Going to the Moon

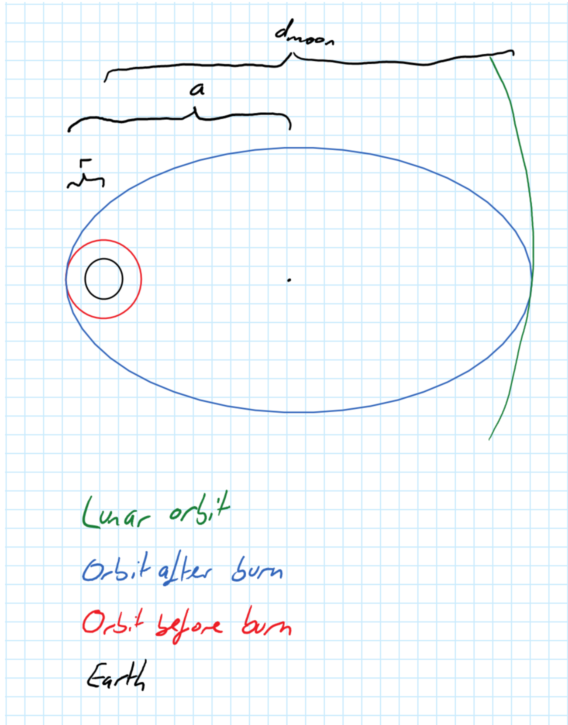
In this document I will provide a short calculation how one gets on a trajectory towards the moon, starting in low earth orbit. For this to be possible without tedious numerical calculations I will use the following simplifications:

- The mass m of the spacecraft is neglible compared to the mass of the earth, this would account for an error of 10^{-20}
- The moons orbit around earth is circular, which is true by an error of up to 0.1. However, the distance itself only plays a minor role in the calculations and can easily be adjusted.
- Tidal forces from the sun and other planets are neglible, which would account for an error of 10^{-5}.
- The spacecraft can change its speed instantaneously. This might induce a slight error of the factor 10^{-3}.
- The moon does not exert a gravitational pull on the spacecraft. This is by far the most grave simplification. However, an analytical solution without this assumption can not be obtained and one would be forced to use numerical calculations. However, if we manage to hit the moon without it pulling the spacecraft, it is to believe that with the moons additional pull we have an even better chance to hit.

The following values are used:

- Gravitational constant $G = 6.674 \cdot 10^{-20} \frac{km^3}{kg\, s^2}$
- Distance to moon $d_{Moon} = 384402km$
- Radius and Mass of earth: $r_{Earth} = 6371km$, $M_{Earth} = 5.972 \cdot 10^{24}$

We start in a circular orbit around earth at 200km height, our orbital velocity is $v = \sqrt{\frac{GM}{r}} \approx 7.788 \frac{km}{s}$. If we now boost forward, our speed changes and our circular orbit becomes elliptical, with the earth in one focal point of the ellipse. The situation is depicted in the following sketch:



For the energy of the system immediately after the boost we have: $E = \frac{1}{2}mv^2 - G\frac{mM}{r}$. The semi major axis is $a = G\frac{mM}{-2E}$. Now we want $2a - r = d_{Moon}$. Inserting this into the equations and solving for v yields:

$$v = \sqrt{2\left(\frac{GM}{r} - \frac{GM}{d_{Moon} + r}\right)} \approx 10.921\frac{km}{s}$$

So we get $\Delta v = 3.13\frac{km}{s}$. Inserting this into Tsiolkovsky's rocket equation $\Delta v = v_{Exhaust} \ln \frac{m_0}{m_f}$ we can, depending on our used engine and masses, calculate the used fuel.

The remaining question is: when should we boost? For this we use keplers third law $\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$ yielding:

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} \approx 856984.66s \approx 9.9d$$

This deviates a little from the actual flying time due to the fact that our speed at apoapsis is very low, as we neglect gravitational influence from the moon. However, the journey to the moon we need only half that time, with which we now can calculate where the moon should be when we start boosting. The velocity of the moon is $v = \frac{GM}{d_{Moon}} \approx 1.018\frac{km}{s}$, yielding an angular velocity of $\omega = \frac{v}{d_{Moon}} \approx 2.649 \cdot 10^{-6}s^{-1}$. The angular change of the moon would be $\phi = \omega\frac{T}{2} \approx 2.27rad \approx 130°$.