quite independent of any motion of the source itself. In our first discussion of the Michelson-Morley experiment (in Chapter 2) we stated that this was indeed the case. For a long time it was believed that this was proved by observations on the light from close binary stars. The two members of any such binary system have large relative velocities, and when one star has a component of velocity toward the earth the other will be moving away. It was argued that if these velocities were communicated to the emitted light, the *apparent* motions of the stars would be distorted away from the Newtonian orbits required by the law of gravitation. No such distortions were observed. It has been more recently argued, however, that since these binary star systems are usually surrounded by a gas cloud, which absorbs and then reradiates the light from the stars, the speed of the light that crosses interstellar space may in any case be independent of any possible influence of the original moving sources.[1] Subsequently, however, experiments have been made on rapidly moving terrestrial sources of radiation which verify this aspect of Einstein's second postulate in a convincing way. In one such experiment made with high-energy photons, not visible light, the source consisted of unstable particles (neutral π mesons) traveling at 99.975% of the speed of light. The measured speed of the photons emitted forward with respect to this motion was $(2.9977 \pm 0.0004) \times 10^8$ m/sec.[2] Reference to Table 1–2 will show that this is in excellent agreement with the best values of c obtained for stationary sources. In Chapters 5 and 6 we shall discuss in more detail the radiation from moving sources, in connection with the relativistic law of addition of velocities and related phenomena.

THE RELATIVITY OF SIMULTANEITY

An immediate consequence of Einstein's prescription for synchronizing clocks at different locations is that simultaneity is relative, not absolute. Let us see how this follows.

Suppose that three observation stations A, B, and C are equally spaced along the x axis of an inertial frame S in which they are all at rest. We can construct a simple x-t coordinate system, on which we draw "world lines" (to use the accepted phraseology) showing the development of the system in space

[1]J. G. Fox, *Am. J. Phys.*, **30**, 297 (1962).

[2]T. Alväger, F. J. M. Farley, J. Kjellman and I. Wallin, *Phys. Letters*, **12**, 260 (1964).

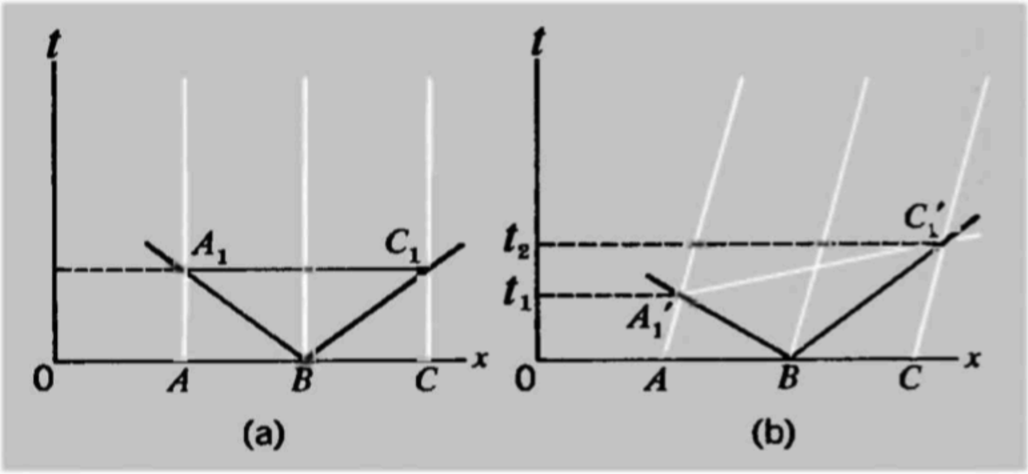

Fig. 3–2 (a) Space-time diagram showing experiment to define simultaneity at stations A and C (at rest in this reference frame) by light signals emitted from a station B midway between them. (b) Equivalent experiment for the case in which A, B, and C all have a velocity with respect to the reference frame.

and time [Fig. 3–2(a)]. The world line of any given particle is just a graph of its position as a function of time; it provides a complete picture of the history of the particle as observed within a given frame of reference. The world lines of A, B, and C are of course just vertical lines parallel to the t axis, corresponding to $x =$ constant. Suppose that a light or radio signal is sent out from B at $t = 0$. It travels at the same speed c forward and backward along the x axis—an assertion that embodies the universality of c. This signal is described by two sloping lines $x = x_B \pm ct$. The arrival of the signal at the positions of A and C is thus given by the intersections A_1, C_1, and simultaneity at the positions of A and C is *defined* by the line A_1C_1, parallel to the x axis, which joins a series of points possessing the same value of t.

