รายงานวิจัย

เรื่อง

ความผันผวนของราคาแบบไดนามิก: การศึกษาเชิงประจักษ์ของอุตสาหกรรมสายการบินต้นทุนต่ำในประเทศไทย

Volatility of Dynamic Pricing: An Empirical Study of the Low Cost Airline Industry in Thailand

โดย

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บทคัดย่อ

วัตถุประสงค์หลักของการศึกษาคือ การหาค่าและเปรียบเทียบความผันผวนของราคาตั๋ว ของสายการบินต้นทุนต่ำทั้งภายในประเทศและระหว่างประเทศ ที่ออกเดินทางจาก กรุงเทพฯ ไปยังจุดหมายปลายทางต่างๆ เช่นเชียงใหม่ ภูเก็ต โตเกียว และเมลเบิร์น รวมทั้งเปรียบเทียบ ความผันผวนสัมพัทธ์ ระหว่างราคาตั๋วเครื่องบินในและต่างประเทศในระยะเวลาที่กำหนด

การวิจัยครั้งนี้มีได้ใช้โมเดลทางเศรษฐมิติเพื่อค้นหาความผันผวนในการกำหนดราคา แบบ ไดนามิก ในระบบจองตั๋วเครื่องบินของสายการบิน ในช่วงระยะเวลาการศึกษาเป็นระยะเวลา 6 เดือน (ตั้งแต่เดือน มกราคม จนถึงเดือน สิงหาคม 2562) ซึ่งข้อมูลได้รวบรวมจากราคาตั๋วรายวันผ่านทางเว็บไซต์ที่เป็นทางการของสายการบินต่างๆ (Nok Air, Thai Air Asia, Nok Scoot และ Jetstar

Microsoft Excel NumXL (addins) เป็นเครื่องมือที่ถูกเลือกใช้เพื่อสร้างโมเดล และตรวจสอบรูปแบบความผันผวน และเพื่อคำนวณผลลัพธ์ของ GARCH (1,1)

โดยการสังเกตการเปลี่ยนแปลงราคาตั๋วในเส้นทางการบินต่างๆ รวมถึงเส้นทางภายในประเทศและระหว่างประเทศเพื่อวิเคราะห์และระบุระดับความผันผวนของการเปลี่ยนแปลง ราคาตั๋วในช่วง 6 เดือน เราพบว่าระดับความผันผวนเพิ่มขึ้นอย่างต่อเนื่อง จากการวิเคราะห์เชิงประจักษ์ ได้แสดงให้เห็นว่า การแจกแจงของการเปลี่ยนแปลงในราคาตั๋วเครื่องบิน มีการเปลี่ยนแปลง ไปจากราคาเริ่มต้น เป็นความผันผวนที่เปลี่ยนแปลงอย่างต่อเนื่อง และผลที่ได้จากการทดสอบความ ผันผวนแสดงให้เห็นว่าราคาตั๋วค่อนข้างผันผวนเมื่อซื้อตั๋วใกล้กับวันออกเดินทาง

คำสำคัญ: ความผันผวน, การกำหนดราคาแบบไดนามิก, การเพิ่มประสิทธิภาพราคา, ธุรกิจสายการบินในประเทศไทย, แบบจำลอง GARCH

Research Title: Volatility of Dynamic Pricing: An Empirical Study of the Low Cost Airline Industry in Thailand

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Abstract

The main purposes of the study are to find and compare volatility in ticket prices of low cost airline companies for domestic and international flights departing Bangkok to various destinations such as Chiang Mai, Phuket, Tokyo and Melbourne as well as to compare relative volatility between domestic and international ticket prices in a particular time frame.

Econometric model is used in this research to find any dynamic pricing volatiliy in airlines ticketing system during the study period of 6 months (January to August 2019). Daily ticket prices were obtained via official airline websites (Nok Air, Thai Air Asia, Nok Scoot, and Jetstar).

Microsoft Excel NumXL (addins) was used to construct and examining volatility model to calculate the GARCH(1,1) results.

By observing ticket price changes in various flight routes including domestic and international routes to analyze and identify volatility level of the change in ticket prices over the 6 months time, we found that level of volatility increases as time to departure approaches. Empirical analyses reveal that distributions of the change in ticket prices deviate from normality with volatility varying over time. The results of the volatility tests show that the ticket prices were quite volatile when purchasing tickets close to departure date.

Keywords: volatility, dynamic pricing, price optimization, airline industry in Thailand, GARCH model

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Sethapong Watanapalachaikul

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Chapter 1

Introduction

1.1 Background of the Study

In recent years, there was a rapid growth in airline industry especially in Asian countries. According to Boeing's business outlook (2017), Southeast Asia's airlines are growing at a fast pace as the region continues to develop economically. Most airlines are growing and gaining more market share, stimulating passenger demand with attractive ticket prices and opening new routes. Some low cost airlines have launched their strategies to expand their operations into other countries in this region. There are an increase in numbers of new entrants in this industry, resulting in competitive environment. Airliners have restructured to expand their product offerings for growth and increased competitiveness in other countries in the Southeast Asian region. Furthermore, this rapid growth in airline industry has varieties of service availability and affordability of air fares to their potential customers. In addition, International Air Transport Association (IATA) (2019) made some prediction that number of passengers would grow from 3.5 billion to 7 billion within 2037 and could create 100 million jobs globally. Thailand is expected to enter the top 10 markets in 2030, according to the forecast.

Initially, airfare pricing started from uncompetitive fares where most international routes were operated by a single national airline and the lack of choice. Subsequently, deregulation of airline industry changed all pricing practices and that was the starting of the implementation of dynamic pricing. Some Airliners lost their share and profits forcing them to merge to stay competitive. Nowadays, airlines price tickets "as much as the customer and market will bear". For typical routes, airlines will start with minimum ticket prices to fill a minimum capacity, and then increase ticket prices abruptly as some passengers may be inevitable to book at the last minute. Hence, the frequent update in ticket prices offered by the airlines to their potential customers is a common practice to maximize firm's profit (called airline dynamic pricing). In addition, uncertainty could reflect the demand volatility that is present in the markets served. Airlines update their airfares frequently utilizing advanced dynamic pricing and revenue management systems. These systems generate forecasts about future demand, consider the corresponding departure/arrival rates of different customers' types and remaining capacity, and offer a varietty of ticket classes to price-discriminate between the different customers in their ongoing effort to maximize the revenue generated from their network of flights (Bertsimas and Perakis 2006; Mantin and Gillen 2009).

Nearly all Airliners that implement dynamic pricing strategy with their revenue management systems often adjust their ticket prices over time as time to departure approaches. This method offers lower prices far from the departure date as to capture demand from leisure passengers to facilitate consumer needs and fills seat availability of the flights. As time advances, they charge higher prices aimed price insensitive which targets business passengers. Due to demand fluctuation, airlines generate booking curves for each flight, which demonstrates the predicted progression of their ticket prices and booking for individual flight. When demand falls short of the booking curve, airlines usually reduce ticket prices. On the other hands, during excess demand may result in an upward change in ticket prices. The sensitivity level of dynamic pricing system illustrates how rapidly prices are adjusted to reflect changes in the forecasted demand. Therefore, the sensitivity of these systems could very well describe the level of uncertainty about demand in airline business. Alternatively, price volatility could be determined by the sensitivity of these systems. Ticket price volatility could be the outcome of other mechanisms employed by airliners.

1.2 Research Problems

1.2.1 There are two categories of passengers: leisure and business. The way each category is priced is very different. Leisure passengers usually book in advance, while business passengers usually search for tickets close to their date of travel because they are willing to pay extra for convenience. This difference in traveling behavior influenced airlines to adopt and use dynamic pricing for airfare charges.

1.2.2 Profit maximization could only be achieved by implementing dynamic pricing in airfare ticketing rather than fixed price.

1.2.3 The information regarding to the change in Airline's ticket prices has been hidden by airline companies and has not been released or announced.

1.2.4 Airlines generally adopted dynamic pricing in airfare ticketing; this method not only influenced revenue but also affects the cost structure of airlines.

1.3 Research Questions

1.3.1 How significance is the change in ticket prices for domestic flights during normal day?

1.3.2 How significance is the change in ticket prices for domestic flights during public holiday/long weekend?

1.3.3 How significance is the change in ticket prices for international flights during normal day?

1.3.4 How significance is the change in ticket prices for international flights during public holiday/long weekend?

1.4 Research Objectives

1.4.1 To find and compare volatility in ticket prices for domestic flights departing Bangkok to Chiang Mai and Phuket during normal working day.

1.4.2 To find and compare volatility in ticket prices for domestic flights departing Bangkok to Chiang Mai and Phuket during public holiday/long weekend.

1.4.3 To find and compare volatility in ticket prices for international flights departing Bangkok to Tokyo and Melbourne during normal working day.

1.4.4 To find and compare volatility in ticket prices for international flights departing Bangkok to Tokyo and Melbourne during public holiday/long weekend.

1.5 Research Hypothesis

1.5.1 If a customer purchases airline ticket at the last moment, the price of the ticket would be at the maximum price that market can bear.

1.5.2 Higher volatility is expected in an international route's ticket price than domestic route's ticket price.

1.5.3 For customer, the best price would be at least 90 days to 180 days before departure flight.

1.6 Research Scope and Limitation

1.6.1 The most popular domestic route destination departing from Bangkok are Chiang Mai and Phuket, these routes has been selected to study. On-line ticket prices had been recorded for Bangkok to Chiang Mai during the study period, ticket price data was obtained from Nok Air website. For the route from Bangkok to Phuket, Thai Air Asia has been selected.

1.6.2 During 2018-2019, there were 6 low cost airlines for domestic route such as Air Asia, Bangkok Airways, Lion Air, Nok Air, Orient Thai and Thai Smile. However, Nok Air and Thai Air Asia were chosen because of their popularity and frequent flights during the day.

1.6.3 This research is not a comparison study; therefore, we will not compare and suggest which airline is more superior to others.

1.6.4 For international route, Bangkok to Tokyo route has been selected, as it is one of the most popular tourist destinations. Bangkok to Melboune is also selected because of author familiarity with this route.

1.6.5 There were many low cost airlines, which operate international route between Bangkok to Tokyo and Bangkok to Melbourne. Scoot is chosen for the flight between Bangkok and Tokyo, while Jetstar is chosen for the route Bangkok to Melbourne.

1.6.6 The study of volatility of ticket prices during Thai national holiday and International holiday such as New Year were also included in this research.

1.6.7 Mathematical software packages such as MathLab, Mathematica and others are unavailable to obtain. Therefore, Microsoft Excel has been used to calculate GARCH results.

1.7 Contribution to Knowledge

1.7.1 This is the first study to consider the relevance and appropriateness of using financial econometric models in the empirical analysis of a developing economy especially in the Thai airline industry.

1.7.2 Empirical studies and analyses of the characteristics of the dynamic pricing and volatility are addressed in this research.

1.7.3 This is probably the first comprehensive research to develop a volatility model, especially for dynamic pricing in airline industry. This volatility model overcomes the limitations of traditional volatility methods.

1.8 Keywords

volatility, dynamic pricing, price optimization, airline industry in Thailand, GARCH model

Volatility – Volatility is a statistical measure of the dispersion of returns for a given security or market index. In most cases, the higher the volatility, the riskier of the security. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index.

Dynamic pricing – Dynamic pricing, also called real-time pricing, is an approach to setting the cost for a product or service that is highly flexible. The goal of dynamic pricing is to allow a company that sells goods or services over the Internet to adjust prices on the fly in response to market demands.

Price optimization – Price optimization is the use of mathematical analysis by a company to determine how customers will respond to different prices for its products and services through different channels. It is also used to determine the prices that the company determines will best meet its objectives such as maximizing operating profit.

Airline industry in Thailand – An airline is a company that provides air transport services for traveling passengers and freight. Airline industry in Thailand utilizes aircraft to supply these services and may form partnerships or alliances with other airlines for codeshare agreements particularly have their operation in Thailand.

GARCH model – The generalized autoregressive conditional heteroskedasticity (GARCH) process is an econometric term to describe an approach to estimate volatility in financial markets. This approach incorporates a moving average component together with the autoregressive component. Specifically, the model includes lag variance terms together with lag residual errors from a mean process.

Chapter 2

Conceptual and Theoretical Framework

In this literature review, we include various published articles of dynamic pricing, optical pricing and volatility models. We separate the literature into two main streams; the first consists of research papers that formulate dynamic and optimal pricing, while the second includes research papers regarding to volatility such as ARCH and GARCH type models.

