CSL-471 : Probability and Computing
Test-2
November 9, 2016

Maximum Time Allowed: 50 minutes
Maximum Marks: 50

Name: _____

Entry Number: _____

1. (10 points) Prove that the Farthest in Future Page replacement policy is an optimal caching policy.

2. (20 points) Let $X_1, X_2, X_3, ..., X_n$ be independent Poisson trials with $Pr(X_i = 1) = p$. Then if X is the sum of X_i and if μ is $E[X]$, for any $\delta \in (0, 1]$:
$Pr(X < (1 - \delta)\mu) < (\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}})^{\mu}$
Also prove that the above bound can be simplified to a smaller weaker bound as:
$Pr(X < (1 - \delta)\mu) < exp(-\mu\delta^2/2)$

3. (10 points) Explain load balancing problem with its analysis.

4. (10 points) Consider the branching process for a contagion. Given that the contact network exists in the form of a tree and every node except the leaves have 4 children each. Moreover, there are 11 levels in the tree from level 0 to level 10. Every level except the last do not have any leaf node. The probability of disease transmission across every link is 0.5. Answer the following questions.

   1. Given a lady Elsa, standing at level 5 in this tree, what is the probability that the infection reaches level 10 by passing through Elsa?

   2. Is the disease going to be an epidemic? If yes, how can you alter the given parameters such that the contagion can not convert into an epidemic?