But now suppose that A, B, and C are at rest in an inertial frame S' which is moving with respect to S at a speed v along the x direction [Fig. 3–2(b)]. The world lines of A, B, and C are now inclined as shown. A signal sent from B at $t = 0$ is again described (in S) by the lines $x = x_B \pm ct$, and the arrival of the signal at the positions of A and C is now given by the intersections A_1' and C_1'. These are clearly not simultaneous for S, because the line $A_1'C_1'$ is manifestly not parallel to the x axis. Or, to put it more concretely, the signal reaches A before it reaches C because, as observed in S, A is running to meet the signal pulse whereas C is running away from it. But we require

B to be midway between A and C in S' as well as in S. Accepting the universality of c and the equivalence of inertial frames, we therefore *demand* that A_1' and C_1' represent simultaneous events in S'. (An *event*, from the standpoint of relativity theory, is completely characterized by its space and time coordinates in a given frame of reference.) *Our judgment of simultaneity is a function of the particular frame of reference we use.*

It is natural to ask why we should base this definition of simultaneity on the velocity c in particular and not on any other possible signal velocity. The simplest answer to this is to point to the obvious uniqueness of c, not merely as the speed of light, but as the ultimate speed in all of dynamics. A more convincing reply (at least in the long run) is that this choice does indeed have the consequence that every known physical law has the same form in all inertial frames.

In the above discussion we have demonstrated in a qualitative way the relativity of simultaneity. Our next step must be to develop the quantitative aspects of time and space measurements according to special relativity.

THE LORENTZ-EINSTEIN TRANSFORMATIONS

Look now at Fig. 3–3. It depicts the operation of defining simultaneity at stations A and C which are moving at speed v with respect to an inertial frame S. We have already discussed such a diagram (cf. Fig. 3–2). But now we have added lines to represent the coordinate axes of the frame S' in which A and C

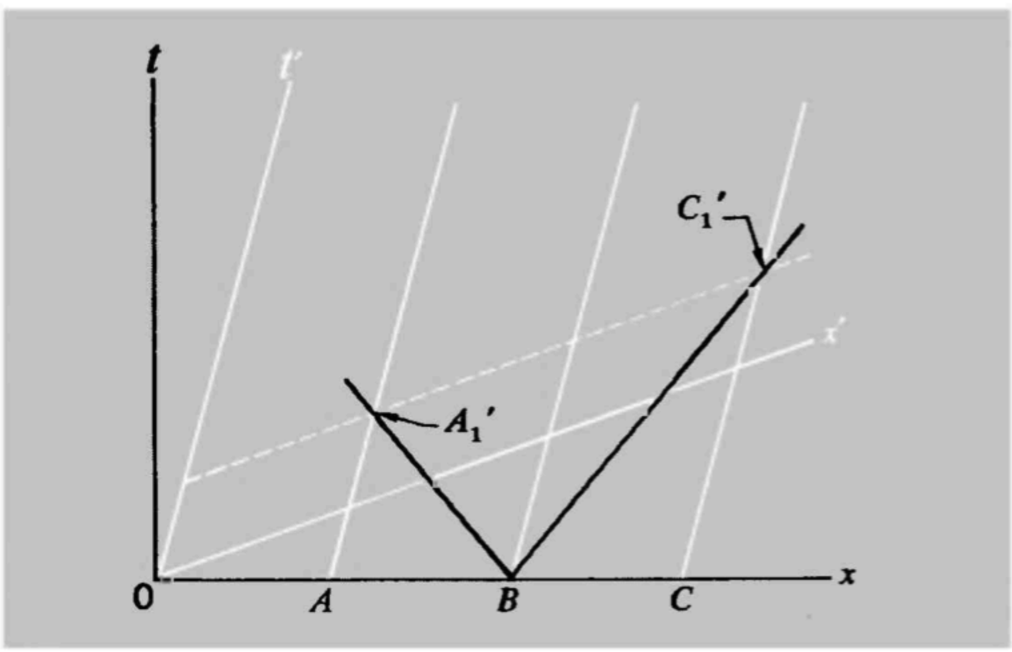

Fig. 3–3 Specification of coordinate axes (x, t) and (x', t') for two reference frames in relative motion.