2.1 Dynamic Pricing Model

For this model, it is assumed that the airline sells only one type of service/product, in this case, flight ticket. In each time period $t \in \mathbb{N}$, airliner sets on a selling price $p_t \in [p_l, p_h]$, where $0 \leq p_l < p_h < \infty$ denote the lowest and highest acceptable price. After selecting the acceptable price, the airline notices demand d_t, which is a apprehension of the random variable $D_t(p_t)$. Conditional on the selling prices, the demand in different time frame is independent. According to Broder and Rusmevichientong (2012), the expected demand in period *t*, against a price *p*, can be formulated as:

$$E[D_t(p)] = M(t) + g_t(p).$$

where$(M(t))_{t \in N}$ is a stochastic process, and this could be unobservable for the airline, and taking values in an interval $M \subset R$.

The function g_g model the dependence of expected demand on selling price. These variable are assumed to be known by airliners. Later, by observing demand, the airline collects revenue $p_t d_t$, and proceeds to the next period. Hence, this process maximizes airline's revenue (Besbes and Saure 2012).

$\mathcal{F}_t$ can be generated by $d_1, p_1, M(1), \dots, d_t, p_t, M(t), \mathcal{F}_o$ the trivial σ-algebra, and write $\epsilon_t = d_t - g_t(pt) - M(t)$; then we assume that $M(t)$ and ϵ_t are $\mathcal{F}_{t-1}$ measurable, for all $t \in N$. Furthermore, it is possible to impose the following mild conditions on the moments of $M(t)$ and ϵ_t: there are positive constants σ_M and σ, such that

$$\sup_{t\in N} E[M(t)^2|\mathcal{F}_{t-1}] \leq \sigma_M^2 \; a.s. \quad \text{and} \quad \sup_{t\in N} E[\epsilon_t^2|\mathcal{F}_{t-1}] \leq \sigma^2 \; a.s.$$

In addition, $r_t(p, M) = p(M + g_t(p))$ denote the expected income in period $t \in N$, when the market procedure equals M and the selling price is set at p. The price that generates the highest value of expected revenue, given that the current market equals M, is denoted by $p_t^*(M) = p \in [p_t, p_h] r_t(p, M)$.

Furthermore, it is possible to add some conditions to ensure that this optimal price exists and is uniquely defined. It is possible to assume that for all admissible prices p and all $t \in N, g_t(p)$ is decreasing in p, and double its value continuously differentiable with first and second derivative denoted by $g'_t(p)$ and $g''_t(p)$. These two properties promptly carry over to the expected demand, and in fact are normal conditions for demand functions to hold (Besbes and Zeevi 2011).

Hence, it is possible to assume that for all $M \in \mathcal{M}$ and all $t \in N$ the revenue function $r_t(p, M)$ is identified as unique optimum $p_t^{\#}(M) \in R$ which satisfying function $r'_t\left(p_t^{\#}(M), M\right) = 0$.

According to Cope (2007), the value of the market process and the corresponding optimal price are unknown to airliners. The goal of the airline is to determine a pricing policy that minimizes this loss of revenue. With a pricing policy we here mean a sequence of random prices $(p_t)_{t \in \mathbb{N}}$ in $[p_l, p_h]$ where each price p_t may depend on all previously chosen prices $p_1,\ldots, p_{t-1}$ and demand realizations $d_1,\ldots, d_{t-1}$. As a result, the policy and decision maker could select sub-optimal prices, which results a loss of income relative to someone who would know the market process and the highest possible price.

In order to assess the quality of a dynamic pricing policy, Φ, Cope (2007) models this pricing policy into the following two fuctions.

$$AR(\Phi, T) = \frac{1}{T-1} \sum_{t=2}^{T} E\left[r_t\left(p_t^*(M(t)), M(t)\right) - r_t(p_t, M(t))\right]$$

$$LRAR(\Phi) = \lim_{T \to \infty} sup \, AR(\Phi, T).$$

These two functions measure the expected revenue loss caused by not using the optimal price in period t. The expectation medium for both p_t and $M(t)$ may be random variables. It is possible to measure the average disconsolate from the second period (Cope 2007, Harrison et al 2012).

Furthermore in the first period, data is unavailable to estimate $M(1)$, and minimizing the immediate regret covered in the first period is not possible. In addition, Average Regret (Φ,T) and Long Run Average Regret (Φ) are not observed by airliners, and therefore, cannot directly be used to determine an optimal pricing policy (Dolan and Jeuland 1981). These models have been tested in Keskin and Zeevi (2014).

2.2 Optimal Profitability Model

Observing at the firm's position, the optimal model for profit in time series over the fixed planning horizon in the airline industry can be explained as $R(t)$. The rate $\lambda(t,p)$ of this process is non-increasing in the current price p, and is a bounded continuously differentiable function of time t for each p. Suppose that the demand of the flight ticket process $N'(t)$ is a non-homogeneous Poisson process.

Risk-neutral objective is to simply max $E[R(0)]$ without constraints, except possibly one on the set of allowed prices, it is possible to incorporate risk by adding a constraint of the above function as below.

$$P[R(0) \geq \zeta] \geq \pi_0$$

where ζ is a desired minimum level of incomes and π_0 is the minimum acceptable probability with which we want this level to be reached. The number of ticket sold $N(t)$ has its value limited by the initial inventory Y_T; thus, $N(t) = \min\{N'(t), Y_T\}$. If the ticket price is $p(t)$, then the revenue process $R(t)$ can be defined as a following function (Gallego and Ryzin 1994).

$$R(t) = \int_T^t p(\tau)\, dN(\tau)$$

Maximizing $E[R(0)]$, as a main objective, is appropriate for a policy maker for airliners who can sell tickets repeatedly in consecutive planning horizons and is fully rational in the long run. Furthermore, reaching the level ζ, demotes a short-term objective of the optimal model.

If π_0 is substituted, the problem will have different optimal solutions resulting in an efficient frontier in function of optimal $P[R(0) \geq \zeta]$ and $E[R(0)]$; that is, the efficient frontier is the set of attainable expected revenue/probability models that are not overruled by any other attainable model.

Alternatively, we can seek this frontier is by solving the problem max $E[R(0)]-CP[R(0)<\zeta]$ for a range of values of the coefficient C $[0,+\infty)$. The parameter C

can be interpreted as the maximum cost associated with not meeting the desired level of income ζ (Lim and Shaunthikumar 2007).

2.3 Demand Function in Airline Industry

Observing at the customer's side, demand function can be modelled by dividing the set of potential customers into different categories, each one has its own set of attributes including budget, needs and preference time to fly and quality expectations. By applying demand function in Dolan and Jeuland's models, the heterogeneity of demand for a ticket of the flight can be explained as follows (Dolan and Jeuland 1981).

$$N(t, \mathcal{H}_t) \square (N_1(t, \mathcal{H}_t), \dots, N_d(t, \mathcal{H}_t))$$

where $N_j(t,\ \mathcal{H}_t)$ is the cumulative potential demand up to time t from factor j given the available information $\mathcal{H}_t$.

In dynamic pricing condition, it is possible for the airlines with the ability to partially serve demand when the airlines produce profit. Hence, airlines could adjust the pricing of flight tickets during any promotion days. Furthermore, airlines are able to reject low-fare reservations even if they have available capacity. Moreover, it is denoted that n-dimensional vector $S(t)$ that represents the cumulative sales up to time t. Given the sale, demand and price processes, the dynamics of the available capacity are functioned by the following conditions.

$$C_t = C_0 - AS(t) \text{ and } S(t) \leq D(t, P,\ \mathcal{H}_t) \text{ for all } t \in [0, T]$$

In other cases, the differences between sales S and demand D is unnecessary. For example, if the ticket price can be changed frequently and unrestrictedly, airliners would prefer to adjust premium price rather than reject customers. Therefore, the ticket price is the only functioned that the airline can implement to reach maximum profit in dynamic pricing strategy (Raman and Chatterjee 1995).

It is important to mention that customer factor considerations are mainly focused to the structure of the matrix *B(P)* whereas its dependence on the product item m. Combining the vector of potential demand $N(t, \mathcal{H}_{t})$ and the matrix *B(P)*, it is possible to denote n-dimentional vector $D(t, P, \mathcal{H}) \equiv B(P)N(t, \mathcal{H}_t)$ that represents the effective cumulative demand process in $[0, t]$ at the product level of the model.

Relying on the price and other attributes such as time of a flight, prospects/potential customers will decide whether or not to purchase a flight ticket for their suit. In order to model this purchasing process, it is denoted that $n \times d$ matrix *B(P)* = $[b_{ij}]$ where b_{ij} represents the units of product $i \in \mathcal{M}$ requested by customer factors $j = 1, \dots, d$.

According to Gallego and Ryzin (1994), in the yield management literature of seat availability, the concept of a $null$ price has been introduced to the function model while accept/reject decision in the context of dynamic pricing policies. It is noted that if the airline is restricted in the way that customer can select the price then the observe changes between sales and demand becomes relevant and the accept/reject decision is not necessarily simulated by using a dynamic pricing strategy.

The use of a price-sensitive demand $D(t, P, \mathcal{H}_t)$ function demonstrated that the airline has monopolistic market power over some buyers. Industry competition could be present in this function, but it could be hidden and only the residual demand $N(t, \mathcal{H}_t)$ faced by the airline is taking noted. Demand in this function is assumed to be given exogenously and prospects are assumed to be price takers. It could be explained that they observe the price list offered by the airline and react by purchasing or not purchasing airline ticket.

To forecast and predict the demand for an appropriate flight schedule, the model take deterministic demand into a set of different factors, which each function addressing a specific aspect of the problem in the dynamic pricing process, where:

$$D^{det}(t, p, \mathcal{H}_t) = \mathcal{D}(t)\, \mathcal{G}(p)\, \mathcal{F}(\mathcal{H}_t)$$

Denoted that $\mathcal{D}(t)$ is an estimate of the market size as a function of schedule/time, $\mathcal{G}(p)$ captures price elasticity and $\mathcal{F}(\mathcal{H}_t)$ shows the influence of the available information on customers buying behavior.

According to Raman and Chatterjee (1995), the notions of consumers' utility, elasticity, and product substitution form the bases of our understanding and modeling of $\mathcal{G}(p)$. That is, $\mathcal{G}(p) = \exp(-\eta p)$, where η is a measure of demand elasticity per unit of ticket price. Other model using constant elasticity, $(p) = p^{-\eta}$, have also been proposed. On the other hand, the modeling of $\mathcal{D}(t)$ depends on the seasonality of demand and product life cycle. In addition, diffusion models are widely used to model fluctuation of demands. In this case, a population of customers of size N gradually purchases the flight ticket. In diffusion model, the rate of purchase/booking a flight at time t is given by:

$$\frac{d\mathcal{D}(t)}{dt} = pN + (q - p)\mathcal{D}(t) - \frac{q}{N}\mathcal{D}^2(t)$$

For the discrete time case, the standard approach is to represent demand as the sum of a deterministic part and a zero-mean stochastic component. The stochastic behavior of the demand has been added to these deterministic models for discrete and continuous time formulations. By implementing the notation $dD(t, p, \mathcal{H})$ for the marginal demand in period t, the stochastic additive noise model is shown as:

$$dD^{stoc}(t,p,\mathcal{H}_t) = dD^{det}\ (p,t,\mathcal{H}_t) + \xi(t,p,\mathcal{H}_t)$$

The random noise, which usually follows a zero-mean normal random variable, which depends on price and time to reflect the changes on demand regarding to its uncertainty over the life cycle. Furthermore, an alternative model is the multiplicative noise model is as follow:

$$dD^{stoc}(t,p,\mathcal{H}_t) = dD^{det}\ (t,p,\mathcal{H}_t)\ \xi(t,p,\mathcal{H}_t)$$

The above model indicates that the expected value random noise is normally set to one. Combinations of the additive and multiplicative models can also be utilized. For the continuous time case, the most common formulation assumes that demand follows a Poisson process with a deterministic intensity that depends on price and time, although it is possible to extend the discrete time formulation above replacing the normally distributed random noise by a continuous time process (Raman and Chatterjee 1995).

2.4 Volatility in Stochastic Process

According to Bertsimas and Perakis (2006), volatility can be measured by the fluctuation between the demands and ticket prices, so called volatility in dynamic pricing, given the demand rate at time t and price level p is $\lambda(t,p)$. We assume that a company sets the ticket at price $p(t,n,r)$ in the state (n,r) at time t. Thus, according to the continuous-time Markov chain, it is possible to make transitions from the state (n,r) to the state $(n-1, r+p)$ with a rate $\lambda(t, p(t,n,r))$. Let $P_{(n,r)}(t)$ be the probability that the function is in the state (n,r) at time t. Then, $P_{(n,r)}(t)$ is governed by the continuous time Markov chain for ordinary differential functions as follow:

$$\frac{dP_{(n,r)}}{dt} = -\lambda\big(t, p(t,n,r)\big)P_{(n,r)}(t) + \sum_{r':r'=r-p(t,n+1,r')} \lambda\big(t, p(t,n+1,r')\big)P_{(n+1,r')}(t)$$

for $0 < n < Y_t,\ 0 \leq r \leq (Y_T - n)P_{max}$

$$\frac{dP_{(0,r)}}{dt} = \sum_{r':r'=r-p(t,1,r')} \lambda\big(t, p(t,1,r')\big)P_{(1,r')}(t)$$

for $0 \leq r \leq Y_{tPmax}$,

$$\frac{dP_{(Y_T,o)}}{dt} = -\lambda\big(t, p(t,Y_T,0)\big)P_{(Y_T,0)}(t),$$

with the initial conditions $P_{(n,r)}(T) = 0$ for $(n,r) \in \bar{\mathcal{P}} \setminus (Y_T, 0)$ and $P_{(Y_T,0)}(T) = 1$.

