions or in a grid
	$4x^2 - 20x + 25$	A1	
	$4x^2 - 20x + 25$ **and** $(2x-5)^2$ or $(2x-5)(2x-5)$ or $4(x-2.5)^2$ or $x = 2.5$ or $b^2 - 4ac = 0$ from quadratic formula	M1dep	factorises or completes the square or uses the quadratic formula correctly. Answer required for M1 dep
	$(2x-5)^2$ or $4(x-2.5)^2$ (are squared terms) and so are always ≥ 0	A1	oe there must be a stated conclusion eg equal roots and positive quadratic so must be greater than or equal to zero
	Alternative method 2 Not GCSE		
	$9x^2 + 15x + 15x + 25 - 5x^2 - 50x$ or $9x^2 + 30x + 25 - 5x^2 - 50x$ or $9x^2 + 15x + 15x + 25$ and $-5x^2 - 50x$ or $5x^2 + 50x$	M1	allow only one error in sign, omission or coefficient but not in more than one of these could be written as 2 separate expansions or in a grid
	$4x^2 - 20x + 25$	A1	
	$4x^2 - 20x + 25$ and $\frac{d}{dx} = 8x - 20$ and is zero when $x = 2.5$	M1dep	uses calculus to find stationary point
	Tests for minimum by using eg $x = 2$ and $x = 3$ or by using 2nd derivative **or** concludes argument by saying this is a positive quadratic curve with minimum point (2.5, 0), hence always ≥ 0	A1	oe there must be a stated conclusion

Q	Answer	Mark	Comments
18a	**Alternative method 1**		
	$(AB =)\ \frac{\sqrt{6}}{\tan 60} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$	B1	oe must see tan60 oe and some evidence of manipulation with $\sqrt{3}$ oe as well as the final answer to award B1
	Alternative method 2		
	Use of 1 : 2 : $\sqrt{3}$ triangle and showing that our triangle is an enlargement scale factor $\sqrt{2}$	B1	oe must see the triangle drawn and labelled or the ratio clearly seen **and** the scale factor clearly stated
	Additional Guidance		

Q	Answer	Mark	Comments
18b	**Alternative method 1**		
	$(DE =)\ \frac{\sqrt{6}}{\sin 30} = \frac{\sqrt{6}}{0.5} = 2\sqrt{6}$	B1	oe must see sin30 oe and some evidence of manipulation with 0.5 oe as well as the final answer to award B1
	Alternative method 2		
	Use of 1 : 2 : $\sqrt{3}$ triangle and showing that our triangle is an enlargement scale factor $\sqrt{2}$	B1	oe must see the triangle drawn and labelled or the ratio clearly seen **and** the scale factor clearly stated
	Additional Guidance		

18c	$AF = \frac{AB}{\cos 60} = \frac{\sqrt{2}}{0.5} = 2\sqrt{2}$ or $AF = \frac{BF}{\sin 60} = \frac{\sqrt{6}}{\frac{\sqrt{3}}{2}} = 2\sqrt{2}$ or $AF^2 = (\sqrt{2})^2 + (\sqrt{6})^2$, so $AF = \sqrt{8}$ or $2\sqrt{2}$	B1	oe allow $2\sqrt{2}$ or $\sqrt{8}$ for this mark seen on the diagram or clearly shown in working
	$CD = \sqrt{6} \times \tan 60 = \sqrt{6} \times \sqrt{3}$ $= \sqrt{18}$ or $3\sqrt{2}$ or $CD = DE \cos 30°$ $= 2\sqrt{6} \times \frac{\sqrt{3}}{2} = \sqrt{6} \times \sqrt{3}$ $= \sqrt{18}$ or $3\sqrt{2}$ or $CD^2 = (2\sqrt{6})^2 - (\sqrt{6})^2 = 18$ so $CD = \sqrt{18}$ or $3\sqrt{2}$	B1	oe allow $\sqrt{6} \times \sqrt{3}$ or $\sqrt{18}$ or $3\sqrt{2}$ for this mark seen on the diagram or clearly shown in working
	$6\sqrt{2} + 4\sqrt{6}$	B1dep	dependent on B1, B1 already awarded
	Additional Guidance		
	Condone brackets missed off if recovered		
	AF and *CD* could be seen in part (a) or part (b) so could be awarded B1 in part (c) if used correctly		

19	$\frac{x-2}{2x+2}$ or $\frac{x+1-3}{2(x+1)}$ or $\frac{2x-3}{4x}$	M1	oe substituting correctly in at least one expression
	$4x(x-2)$ and $(2x+2)(2x-3)$ or $4x(x-2) - (2x+2)(2x-3)$ or $4x^2 - 8x - 4x^2 + 2x + 6$ or $6 - 6x$ or $2x(x-2)$ and $(x+1)(2x-3)$	M1dep	oe (could be from using a different denominator) correct numerators or an expression for both, which need not be simplified do not award any follow through marks from an error in first M mark this one comes from a denominator of $4x(x+1)$
	$4x(x-2) - (2x+2)(2x-3)$ $= 0.5 \times 4x \times 2(x+1)$	M1dep	oe but needs to be the correct equation setting up the quadratic by multiplying the RHS by the product of the denominators could be scored by both sides of the equation still having the same denominator dep on both previous M marks
	$4x^2 + 10x - 6 = 0$ or $2x^2 + 5x - 3 = 0$	A1	
	$(4x-2)(x+3) = 0$ or $(2x-1)(2x+6) = 0$ or $(2x-1)(x+3) = 0$	M1dep	correct factors or correct use of quadratic formula oe
	0.5 **and** -3	A1	both answers needed