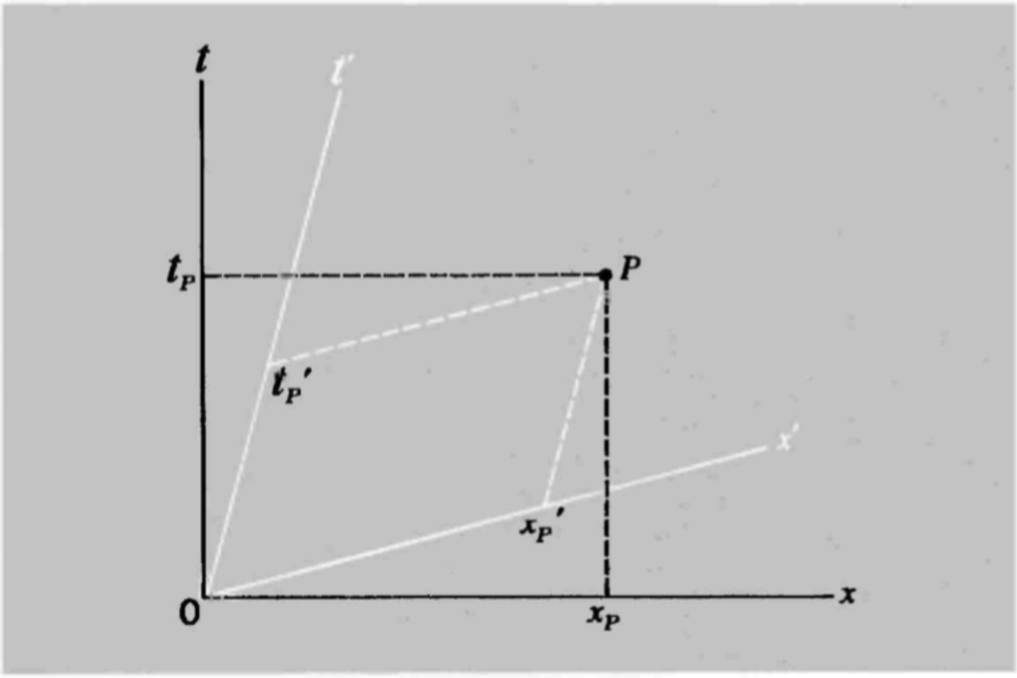

Fig. 3–4 Space-time coordinates of a given point event in two different inertial frames.

are at rest. How have we done this? The axis of t' is readily described; it is the line $x' = 0$, i.e., the world line of the origin of S'. And since the frame S' has a speed v along the x direction with respect to S, the position of this origin is described in S by the equation $x = vt$ if the origins of S and S' coincide at the time $t = 0$.

What about the axis of x'? This is the line that connects all points corresponding to $t' = 0$. Any line of the form t' = constant is parallel to this x' axis. But the line $A_1'C_1'$ is just such a line, since A_1' and C_1' are events by which simultaneity in S' is defined. Hence we construct the x' axis by drawing a line parallel to $A_1'C_1'$, and for convenience we make it pass through O, which is thus described both by $x = 0$, $t = 0$ and by $x' = 0$, $t' = 0$. The noncoincidence of the axes of x and x' does not, of course, imply any geometric tilting of one with respect to the other; it is a purely formal tilting in the abstract space constructed from the x and t coordinates.

Now this type of diagram displays for us a key feature of the kinematic transformations of special relativity. In Fig. 3–4 any point P in the plane of the diagram represents what is called a *point event*, which can be characterized alternatively by the values of x and t or of x' and t'. And our construction implies that x' and t' alike should be linear functions of both x and t. Similarly, x and t are linear functions of x' and t'. This linearity is a fundamental property of the transformation equations. If they did not have this form, a motion recorded as motion at constant velocity along a straight line in one frame (say S) would not be recorded as uniform rectilinear motion in S'. This would therefore conflict with Galileo's law of inertia and with our basic dynamical condition that all inertial frames are equivalent.

The symmetry implied by the relativity principle means that the form of the relationships must be as follows:

$$x = ax' + bt'$$

with (3–8)

$$x' = ax - bt$$

These are set up so as to resemble as closely as possible the Galilean transformation [equations (3–4)] to which they must certainly reduce for sufficiently small values of the speed v of S' relative to S. The motion of the origin of S as measured in S' is defined by putting $x = 0$ in the first of these equations. Similarly, the motion of the origin of S' as measured in S is defined by putting $x' = 0$ in the second equation. The velocities are equal and opposite and both of magnitude v. This gives us the condition

$$b/a = v \qquad (3\text{–}9)$$

Next we consider the descriptions according to S and S' of a light signal traveling in the positive x direction. Let the signal originate at O of Fig. 3–3. It is then described by the following very simple equations in S and S', respectively:

$$x = ct \qquad x' = ct' \qquad (3\text{–}10)$$

Substitute these particular expressions for x and x' in equations (3–8), and we get the following:

$$\begin{aligned} ct &= (ac + b)t' \\ ct' &= (ac - b)t \end{aligned} \qquad (3\text{–}11)$$

Eliminating t and t' between these last equations, and using the condition $b = av$ from Eq. (3–9), we find

$$c^2 = a^2(c^2 - v^2)$$

Therefore,

$$a = \frac{1}{(1 - v^2/c^2)^{1/2}} \qquad (3\text{–}12)$$

It may be noted that this coefficient, a, is precisely the factor $\gamma(v)$ that emerged in our dynamical analysis in Chapter 1—cf. Eq. (1–22). We can now rewrite equations (3–8) in the following