According to Gallego and Ryzin (1994) and Lim and Shanthikumar (2007), it is possible to employ standard results from control theory such as standard existence theorem and optimality conditions. This model could be able to convexify the controls by embedding them into a larger policy space of randomized policies formed by time-dependent measures $\pi(p|t,n,r)$ on the set of allowable prices in all possible states(n,r). Suppose that at time *t*, the company has available stocks *n* and revenue *r*.

Denotes that $n > 0$, the price *p* is offered to a buyer with probability $\pi(p|t,n,r)$. When $n = 0$, no price is offered the zero inventory state is absorbing. Both situations can be captured using normalizations $\sum_p \pi(p|t,n,r) = 1$ for $(n,r) \in \mathcal{P}, \sum_p \pi(p|t,0,r) = 0$, where the summation range condition $\in \mathcal{P}$.

While this extended form of controls could assist us establish existence, we emphasize that the extension is merely an analytical device, and the original space of discrete price controls remains our primary interest. The corresponding rate of transition out of function (n, r) is $\sum_p \lambda(t,p)\pi(p|t,n,r)$; thus, for each $p, \lambda(t,p)\pi(p|t,n,r)$ transitions per unit of time are directed into the state $(n-1, r+p)$ (Mantin and Gillen 2009).

2.5 Volatility Models

2.5.1 Autoregressive Conditional Heteroscedasticity (ARCH) Models

Engle (1982) developed a model to describe time-varying variance. The methodology is called Autoregressive Conditional Heteroscedasticity (ARCH) (see Mills 1999). The concept of the ARCH model has led to the development of other related formulations in order to identify and explain the variance of time series. Engle introduced the linear ARCH(q) model where the time varying conditional variance is postulated to be a linear function of the past q squared innovations. The ARCH (q) model is defined by:

$$r_t = \mu + \sigma_t \varepsilon_t$$

and:

$$\sigma_t^2 = \lambda + \alpha_1 (r_{t-1} - \mu)^2 +,\ldots,+ \alpha_q (r_{t-q} - \mu)^2$$

where r_t is the returns, μ is the conditional mean of the return process and is constant, $\varepsilon_t \sim NID(0,1)$ is conditionally Gaussian (*NID* denotes normally and independently distributed), σ_t is the first alternative of the stochastic volatility models and is modelled by a stochastic process, λ_1 and α are real constants, and ε_t are zero mean, uncorrelated, random variables or white noise.

The model could also be represented as:

$$\sigma_t^2 = \lambda + \sum \alpha_1 r_{t-1}^2 + \varepsilon_t \,.$$

Hence the volatility σ_{t+1}^2 can be represented by:

$$\sigma_{t+1}^2 = E((r_{t+1} - \mu)^2 \mid \Phi_t)$$

$$\sigma_{t+1}^2 = \lambda + \alpha_1 (r_{t-1} - \mu)^2 +, \ldots, + \alpha_q (r_{t-q} - \mu)^2$$

where Φ_t is the information set at the end of period *t*. This is an AR(*q*) model in terms of $(r_t - \mu)^2$. Therefore, the optimal one-day ahead forecast of period *t+1* volatility can be obtained based on the returns on the most recent *q* days. In general, an *h*-day ahead step forecast can be formed as follows:

$$\hat{\sigma}_{t+h}^2 = \lambda + \alpha_1 (\hat{r}_{t+h-1} - \mu)^2 +, \ldots, + \alpha_q (\hat{r}_{t+1-q} - \mu)^2$$

where $\hat{r}_{t+h-1} = r_{t+h-j}$ if $1 \leq h \leq j$ and ($\hat{\sigma}_{t+h-j}^2 = (\hat{r}_{t+h-1} - \mu)^2$ if $h > j$.

The ARCH (1) Model

This simple ARCH model exhibits constant unconditional variance but non-constant conditional variance:

$$r_t = \mu + \sigma_t \varepsilon_t$$

given that:

$$\varepsilon_t = u_t \sqrt{(\lambda + \alpha \varepsilon_{t-1}^2)}$$

where $u_t \sim IID(0,1)$ (IID, Independent and Identically Distributed, or strict white noise); and λ and $\alpha > 0$.

Note that $\sqrt{(\lambda + \alpha \varepsilon_{t-1}^2)}$ is the conditional standard deviation; and σ_t is defined as:

$$\sqrt{E(\varepsilon_t^2 \mid \varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \ldots, \varepsilon_{t-i}^2)} \,.$$

The simplest form of ARCH (1) model for the:

a) *conditional expectation* of ε_t given that ε_t is equal to zero, is defined as:

$$E(\varepsilon_t \varepsilon_{t-1}) = E(u_t \mid \varepsilon_{t-1})\sqrt{\lambda + \alpha\varepsilon_{t-1}^2} = 0$$

note that $E(u_t \mid \varepsilon_{t-1}) = E(u_t) = 0$ since $u_t \sim IID(0,1)$;

b) *conditional variance* is defined as:

$$Var(\varepsilon_t \mid \varepsilon_{t-1}) = E(u_t^2 \mid \varepsilon_{t-1})(\lambda + \alpha\varepsilon_{t-1}^2)$$

note that $E(u_t^2 \mid \varepsilon_{t-1}) = E(u_t^2) = 1$ since $u_t \sim IID(0,1)$.

Thus, the conditional mean and variance of r_t are given by the following formulae:

$$E(r_t \mid r_{t-1}) = \mu$$

and:

$$Var(r_t \mid r_{t-1}) = (\lambda + \alpha\varepsilon_{t-1}^2).$$

Therefore, the conditional variance of r_t is time varying. However, it can be easily seen that the unconditional variance is time invariant given that ε_t^2 is stationary:

$$Var(r_t) = Var(\varepsilon_t) = \frac{\lambda}{(1-\alpha)}.$$

First Order Autoregressive Process with ARCH Effects

For more complicated models such as AR(1)-ARCH(1), we obtain similar results provided that the process for *t* is stationary given that the autoregressive parameter is smaller than one in absolute value.

Assume the following first order autoregressive process:

$$r_t = \theta r_{t-1} + \varepsilon_t$$

where $\varepsilon_t = u_t\sqrt{\lambda + \alpha\varepsilon_{t-1}^2}$, $u_t \sim$ IIN(0,1), and $\lambda > 0$, $\alpha = 0$.

a) The *conditional expectation* of ε_t given that ε_t is equal to zero is:

$$E(\varepsilon_t \varepsilon_{t-1}) = E(u_t^2 \mid \varepsilon_{t-1})(\lambda + \alpha\varepsilon_{t-1}^2) = 0$$

note that $E(u_t \mid \varepsilon_{t-1}) = E(u_t) = 0$.

b) The *conditional variance* is given by the following formula:

$$Var(\varepsilon_t \mid \varepsilon_{t-1}) = E(u_t^2 \mid \varepsilon_{t-1})(\lambda + \alpha\varepsilon_{t-1}^2) = \lambda + \alpha\varepsilon_{t-1}^2$$

note that $E(u_t^2 \mid \varepsilon_{t-1}) = E(u_t) = 1$ since $u_t \sim$ IIN(0,1).

Then the conditional mean and variance of r_t are given by the following formulae:

$$E(r_t \mid r_{t-1}) = \theta r_{t-1}$$

and:

$$Var(r_t \mid r_{t-1}) = (\lambda + \alpha\varepsilon_{t-1}^2).$$

To find the unconditional variance of r_t we recall the following property for the variance:

$$Var(r_t) = E(Var(r_t \mid r_{t-1})) + Var(E(r_t \mid r_{t-1})).$$

The left hand-side formula $E(Var(r_t \mid r_{t-1}))$ is equal to $E(\lambda + \alpha\varepsilon_{t-1}^2)$, $\lambda + \alpha E(\varepsilon_{t-1}^2)$ and $\lambda + \alpha Var(\varepsilon_{t-1})$. The right hand-side formula $Var(E(r_t \mid r_{t-1}))$ is equal to $\theta^2 Var(r_{t-1})$. Then if the process is covariance stationarity, we have:

$$Var(r_t) = \frac{\lambda + \alpha Var(\varepsilon_{t-1})}{1-\theta^2}$$

or:

$$Var(r_t) = \frac{1}{(1-\alpha)(1-\theta^2)}$$

since:

$$Var(\varepsilon_{t-1}) = \frac{\lambda}{(1-\alpha)}.$$

According to Aydemir (1998), the important property of ARCH models is their ability to capture the tendency for volatility clustering in stock prices data, i.e. a tendency for large or small swings in prices to be followed by large or small swings in random direction. In addition, Barndorff-Nielsen, Nicolato and Shephard (2001) and Aydemir (1998) also found that the ARCH/GARCH type models are significantly outperformed by other models including the ARMA and SV models.

2.5.2 Autoregressive Moving Average (ARMA) Models

The ARMA models, where autoregressive in order *p*, [AR(*p*)] can be expressed as:

$$y_t = \gamma_1(y_{t-1}) + \gamma_2(y_{t-2}) +,\ldots,+ \gamma_p(y_{t-p}) + \varepsilon_t$$

where y_t = the actual or data value at time *t*, γ = the constant value, and ε_t = the residual or error term.

Moving average of order *q*, [MA(*q*)] can be expressed as:

$$y_t = \varepsilon_t - \theta_1(\varepsilon_{t-1}) - \theta_2(\varepsilon_{t-2}) -,\ldots,- \theta_q(\varepsilon_{t-q}) .$$

The general presentation for ARMA models is:

$$y_t = \gamma_{0,1} + \sum_{j=1}^{p} \gamma_j y_{t-j} + \sum_{j=0}^{q} \theta_j \varepsilon_{t-j} .$$

These models are widely used in the finance literature especially during the last decade. Some studies such as Schwert (1990), French, Schwert and Stambaugh (1987) and Poterba and Summer (1986) use the ARMA process for modelling volatility of the stock market. According to Aydemir (1998), the advantages of these models include the following: 1) the theory of the Gaussian model is well understood, therefore, the ARMA models are well developed; 2) modelling data within an ARMA structure is considerably easy; and 3) these models are capable of data analysis, forecasting and control.

However, several limitations of the ARMA models include: 1) these models have definite limitations in mimicking the properties where sudden bursts of the data at irregular time intervals, and periods of high and low volatility are detected; and 2) the ARMA type models are based on the assumption of constant variance. Most financial data exhibit changes in volatility and this feature of the data cannot be captured due to this assumption.

2.5.3 Stochastic Volatility (SV) Models

There are many types of Stochastic Volatility (SV) models, one the most popular being the discrete-time SV model, the continuous-time SV model and the jump diffusion model with SV. The relevant type of SV model applicable to Thai stock data is the discrete-time SV model, where s_t denotes the stock price at time *t* and the return process y_t is defined as (Jiang 1998):

$$y_t = \ln\left(\frac{s_t}{s_{t-1}}\right) - \mu_t \,.$$

The SV model of stock return may be written as:

$$y_t = \sigma_t \varepsilon_t$$

where $\varepsilon_t \sim$ *IID*. The most popular SV specification assumes that h_t follows an AR(1) process as:

$$h_{t+1} = \phi h_t + \eta_t \,,\ |\phi| < 1$$

where η_t is an innovation. This process is satisfied using the idea of Exponential GARCH (EGARCH) and this specification ensures that the conditional variance remains positive.

According to Barndorff-Nielsen, Nicolato and Shephard (2001) and Aydemir (1998), there are several advantages in using SV models. SV properties can be found and manipulated much easier than ARCH/GARCH type models and they can also mimic the fat tail property observed in the data. Finally, they also induce an incomplete market. In contrary, Hansen and Lunde (2001) disagree that these SV models are superior to the ARCH/GARCH type model when using returns of stock indices or bonds. Furthermore, in SV models, the persistence in volatilities can be captured by specifying a random walk process. This specification is analogous to the IGARCH specification.

2.6 GARCH Type Models

The implementation of univariate parametric models such as ARCH and GARCH type models in estimating and forecasting the financial market volatility has been growing in popularity, especially when dealing with incomplete or emerging financial markets. A most commonly used modified ARCH model has been the Generalized ARCH (GARCH) model developed by Bollerslev (1986). Other ARCH-type models are characterized by Nelson (1991), who introduced the Exponential GARCH (EGARCH). Glosten, Jagannathan and Runkle (1993) have developed the GJR-GARCH(p,q) model to estimate the relationship between the expected value and the volatility of nominal excess return on stocks. Ding, Granger and Engle (1993) developed a model which extends the ARCH class of models to identify a wider class of power transformations, called Power Generalized ARCH or PGARCH.

These models consist of linear and non-linear types – non-linear models are EGARCH, GJR-GARCH and PGARCH. Franses and Dijk (2000) conclude that linear time series models do not yield reliable forecasts. However, this does not imply that linear models are not useful, and these models are used in comparing the results for the index price of the Stock Exchange of Thailand.

2.6.1 GARCH(p,q)

In empirical applications of the ARCH(q) model, it is often difficult to estimate models with a large number of parameters. This motivates Bollerslev (1986) to use the Generalized ARCH or GARCH(p,q) specification to circumvent this problem.
The GARCH(p,q) model is defined as:

$$r_t = \mu + \sigma_t \varepsilon_t$$

and:

$$\sigma_t^2 = \lambda + \sum_{i=1}^{q} \alpha_i (r_{t-i} - \mu)^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \ .$$

The model could also be represented as:

$$\sigma_t^2 = \lambda + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

or:

$$\sigma_t^2 = \lambda + \alpha(L)\varepsilon_{t-1}^2 + \beta(L)\sigma_{t-1}^2 .$$

A sufficient condition for conditional variance in the GARCH(p,q) model to be well defined is that all the coefficients in the infinite order linear ARCH model must be positive. Given that $\alpha(L)$ and $\beta(L)$ have no common roots and that the roots of the polynomial in L, 1 - $\beta(L)$ = 0 lie outside the unit circle, this positive constraint is satisfied, if and only if, the coefficients of the infinite power series expansion for $\frac{\alpha(L)}{1-\beta(L)}$ are non-negative.

Rearranging the GARCH(p,q) model by defining $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, it follows that:

$$\varepsilon_t^2 = \lambda + (\alpha(L) + \beta(L))\varepsilon_{t-1}^2 - \beta(L)v_{t-1} + v_t$$

which defines an ARMA (Max(p,q),p) model for ε_t^2.

In addition, the model is covariance stationary if and only if all the roots of $(1-\alpha(L)-\beta(L))$ lie outside the unit circle. If all the coefficients are non-negative, this is equivalent to the sum of the autoregressive coefficients being smaller than 1. The analogy to the ARMA class of models also allows for the use of standard time series techniques in the identification of the order of p and q. In most empirical applications with finitely sampled data, the simple GARCH(1,1) is found to provide a fair description of the data.

The GARCH(1,1) is used to construct multi-period forecasts of volatility. When $\alpha + \beta$ <1, the unconditional variance of ε_{t+1} is $\frac{\lambda}{1-\alpha-\beta}$. If we rewrite the following GARCH(1,1) as:

$$\begin{aligned}\sigma_t^2 &= \lambda + \alpha(\varepsilon_{t-1}^2) + \beta(\sigma_{t-1}^2) \\ &= \lambda + \alpha(\varepsilon_{t-1}^2 - \sigma_{t-1}^2) + (\alpha+\beta)\sigma_{t-1}^2 .\end{aligned}$$

The coefficient measures the extent to which the impact of volatility will extend into the next period's volatility, while $(\alpha + \beta)$ measures the rate at which this effect reduces over time. Recursively substituting and using the law of iterated expectation, the conditional expectation of volatility *j* periods ahead is:

$$E_t[\sigma_{t+j}^2] = (\alpha + \beta)^j \left[\frac{\sigma_t^2 - \lambda}{1 - \alpha - \beta} \right] + \left[\frac{\sigma_t^2 - \lambda}{1 - \alpha - \beta} \right].$$

Note that the multi-period volatility forecast reverts to its unconditional mean at rate $(\alpha + \beta)$.

2.6.2 EGARCH

Even though the GARCH model has the capability to capture thick tailed returns, volatility clustering is not well suited to capture the leverage effect since the conditional variance is a function only of the magnitudes of the lagged residuals and not their signs. Nelson (1991) introduced the exponential GARCH (EGARCH) where σ_t^2 depends on both the sign and the size of lagged residuals.

The EGARCH(1,1) model is represented as follows:

$$\ln \sigma_t^2 = \lambda_1 + \beta_1 \ln \sigma_{t-1}^2 + \gamma_1 \left(\left[\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - (2/\pi)^{1/2} \right] + \delta \left[\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right] \right).$$

In fact, the EGARCH model always produces a positive conditional variance σ_t^2 for any choice of λ_1, β_1, γ_1 so that no restrictions need to be placed on these coefficients (except $|\beta_1| < 1$). Because of the use of both $|\varepsilon_t / \sigma_t|$ and $(\varepsilon_t / \sigma_t), \sigma_t^2$, it will also be non-symmetric in ε_t and, for negative δ, it will exhibit higher volatility for large negative ε_t. In addition, the EGARCH model is capable of capturing any asymmetric impact of shocks on volatility. This model allows volatility to be affected differently by good and bad news.

2.6.3 GARCH-M

A number of theories in finance assume some kind of relationship between the mean of a return and its variance. A way to take this into account is to explicitly write the returns as a function of the conditional variance or, in other words, to include the conditional variance as another regressor. The GARCH in Mean Model (GARCH-M) allows for the conditional variance to have mean effects. Most of the time this conditional variance term will have the interpretation of time varying risk premium.

Recall the equation:

$$\begin{aligned}\sigma_t^2 &= \lambda + \alpha(\varepsilon_{t-1}^2) + \beta(\sigma_{t-1}^2) \\ &= \lambda + \alpha(\varepsilon_{t-1}^2 - \sigma_{t-1}^2) + (\alpha + \beta)\sigma_{t-1}^2\end{aligned}$$

and ARCH-M:

$$r_t = \psi\sigma_t^2 + \varepsilon_t$$

where $\varepsilon_t = v_t\sigma_t$, and $v_t \sim N(0,1)$:

$$\sigma_t^2 = w + \lambda + \alpha\varepsilon_{t-1}^2 .$$

Then r_t may be expressed as:

$$r_t = \psi(\lambda + \alpha\varepsilon_{t-1}^2) + \varepsilon_t .$$

Consider the following formula (extension form of the above equation):

$$r_t = \theta x_t + \psi\sigma_t^2 + \varepsilon_t .$$

Therefore, GARCH-M could be defined as:

$$\sigma_t^2 = \lambda + \alpha(L)\varepsilon_{t-1}^2 + \beta(L)\varepsilon_{t-1}^2 .$$

Consistent estimation of θ and ψ is dependent on the correct specification of the entire model. The estimation of GARCH in mean type of models is numerically unstable and many empirical applications have used the ARCH-M type of models which are easier to estimate.

2.6.4 GJR-GARCH

Glosten, Jagannathan and Runkle (1993) have extended the GARCH(*p*,*q*) model to estimate the relationship between the expected value and the volatility of nominal excess return on stocks. Their GJR-GARCH is an alternative model capturing asymmetries in financial data. A univariate regression GJR-GARCH(*p*,*q*) process, with *q* coefficients α_i, ..., *q*, *p* coefficients, β_i, for *i*=1, ..., *p* and *k* linear regression coefficients b_i, for *i*=1,..., *k*, can be represented by:

$$r_t = \mu + x_t^T b_i + \varepsilon_t$$

and:

$$\sigma_t^2 = \lambda + \sum_{i=1}^{q}\alpha_i + (\gamma S_{t-1})\varepsilon_{t-1}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-1} .$$

This model allows the impact of the squared residual on conditional volatility to be different when the residuals are negative (first lagged) than when the residuals (first lagged) are positive. For $\gamma > 0$, all negative residuals are weighted and thus generate a different volatility in subsequent periods than do positive residuals of equal magnitude. In other words, negative shocks increase volatility more than positive shocks. Thus, the leverage of firm increases with negative return, inducing a higher volatility.

2.6.5 PGARCH

Ding, Granger and Engle (1993) suggest a model which extends the ARCH class of models to identify a wider class of power transformations than simply taking the absolute value or squaring the data as in the conventional models. This class of models is called Power ARCH (PARCH) and Power Generalized ARCH (PGARCH).
PGARCH is defined as:

$$\sigma_t^2 = \lambda + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2 + \sum_{i=1}^{q}\alpha(|\varepsilon_{t-i}| + \lambda\varepsilon_{t-i})^2 .$$

It has been found that the sample autocorrelation function for absolute returns and squared returns remains significantly positive for very long lags. The pattern of the sample autocorrelation for various speculative returns is quite different from that of the theoretical autocorrelation functions given by the GARCH(p,q) or EGARCH(p,q) process. Ding and Granger (1996) propose a two-component GARCH model which gives a much better description of the real data:

$$\sigma_t^2 = \frac{\lambda}{(1-\beta_1)(1-\beta_2)} + \sum_{i=1}^{p} \alpha_1 \beta_1^{i-1} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \alpha_2 \beta_2^{j-1} \varepsilon_{t-j}^2 .$$

The intuition behind this two-component model is that one can use two different variance components, each of them having an exponentially decreasing autocorrelation pattern, to model the long-term and short-term movements in volatility.

Chapter 3
Research Methodology

We use Excel as a method of volatility estimation and forecasting. Based on the daily airline ticket prices, the GARCH, or Generalized Autoregressive Conditionally Heteroscedastic, process is widely accepted method in estimating and forecasting the volatility. The Excel program on which the estimation and forecasting is constructed by using appropriate calculation and functions to ensure its reliability in volatility estimation. Moreover, the most commonly used method for comparing the evaluation of econometric models is presented.

3.1 Data Collection

3.1.1 Data of ticket prices in term of time series for both domestic and international flights departing from Bangkok had been observed.

3.1.2 For domestic routes, the most popular routes have been selected, i.e. Bangkok to Chiang Mai, and Bangkok to Phuket.

3.1.3 For international routes, Bangkok to Tokyo, and Bangkok to Melbourne have been selected.

3.1.4 Airlines ticket prices are collected from Nok Air, Thai AirAsia, NokScoot and Jetstar webpages.

3.1.5 We compared ticket prices and volatility between normal day and long weekends (including public holidays) in this study.

3.1.6 All prices with each departing schedule within a day are recorded up to six months in advance during the study period between January 2019 to August 2019. We also recorded price changes of the same flight when booking at a different day compared to the base day (t) up to 180 days ahead (t_{+180}).

3.2 Research Methodology

By applying a volatility model, GARCH(1,1) developed and modified by Islam and Watanapalachaikul, we can detect the sensitivity and volatility level of dynamic pricing. Furthermore, by understanding consumer behavior, airlines may be constantly changing prices as a tool to manipulate consumer behavior, as price volatility affects the range of prices that consumers consider to be acceptable for a given service for the profit maximization purpose.

For a volatility model being considered to be reliable, it should provide accurate risk or volatility results across different time horizons and risk levels within the same class. We have tested various volatility models to describe time-varying variance. The GARCH type model has led to the development of other related formulations in order to identify and explain the variance of time series. In this research, a new approach to measure volatility of the dynamic pricing was introduced by adopting a modified model of the original GARCH-type (Generalized Autoregressive Conditional Heteroskedasticity) model. We hypothesize that given the variance of time closer to the departure date; significant volatility in dynamic pricing would be detected.

There are various mathematical tools and software packages that can be used to calculate the volatility results such as Math Lab, Mathematica, etc. However, this research used Microsoft Excel to calculate the GARCH results because of unavailability of those software packages.

The general process for a GARCH model involves three steps. The first is to estimate a best-fitting autoregressive model. The second is to compute autocorrelations of the error term. The third step is to test for significance. Two other widely used approaches to estimating and predicting financial volatility are the classic historical volatility (VolSD) method and the exponentially weighted moving average volatility (VolEWMA) method. In general, heteroskedasticity describes the irregular pattern of variation of an error term, or variable, in a statistical model.

Essentially, where there is heteroskedasticity, observations do not conform to a linear pattern. Instead, they tend to cluster. The result is that the conclusions and predictive value one can draw from the model will not be reliable. GARCH is a statistical model that can be used to analyze a number of different types of financial data.

3.3 Research Model and Tool

Generalized Autoregressive Conditional Heteroskedasticity, or GARCH, is an extension of the ARCH model that incorporates a moving average component together with the autoregressive component.

Specifically, the model includes lag variance terms (e.g. the observations if modeling the white noise residual errors of another process), together with lag residual errors from a mean process.

The introduction of a moving average component allows the model to both models the conditional change in variance over time as well as changes in the time-dependent variance. Examples include conditional increases and decreases in variance. As such, the model introduces a new parameter “p” that describes the number of lag variance terms:

- *p*: The number of lag variances to include in the GARCH model.
- *q*: The number of lag residual errors to include in the GARCH model.

A generally accepted notation for a GARCH model is to specify the GARCH function with the *p* and *q* parameters GARCH(p, q); for example GARCH(1, 1) would be a first order GARCH model.

GARCH(p,q) by Using Excel – NumXL (add-ins)

If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedasticity (GARCH in Excel, Bollerslev 1986) model.

$$x_t = \mu + a_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \, \alpha_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$$

$$\alpha = \sigma_t \times \epsilon_t$$
$$\epsilon_t = P_v(0,1)$$

Where:

- x_t is the time series value at time t.
- μ is the mean of GARCH in Excel model.
- a_t is the model's residual at time t.
- σ_t is the conditional standard deviation (i.e. volatility) at time t.
- P is the order of the ARCH component model.
- $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_p$ are the parameters of the the ARCH component model.
- q is the order of the GARCH component model.
- $\beta_0, \beta_1, \beta_2, \dots, \beta_q$ are the parameters of the the GARCH component model.
- $[\epsilon_t]$ are the standardized residuals:
- $[\epsilon_t] \sim i.i.d$
- $E[\epsilon_t] = 0$
- $VAR[\epsilon_t] = 1$
- P_v is the probability distribution function for ϵ_t . Currently, the following distributions are supported:

  Normal distribution $P_v = N(0,1)$

  Student's t-distribution $P_v = t_v(0,1)_{v>4}$

Generalized error distribution (GED[i]) $P_v = GED_v(0,1)_{v>1}$

- *Clustering*: a large a_{t-1}^2 or σ_{t-1}^2 gives rise to a large σ_t^2. This means a large a_{t-1}^2 tends to be followed by another large a_t^2, generating, the well-known behavior, of volatility clustering in financial time series.
- *Fat-tails*: The tail distribution of a GARCH in Excel (p,q) process is heavier than that of a normal distribution.
- *Mean-reversion*: GARCH in Excel provides a simple parametric function that can be used to describe the volatility evolution. The model converge to the unconditional variance of α_t , where $a_\infty^2 \rightarrow V_L = \frac{\alpha_0}{1-\sum_{i=1}^{\max(p,q)}(\alpha_i+\beta_i)}$

Chapter 4

Results

In GARCH(1,1) model estimates $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \, \alpha_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$, we used data of ticket prices with each departing schedule within a day up to 6 months in advance. M_6 represents the volatility of ticket price for a period of six months (or 180 days, D_{t+180}) in advance, where $M_6 = D_{t+180}$, $M_5 = D_{t+150}$, $M_4 = D_{t+120}$, … , $M_1 = D_{t+30}$. M_2 and M_3 represent the fluctuation of ticket price changes for a period of two and three months in advance respectively. D_{t+1}…D_{t+7} are price changes of the same flight when booking at a different day compared to the base day up to 7 days ahead.

As anticipated, volatility of the ticket prices in dynamic pricing strategy in low-cost domestic airlines such as Nok Air and Thai Air Asia, measured by Alfa α_i and Beta β_j, is considerably low. On the other hand, international flights operated by Nok Scoot and Jetstar, volatility of the tickets prices is arguably moderate to high. The results of this study, the ticket price volatility is more significant at M_1, while M_2 and M_3 were less volatile as time to departure is further away up to M_6. Moreover, the results of volatility of D_{t+1} prove to be highest followed by D_{t+3} and D_{t+7} respectively.

It is observed that an increase in price volatility was evident as time to departure approaches which resulted in a positive Alfa and Beta (increase in price of the ticket). A perspective on the volatility of ticket prices in dynamic pricing model is shown in table below for the following test results.

A. Domestic Flights (Normal Day)

i. Bangkok to Chiang Mai (operated by Nok Air) departure on Tuesday, 23rd July 2019, 16.00-17.40

ii. Bangkok to Phuket (operated by Thai Air Asia) departure on Tuesday, 23rd July 2019, 19.35-21.00

B. Domestic Flights (Long Weekend / Mother day)

i. Bangkok to Chiang Mai (operated by Nok Air) departure on Friday, 9th August 2019, 16.00-17.40

ii. Bangkok to Phuket (operated by Thai Air Asia) departure on Friday, 9th August 2019, 19.35-21.00

C. International Flights (Normal Day)

i. Bangkok to Tokyo (operated by Nok Scoot) departure on Tuesday, 23rd July 2019, 00.45-09.05

ii. Bangkok to Melbourne (operated by Jetstar) departure on Tuesday, 23rd July 2019, 21.25-10.30(D+1)

D. International Flights (Long Weekend / Mother day)

i. Bangkok to Tokyo (operated by Nok Scoot) departure on Saturday, 10th August 2019, 00.45-09.05

ii. Bangkok to Melbourne (operated by Jetstar) departure on Friday, 9th August 2019, 21.25-10.30(D+1)

4.1 Domestic Flights (Normal Day)

4.1.1 Bangkok to Chiang Mai (operated by Nok Air)

Table 4.1: Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Chiang Mai (operated by Nok Air) departure on Tuesday, 23rd July 2019, 16.00-17.40

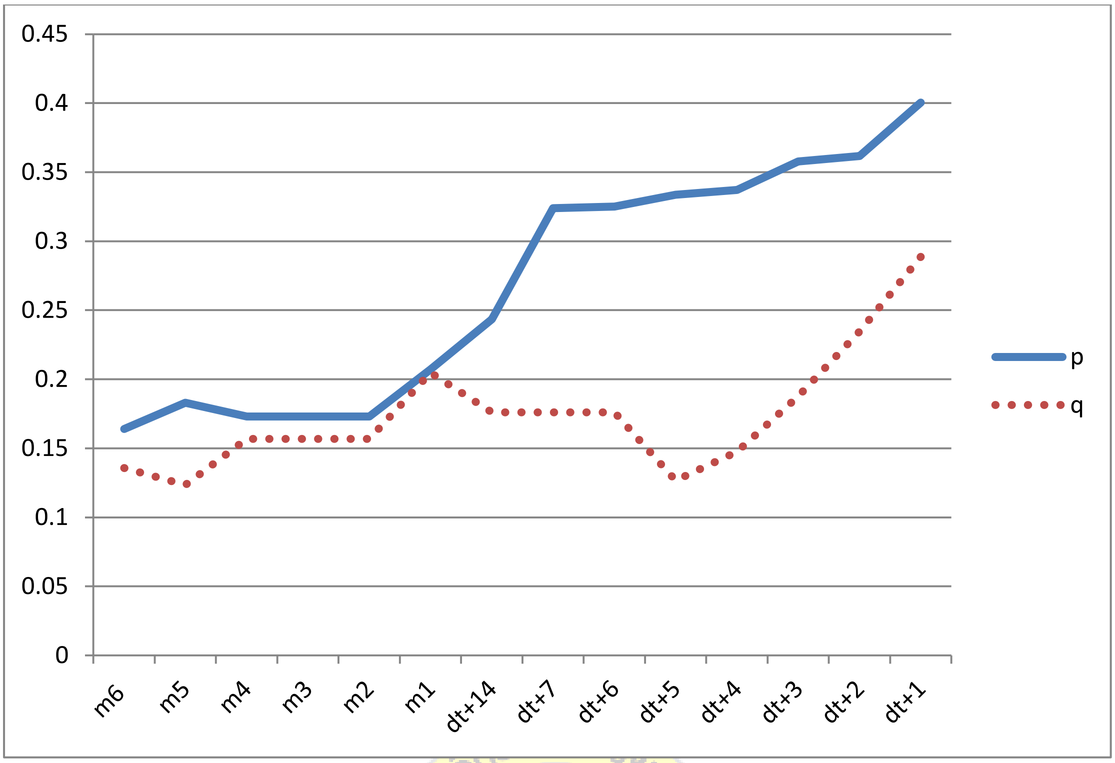
Explanatory variables	$\sum_{i=1}^{p} \alpha_i \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1639	0.1356	0.1456
m_5	0.1830	0.1234	0.1657
m_4	0.1730	0.1567	0.1890
m_3	0.1730	0.1567	0.1890
m_2	0.1730	0.1567	0.1890
m_1	0.2070	0.2048	0.0970
d_{t+14}	0.2435	0.1760	0.0998
d_{t+7}	0.3240	0.1760	0.0996
d_{t+6}	0.3250	0.1760	0.0994
d_{t+5}	0.3337	0.1267	0.1218
d_{t+4}	0.3372	0.1476	0.1134

d_{t+3}	0.3577	0.1876	0.1654
d_{t+2}	0.3616	0.2347	0.1222
d_{t+1}	0.4004	0.2886	0.1143

Moving Average = 10, Iteration = 10, Log-Likelihood = 552, Wald Chi-Square Test = 62.21

According to the result of GARCH(1,1), the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant. The results for the coefficient, standard error, α, and β are found using an *iterative procedure*. Under this iterative procedure, we assume the given value of λ, and estimated parameters α and β, we then use the estimate of λ to re-estimate α and β. Larger value of coefficients α_i and β_j imply that higher volatility is expected for the variables.

Figure 4.1 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Chiang Mai (operated by Nok Air) departure on Tuesday, 23rd July 2019, 16.00-17.40



According to the results, we noticed low volatility during M_2 to M_4. During this period, Nok Air had launched the promotion campaign "flying everyday low price at 750 Baht including taxes (May-July 2019)". However, the volatility was significant higher when the departure time was less than one month, which the ticket prices started to increase with promotional exclusive, especially a week before departure. The ticket prices gradually increased everyday for the last 7 days prior to departure, which observed by higher coefficients α_i.

4.1.2 Bangkok to Phuket (operated by Thai Air Asia)

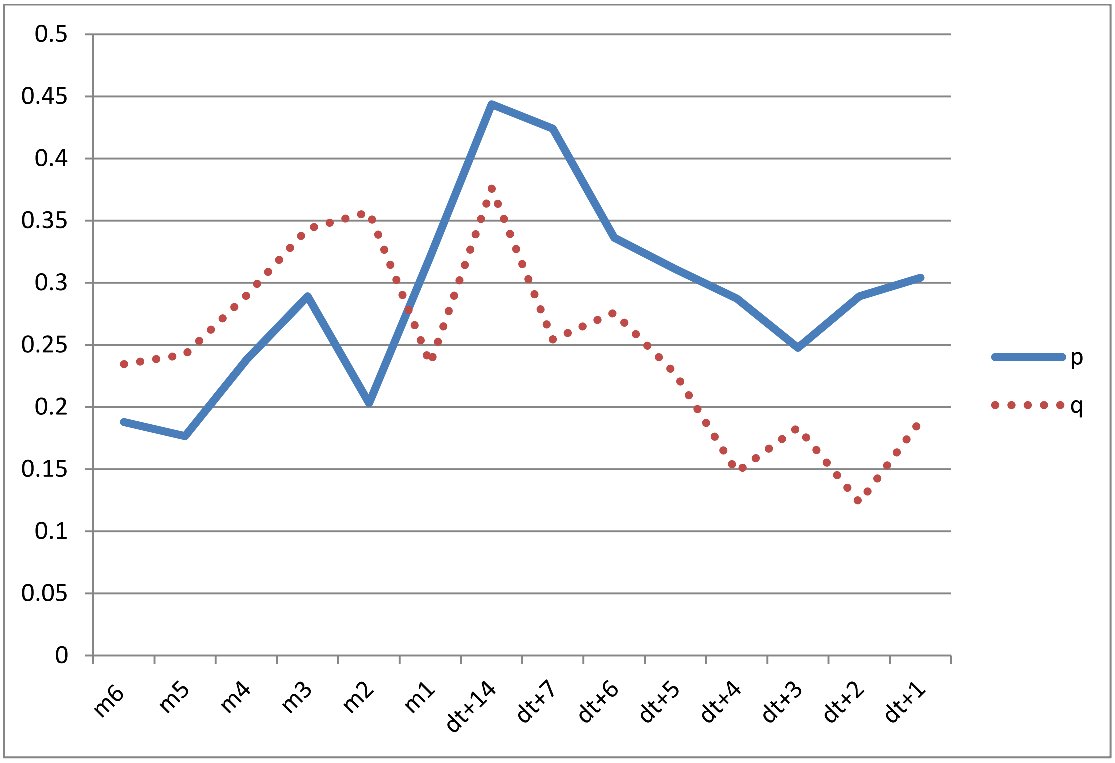
According to U.S. News (2019), Phuket is ranked 8th in world's best tourist destination. It is also popular among the domestic visit as well. In this study, the result of GARCH (1,1) for this route indicates low volatility of ticket prices from m_4 to m_6 (February – April 2019) for domestic departure on 23[th] July 2019 by Thai Air Asia. The volatility of ticket prices was increased during 3 months before departure, which resulted from a gradual increase in ticket prices over time. GARCH(1,1) shows the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant (see table 4.2).

Table 4.2 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Phuket (operated by Thai Air Asia) departure on Tuesday, 23[rd] July 2019, 19.35-21.00

Explanatory variables	$\sum_{i=1}^{p} \alpha_i \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1879	0.2346	0.1837
m_5	0.1766	0.2422	0.1697
m_4	0.2380	0.2899	0.1769
m_3	0.2890	0.3433	0.0876
m_2	0.2030	0.3567	0.1899
m_1	0.3211	0.2348	0.1970
d_{t+14}	0.4435	0.3760	0.1998
d_{t+7}	0.4240	0.2544	0.1654
d_{t+6}	0.3361	0.2760	0.1994
d_{t+5}	0.3109	0.2267	0.1111
d_{t+4}	0.2872	0.1476	0.1134
d_{t+3}	0.2477	0.1834	0.1354
d_{t+2}	0.2890	0.1232	0.1543
d_{t+1}	0.3040	0.1890	0.1664

Moving Average = 10, Iteration = 10, Log-Likelihood = 343, Wald Chi-Square Test = 78.82

Figure 4.2 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Phuket (operated by Thai Air Asia) departure on Tuesday, 23rd July 2019, 19.35-21.00



According to figure 4.2, the results for the coefficient, standard error, α, and β are found by using an iterative procedure were positive. Under this iterative procedure in the sequence of 10, we found larger value of coefficients α_i and β_j imply that higher volatility is expected for the variables. The result also shows high volatility during the last month (June – July 2019) before departure M_1, especially two weeks d_{t+14} before departure where coefficients α_i were greater than 0.4435.

By comparing figures 4.1 and 4.2, low volatility was found during 2-6 months before departure. The ticket prices' volatility of Nok Air airline had risen just a month before the departure especially a week before travel. Thai Air Asia ticket prices rose significantly during last two weeks before departure but not as frequent as Nok Air's. Therefore, Thai Air Asia ticket prices were less volatile than Nok Air during the last two weeks before departure.

4.2 Domestic Flights (Long Weekend / Mother day)

4.2.1 Bangkok to Chiang Mai (operated by Nok Air)

Table 4.3 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Chiang Mai (operated by Nok Air) departure on Friday, 9th August 2019, 16.00-17.40

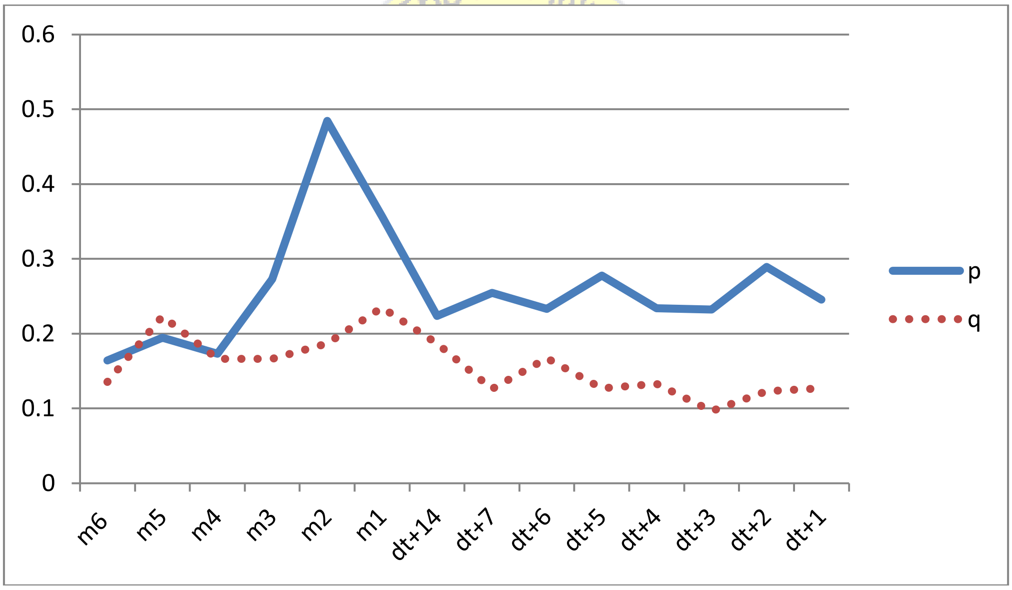
Explanatory variables	$\sum_{i=1}^{p} \alpha_i \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$	Standard error $[\epsilon_t]\sim i.i.d$
m_6	0.1639	0.1356	0.1456
m_5	0.1944	0.2232	0.1657
m_4	0.1730	0.1664	0.1766
m_3	0.2730	0.1664	0.1766
m_2	0.4845	0.1867	0.0990
m_1	0.3570	0.2348	0.1970
d_{t+14}	0.2237	0.1867	0.0818
d_{t+7}	0.2543	0.1262	0.1187
d_{t+6}	0.2332	0.1670	0.0818
d_{t+5}	0.2776	0.1267	0.1890
d_{t+4}	0.2337	0.1327	0.0987
d_{t+3}	0.2321	0.0966	0.1232
d_{t+2}	0.2893	0.1231	0.1654

d_{t+1}	0.2455	0.1269	0.1552

Moving Average = 10, Iteration = 10, Log-Likelihood = 699, Wald Chi-Square Test = 102.33

Similarly to the results of GARCH(1,1) in previous findings, the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant. According to the results, we noticed low volatility during M_4 to M_6.

Figure 4.3 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Chiang Mai (operated by Nok Air) departure on Friday, 9th August 2019, 16.00-17.40



This explains that travelers were not really anticipated in planning for long weekend/holiday. Another reason would be an issue regarding to environment issue at that time (PM2.5), which refers to atmospheric particulate matter that have a diameter of less than 2.5 micrometers.

According to figure 4.3, during up to three months before departure M_1 to M_3, there was an evidence of highest volatility, which measured by coefficient in α_i of 0.4845 particularly during two months before departure. This explains that travellers were planning for spending their times around 30-60 days before departure date. During two weeks before departure, d_{t+14} to d_{t+14}, moderate volatility was found where α_i were between 0.22-0.28. The ticket prices during this period increased but not significantly.

4.2.2 Bangkok to Phuket (operated by Thai Air Asia)

Table 4.4 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Phuket (operated by Thai Air Asia) departure on Friday, 9th August 2019, 19.35-21.00

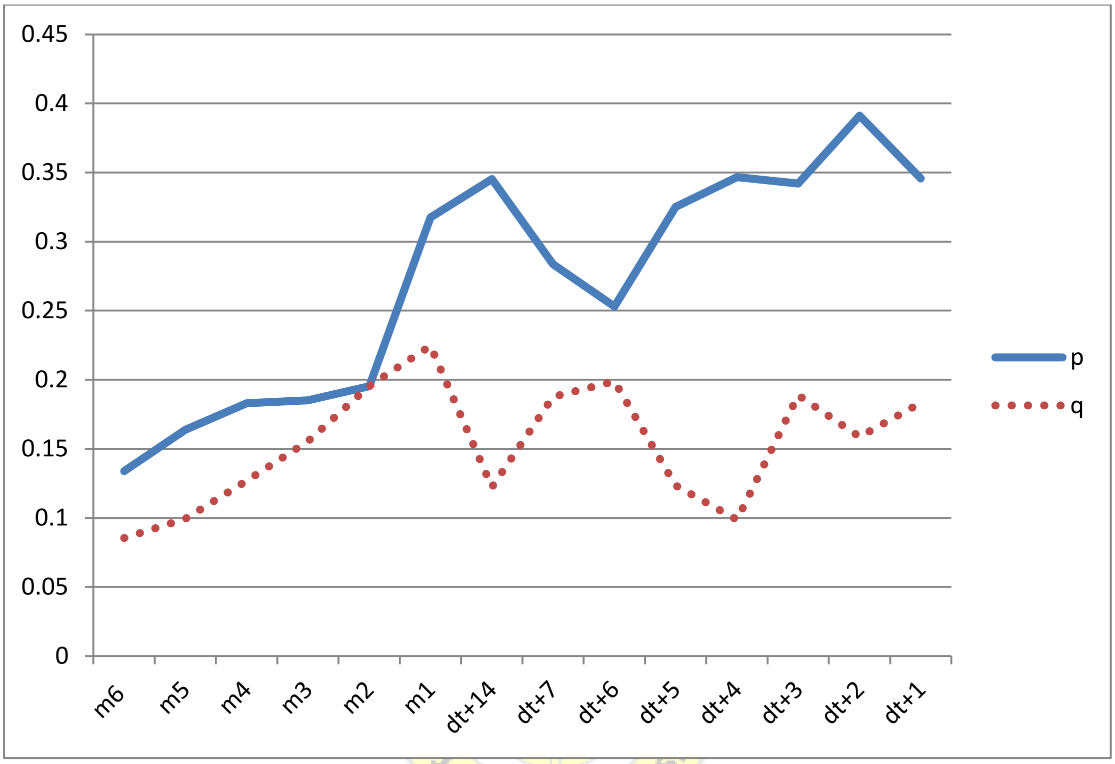
Explanatory variables	$\sum_{i=1}^{p} \alpha_i \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_{\square} \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1339	0.0855	0.1456
m_5	0.1637	0.0994	0.1657
m_4	0.1831	0.1268	0.1830
m_3	0.1850	0.1555	0.1301
m_2	0.1952	0.1956	0.1591
m_1	0.3174	0.2244	0.0977
d_{t+14}	0.3452	0.1223	0.1341
d_{t+7}	0.2835	0.1876	0.1215
d_{t+6}	0.2530	0.1987	0.1521
d_{t+5}	0.3251	0.1234	0.1843
d_{t+4}	0.3466	0.0987	0.1286
d_{t+3}	0.3421	0.1888	0.1634
d_{t+2}	0.3912	0.1587	0.1356
d_{t+1}	0.3458	0.1833	0.1632

Moving Average = 10, Iteration = 10, Log-Likelihood = 482, Wald Chi-Square Test = 45.29

GARCH (1,1) shows the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant. The results of GARCH (1,1) indicate that the on-line ticket prices between Bangkok to Phuket during m_2 to m_6 (February – June 2019) were rather stable with slight increase in ticket prices, which were evident by low volatility of ticket prices during this period for domestic departure on 23th July 2019 by Thai Air Asia.

On the other hand, higher volatility of ticket prices was increased just a month before departure, we found a sharp increase in ticket prices during a month before departure. The results for the coefficient, under this iterative procedure, we found larger value of coefficients α_i and β_j imply that higher volatility is expected for the variables during a week before departure. Especially during a week before departure where d_{t+1} to d_{t+5} where coefficients α_i were 0.3458, 0.3912, 0.3421, 0.3466, 0.3251 and β_j were 0.1833, 0.1587, 0.1888, 0.0987, 0.1234 respectively (see figure 4.4).

Figure 4.4 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Phuket (operated by Thai Air Asia) departure on Friday, 9th August 2019, 19.35-21.00



According to figures 4.3 and 4.4, high volatility was detected during two to three months before departure, especially Nok Air flight to Chiang Mai. This could be explained that travellers are planning their trip to Chiang Mai just 2 or 3 months prior to their travels over holiday/long weekend. However, we found incremental volatility in ticket prices from Thai Air Asia, flight to Phuket. The ticket prices had increase steadily over the study period especially a week before travel over their long weekend. We also notice the ticket prices of Thai Air Asia had increased substantially during last week before departure, which led to higher volatility during this period.

4.3 International Flights (Normal Day)

4.3.1 Bangkok to Tokyo (operated by Nok Scoot)

Table 4.5 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Tokyo (operated by Nok Scoot) departure on Tuesday, 23rd July 2019, 00.45-09.05

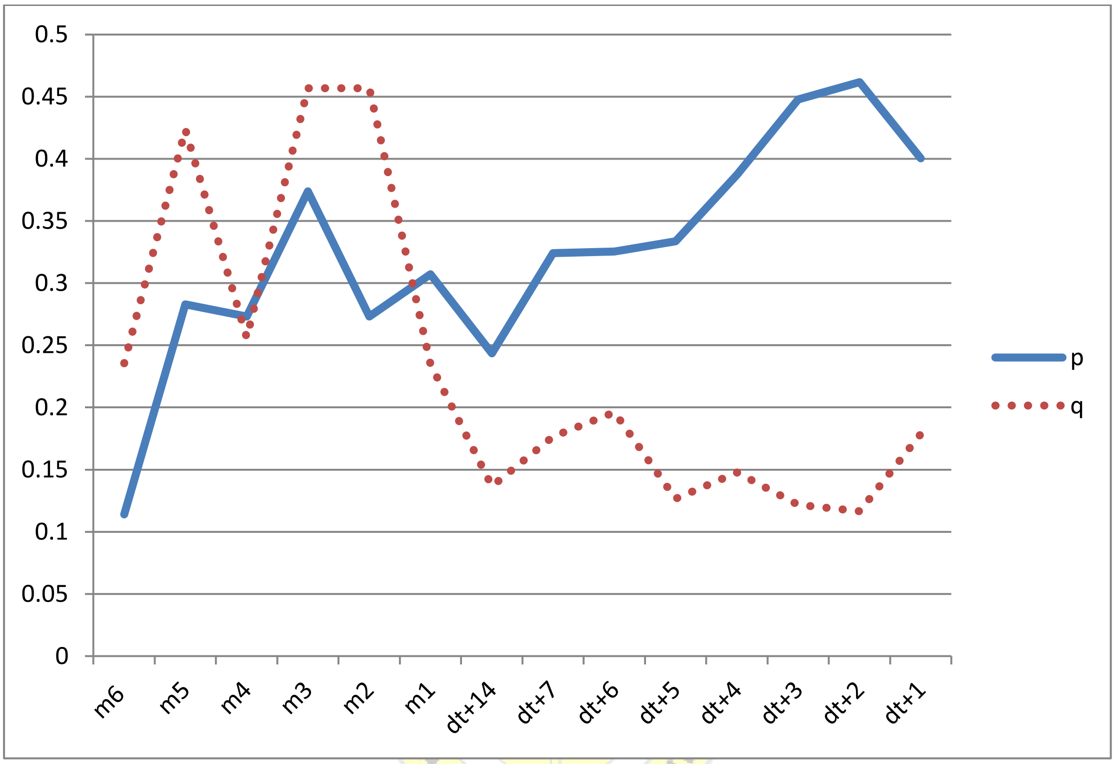
Explanatory variables	$\sum_{i=1}^{p} \alpha_i \, \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1139	0.2356	0.1456
m_5	0.2830	0.4234	0.1657
m_4	0.2732	0.2562	0.1890
m_3	0.3738	0.4567	0.1890
m_2	0.2731	0.4567	0.1890
m_1	0.3070	0.2348	0.0970
d_{t+14}	0.2435	0.1367	0.0998
d_{t+7}	0.3240	0.1765	0.0996
d_{t+6}	0.3255	0.1961	0.0994
d_{t+5}	0.3337	0.1267	0.1218
d_{t+4}	0.3872	0.1476	0.1134
d_{t+3}	0.4477	0.1216	0.1654
d_{t+2}	0.4616	0.1165	0.1222
d_{t+1}	0.4004	0.1786	0.1143

Moving Average = 10, Iteration = 10, Log-Likelihood = 331, Wald Chi-Square Test = 34.02

Bangkok to Tokyo route is considered one of the most popular for Thai people. According to Bangkok Post (2018), Japan is the top travel destination for Thai travellers, followed by China, Singapore and South Korea.

According to table 4.5, the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant. The results for the coefficient, standard error, α, and β are found using an *iterative* *procedure*. Under this iterative procedure, we assume the given value of λ, and estimated parameters α and β, we then use the estimate of λ to re-estimate α and β. Larger value of coefficients α_i and β_j imply that higher volatility is expected for the variables.

Figure 4.5 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Tokyo (operated by Nok Scoot) departure on Tuesday, 23rd July 2019, 00.45-09.05



The results show some fluctuation of volatility in ticket prices over the study period. We noticed low volatility during M_6. However, the ticket prices steadily rose overtime during the study period. Medium volatility in ticket prices occurred three months before the departure. High volatility was evident during three days before departure, where α_i was above 0.4.

4.3.2 Bangkok to Melbourne (operated by Jetstar)

Melbourne is not considered as a popular destination for Thai travellers. Jetstar operates flight between Bangkok and Melbourne every two days. The result of

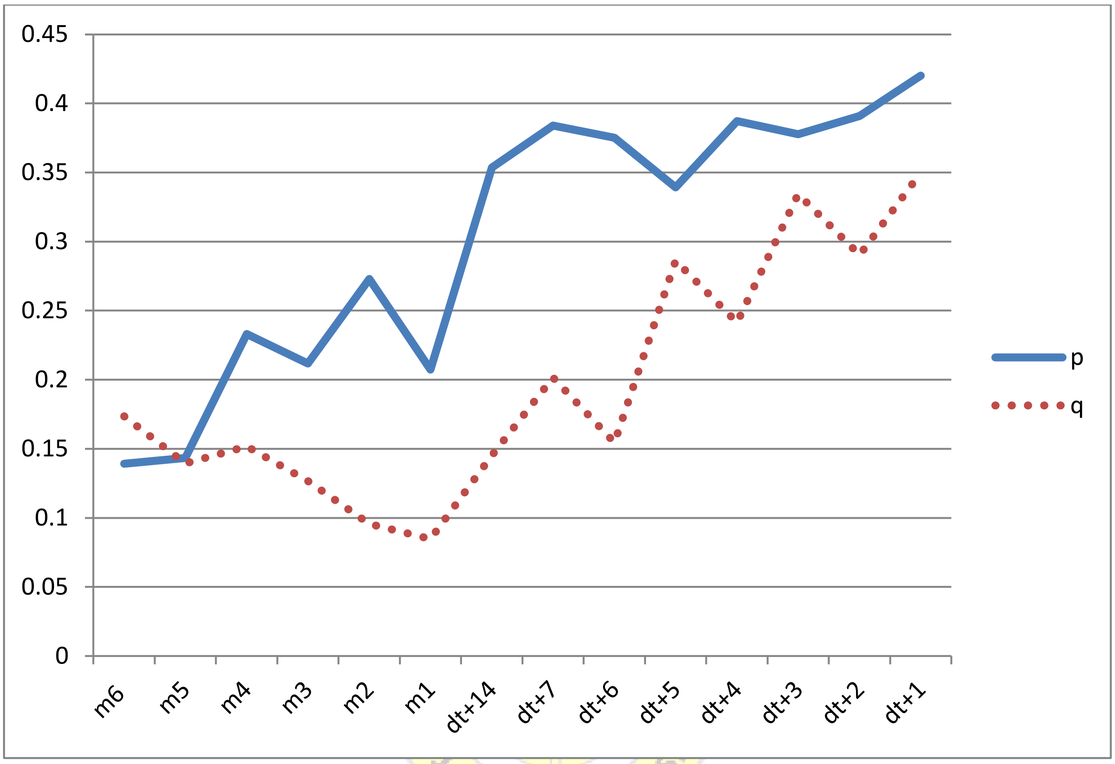
GARCH (1,1) for this route indicates low volatility of ticket prices from m_1 to m_6 (February – July 2019) for international departure on 23^{th} July 2019 by Thai Jetstar and arrives on the following day.

Table 4.6 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Melbourne (operated by Jetstar) departure on Tuesday, 23rd July 2019, 21.25-10.30(D+1)

Explanatory variables	$\sum_{i=1}^{p} \alpha_i \, \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1392	0.1735	0.1237
m_5	0.1434	0.1394	0.0923
m_4	0.2330	0.1518	0.0845
m_3	0.2117	0.1266	0.1443
m_2	0.2730	0.0956	0.1890
m_1	0.2074	0.0848	0.2312
d_{t+14}	0.3535	0.1453	0.3124
d_{t+7}	0.3840	0.2015	0.3341
d_{t+6}	0.3751	0.1545	0.3243
d_{t+5}	0.3393	0.2862	0.2532
d_{t+4}	0.3871	0.2413	0.2354
d_{t+3}	0.3777	0.3343	0.1656
d_{t+2}	0.3910	0.2904	0.1323
d_{t+1}	0.4202	0.3498	0.1433

Moving Average = 10, Iteration = 10, Log-Likelihood = 607, Wald Chi-Square Test = 37.60

Figure 4.6 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Melbourne (operated by Jetstar) departure on Tuesday, 23rd July 2019, 21.25-10.30(D+1)



The volatility of ticket prices was significant during 2 weeks before departure, which resulted from an increase in ticket prices during the last 2 weeks before departure. In this study, GARCH(1,1) result shows the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant. The results for the coefficient, standard error, α, and β are found by using an iterative procedure were positive.

Under this iterative procedure in the sequence of 10, we found larger value of coefficients α_i and β_j imply that higher volatility is expected for the variables. The result also shows high volatility during the last day before departure d_{t+1}, where coefficients α_i was 0.4202.

According to figure 4.5, it was interesting to see that volatility of Nok Scoot ticket was moderate to high over 5 months before travel (flight from Bangkok to Tokyo). This could be explained that the ticket price was initially set a low price and the price was increase over time. Comparing to volatility of ticket prices by Jetstar, flight from Bangkok to Melbourne, that the initial price was set rather high (see figure 4.6). Therefore, Jetstar's ticket price changes were less volatile when we compared to those by Nok Scoot.

In addition, both figures (4.5 and 4.6) share some commonality that the ticket price volatility was significant during the last two weeks before departure on normal day (the closer to the departure time, the higher the volatility of ticket prices).

4.4 International Flights (Long Weekend / Mother day)

4.4.1 Bangkok to Tokyo (operated by Nok Scoot)

Table 4.7 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Tokyo (operated by Nok Scoot) departure on Saturday, 10th August 2019, 00.45-09.05

Explanatory variables	$\sum_{i=1}^{p} \alpha_i \, \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1923	0.1587	0.0983
m_5	0.1343	0.0843	0.0741
m_4	0.2234	0.0957	0.1436
m_3	0.3330	0.1945	0.1375
m_2	0.4230	0.2936	0.1629
m_1	0.2070	0.2274	0.2547
d_{t+14}	0.3435	0.1322	0.1918
d_{t+7}	0.1240	0.1128	0.0823
d_{t+6}	0.1240	0.1128	0.0823
d_{t+5}	0.1240	0.1128	0.0823
d_{t+4}	0.1240	0.1128	0.0823
d_{t+3}	0.1240	0.1128	0.0823
d_{t+2}	0.1240	0.1128	0.0823
d_{t+1}	0.1240	0.1128	0.0823

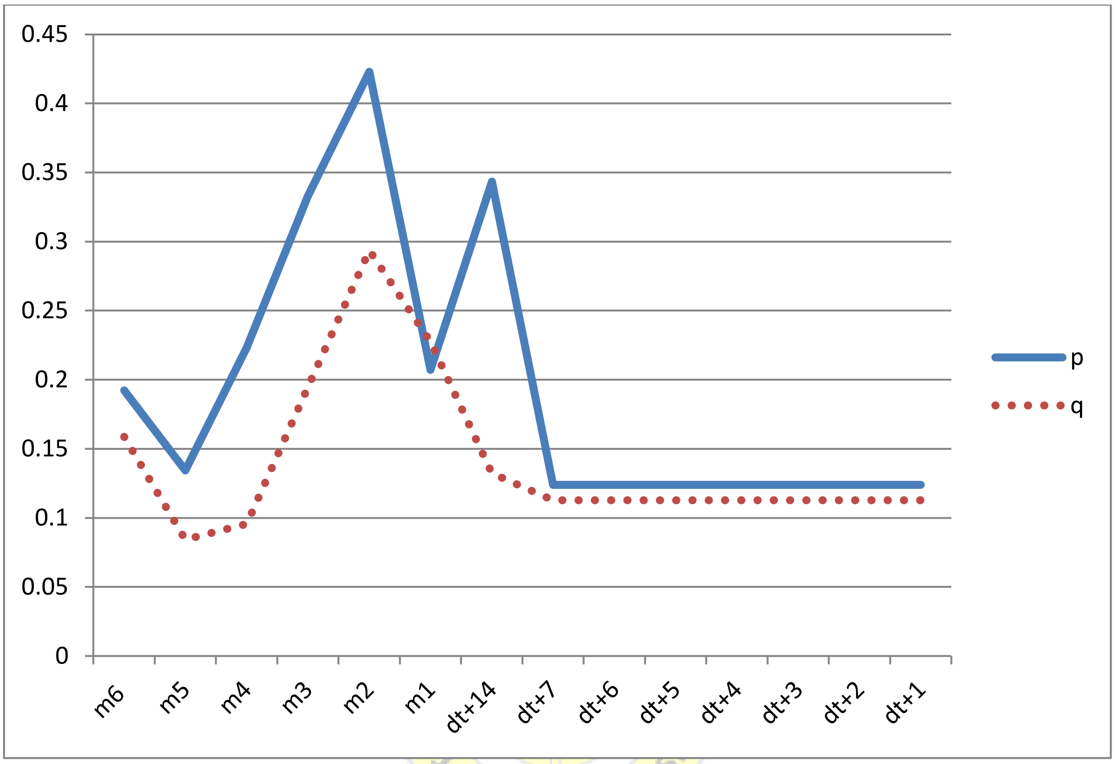
Moving Average = 10, Iteration = 10, Log-Likelihood = 794, Wald Chi-Square Test = 62.92

GARCH(1,1) results shows high statistically significant of coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation. According to the results, we noticed low volatility during M_5 to M_6 and moderate volatility during M_3. However, during up to two months before departure M_2, there was an evidence of highest volatility, which measured by coefficient in α_i of 0.4230 and α_i of 0.2936. This explains that travellers were busy planning holidays two months before departure date.

In addition, during a week before departure, d_{t+1} to d_{t+7}, we observed the constant volatility during this period. This means that the flight was fully booked and no ticket was available.

We believed that the flight between Bangkok and Tokyo during long weekends/ holiday was highly demanded. Traveller anticipated in advance booking, which resulted in a sharp increase up in ticket prices overtime.

Figure 4.7 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Tokyo (operated by Nok Scoot) departure on Saturday, 10th August 2019, 00.45-09.05



4.4.2 Bangkok to Melbourne (operated by Jetstar)

Even though, the flight between Bangkok and Melbourne are not popular comparing to Tokyo, however, the flight on 9th August was almost fully booked. GARCH (1,1) shows the coefficients on both the lagged squatted residual and lagged conditional variance in the Variance Equation are highly statistically significant. The results of GARCH (1,1), in table 4.7, indicate that the on-line ticket prices between Bangkok to Melbourne during m_1 to m_6 (February – July 2019) were gradual increase in ticket prices, which were evident by low volatility of ticket prices α_i where the coefficient is less than 0.3.

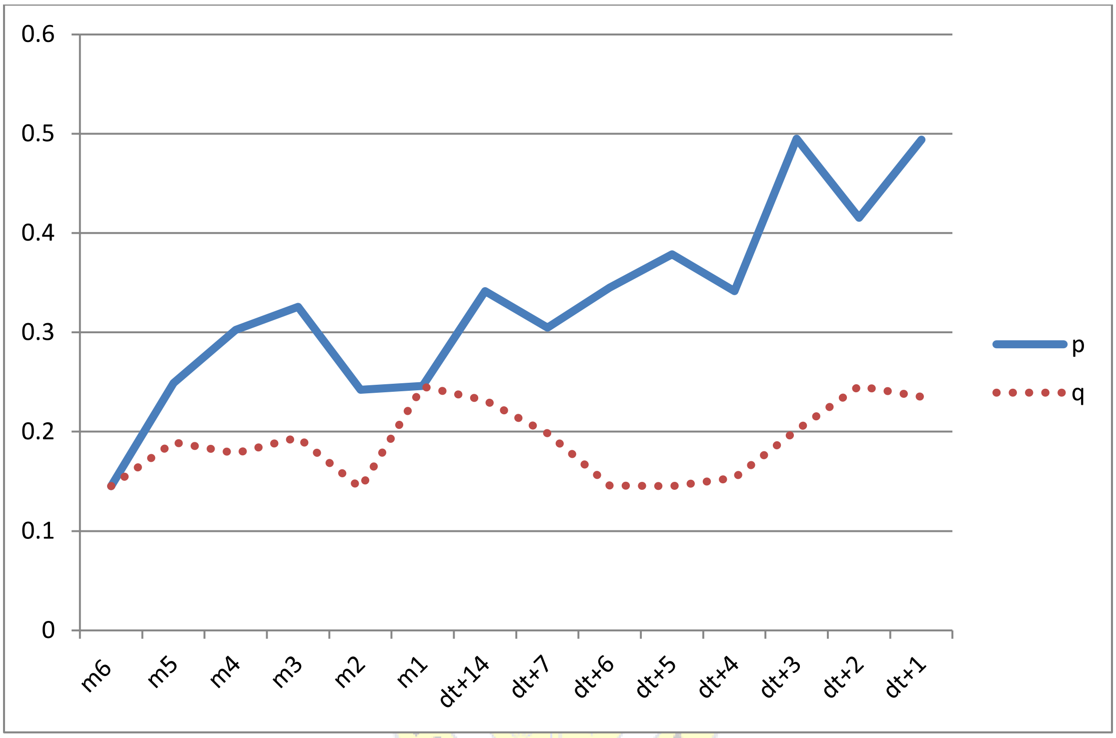
The higher volatility of ticket prices was increased just about 14 days before departure (see figure 4.8). We found a sharp increase in ticket prices during two weeks before departure. The results for the coefficient, under this iterative procedure, we found larger value of coefficients α_i and β_j imply that higher volatility is expected for the variables during last two week before departure.

Table 4.8 Estimation results of GARCH(1,1) in dynamic pricing, Bangkok to Melbourne (operated by Jetstar) departure on Friday, 9th August 2019, 21.25-10.30(D+1)

Explanatory variables	$\sum_{i=1}^{p} \alpha_i \, \alpha_{t-i}^2$	$\sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2$	Standard error $[\epsilon_t] \sim i.i.d$
m_6	0.1453	0.1451	0.0843
m_5	0.2489	0.1894	0.1334
m_4	0.3024	0.1784	0.1743
m_3	0.3258	0.1941	0.1695
m_2	0.2423	0.1431	0.0897
m_1	0.2459	0.2453	0.0787
d_{t+14}	0.3415	0.2315	0.1652
d_{t+7}	0.3046	0.1984	0.1435
d_{t+6}	0.3451	0.1458	0.1143
d_{t+5}	0.3784	0.1453	0.1574
d_{t+4}	0.3415	0.1535	0.1659
d_{t+3}	0.4948	0.2015	0.1658
d_{t+2}	0.4153	0.2457	0.1465
d_{t+1}	0.4940	0.2351	0.1524

Moving Average = 10, Iteration = 10, Log-Likelihood = 413, Wald Chi-Square Test = 31.42

Figure 4.8 Volatility fluctuation in time series by GARCH(1,1) in dynamic pricing, Bangkok to Melbourne (operated by Jetstar) departure on Friday, 9th August 2019, 21.25-10.30(D+1)



By comparing figures 4.7 and 4.8, low volatility was found during five to six months before departure and then rose steadily. The ticket prices' volatility of Jetstar airline had risen just a month before the departure especially a week before travel. However, we cannot compare volatility of ticket prices during the last two weeks between Nok Scoot and Jetstar, because flight tickets to Tokyo were already sold out just a week before travel. This could be explained that flight during holiday/long weekend are more demanded than during normal working day.

Chapter 5

Conclusion and Recommendation

Airline industry has experience explosive growth over the last decade especially in a low cost airline. They are getting more market share by stimulating passenger demand with attractive fares and new routes using their dynamic pricing strategies. Thailand is expected to enter the top ten markets in the near future as the top tourist destination. For dynamic pricing strategy, low cost airliners frequently update their ticket prices to their prospects in order to maximize their profit. They use revenue-generating system to forecast future demand corresponding to departure/arrival rates of different customers' types and remaining capacity, and offer a large pool of ticket classes to price-discriminate. In theory, as time progresses, the ticket prices would increase substantially.

5.1 Summary of Findings

5.1.1 Main Purpose of the Study

The main purposes of the study are to find and compare volatility in ticket prices for domestic and international flights departing Bangkok to various destinations such as Chiang Mai, Phuket, Tokyo and Melbourne as well as to compare relative volatility between domestic and international ticket prices in a particular time frame. Finally, this research is conducted to find any dynamic pricing behaviors in airlines ticketing system during the study period of 6 months.

We construct and test volatility model by using GARCH(1,1) by observing ticket price changes in various flight routes including domestic and international routes. We test volatility by examining the determinants of movements for the volatility of ticket prices in time series with seasonal factors such as normal day and long weekend/holiday effect. Ticket prices have been obtained by four Airline companies' websites such as Nok Air, Thai Air Asia, Nok Scoot and Jetstar.

5.1.2 Data Collection

We collect data of ticket prices in term of time series for both domestic and international flights departing from Bangkok via Nok Air, Thai AirAsia, NokScoot and Jetstar websites. Domestic routes, i.e. Bangkok to Chiang Mai, and Bangkok to Phuket ticket prices has been gathered; and international routes, i.e. Bangkok to Tokyo, and Bangkok to Melbourne, ticket prices are collected from their respective websites. This research compared ticket prices and volatility between normal day and long weekends/holiday.

5.1.3 Period of the Study

All prices with each departing schedule within a day are recorded up to six months in advance during the study period between January 2019 to August 2019 from four airline companies' websites. We also recorded price changes of the same flight when booking at a different day compared to the base day (t) up to 180 days ahead (t_{+180}).

5.1.4 Method of Research Used and Instrument

To construct a volatility model, GARCH(1,1), we uses a model developed by Islam and Watanapalachaikul to detect the sensitivity and volatility level of dynamic pricing. The GARCH type model has led to the development of other related formulations in order to identify and explain the variance of time series. We hypothesize that given the variance of time closer to the departure date; significant volatility in dynamic pricing would be detected.

We use Microsoft Excel NumXL (addins) to calculate the GARCH results. The general process for GARCH model involves three steps. The first is to estimate a best-fitting autoregressive model. The second is to compute autocorrelations of the error term. The third step is to test for significance. Two other widely used approaches to estimating and predicting financial volatility are the classic historical volatility method and the exponentially weighted moving average volatility method.

5.2 Conclusion

Empirical analyses reveal that distributions of the change in ticket prices deviate from normality with volatility varying over time and being highly correlated. The results of the volatility tests show that the ticket prices were quite volatile when purchasing tickets close to departure date.

5.2.1 Domestic Flights

Most of the studied periods, we found that volatility was significant higher when the departure time was less than one month, which the ticket prices started to increase with promotional exclusive, especially a week before departure. The ticket prices gradually increased everyday for the last 7 days prior to departure on normal days.

On normal working day, low volatility was found during *m2 to m6* before departure. In addition, the ticket prices' volatility of Nok Air airline had risen just a month before the departure especially a week before travel. Thai Air Asia ticket prices rose significantly during last two weeks before departure but not as frequent as Nok Air's. Therefore, Thai Air Asia ticket prices were less volatile than Nok Air during the last two weeks before departure. However, low volatility of ticket prices was evident during 4-6 months prior to departure and it was gradually increased during 3 months before departure. This explains that travelers were not really anticipated in planning to travel during long weekend. We also found a sharp increase in ticket prices during a month just before departure.

On the other hand, during holiday/long weekend, high volatility was detected during two to three months before departure, especially flight to Chiang Mai operated by Nok Air. It could be seen that passengers are planning their trip to Chiang Mai early for their holiday/long weekend. We also notice the ticket prices had increased significantly during last week before departure resulting in higher volatility.

5.2.2 International Flights

The results show some fluctuation of volatility in ticket prices over the study period. In addition, low volatility is evident further the departure date. The ticket prices steadily rose overtime and medium volatility in ticket prices was found during three months before the departure. In fact, the volatility of ticket prices was increased noticeably around three months before departure. On normal day, the ticket price volatility was significant during the last two weeks before departure, which resulted from an increase in ticket prices during this period.

For long weekends/ holiday, traveller anticipated in advance booking, which resulted in a sharp increase up in ticket prices overtime just around three to four months before travel date. We also found that the flight between Bangkok and Tokyo during long was highly demanded. On the other hand, the flight between Bangkok and Melbourne are not popular comparing to Tokyo but the result has the same direction where higher volatility is evident when closer to departure date.

5.3 Recommendation and Suggestion for Future Research

In this research, we study the volatility of the dynamic pricing in the low-cost airline industry in Thailand. The modified GARCH model is used to analyze and identify volatility level of the change in ticket prices over the 6 months time. The results clearly show that level of volatility increases as time to departure approaches.

Implications of the research are as follows: 1) Airlines could update their ticket prices constantly to maximize their potential profits; 2) Customers could book flight tickets in advance to avoid confusion and hence save some money.

Instead of running a model on historical data, this research has attempted to use future (pre-booking) ticket prices data. This innovative way of using the modified GARCH model on the future data may cause some doubt regarding to reliability and accuracy of the results. Therefore, similar tests with longer time duration could be conducted to verify the model's validity and reliability. Furthermore, we suggest a usage of historical data when information is available and applicable in comparison to this modified GARCH method. Forecasting ticket prices in a dynamic pricing policy by the use of volatility models could be attempted to provide more accurate prediction of ticket prices to maximize airline's profit at different time horizon.

For future research, we suggest some issues need an in-depth investigation such as the techniques used to study volatility. Different order levels and lag times could be employed to compare these results with current findings. Future studies may also focus on a stochastic process for ticket pricing with economic variables. The use of GARCH models with macroeconomic variables could also be an interesting area to investigate.

The usefulness of assuming a normal distribution and finding alternatives could also be tested. In addition, other Thai and international important holidays such as Songkran Festival, Chrismas and New Year could be included in the further study to find the volalitity of the ticket prices.

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