(1) $A(x) = \cos(2013\pi - x) + \sin\left(\frac{314\pi}{6}\right) + \cos(1439\pi + x) - \sin\left(\frac{960\pi}{4} - x\right)$

$A(x) = \cos[1009(2\pi) - x] + \sin\left[26(2\pi) + \frac{\pi}{3}\right] + \cos[719(2\pi) + \pi - x] - \sin[120(2\pi) - x]$

$A(x) = \cos(-x) + \sin\left(\frac{\pi}{3}\right) + \cos(\pi - x) - \sin(-x)$

$= \cancel{\cos(x)} + \frac{\sqrt{3}}{2} - \cancel{\cos x} - (-\sin x)$

$\boxed{A(x) = \frac{\sqrt{3}}{2} + \sin x}$

, $\boxed{\sin(-x) = -\sin x}$ $\boxed{\cos(-x) = \cos x}$ ع

$\boxed{\cos(\pi - x) = -\cos x}$

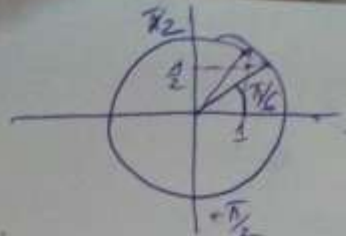
(2) $A\left(-\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} + \sin\left(-\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$

$A\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} + \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

(3) $\begin{cases} A(x) = \frac{\sqrt{3}+1}{2} \\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ ح} \end{cases} \Leftrightarrow \begin{cases} \sin(x) + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}+1}{2} \\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ ح} \end{cases}$

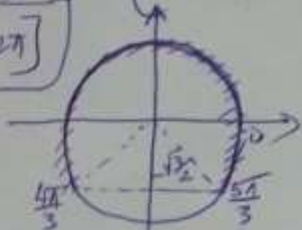
$\Leftrightarrow \begin{cases} \sin(x) = \frac{1}{2} \\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ ح} \end{cases}$

$\boxed{x = \frac{\pi}{6}}$ حل



(4)
$\begin{cases} A(x) > 0 \\ x \in [0, 2\pi] \text{ ح} \end{cases} \Leftrightarrow \begin{cases} \sin(x) + \frac{\sqrt{3}}{2} > 0 \\ x \in [0, 2\pi] \text{ ح} \end{cases} \Leftrightarrow \begin{cases} \sin(x) > -\frac{\sqrt{3}}{2} \\ x \in [0, 2\pi] \text{ ح} \end{cases}$

$\Leftrightarrow \boxed{x \in \left[0, \frac{4\pi}{3}\right[\cup \left]\frac{5\pi}{3}, 2\pi\right]}$



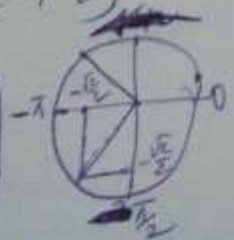
(5)
$\begin{cases} A(x) = \frac{\sqrt{3} - \sqrt{2}}{2} \\ x \in \left[-\pi, -\frac{\pi}{2}\right] \text{ ح} \end{cases} \Leftrightarrow \begin{cases} \frac{\sqrt{3}}{2} + \sin x = \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \\ x \in \left[-\pi, -\frac{\pi}{2}\right] \end{cases}$

$\Leftrightarrow \begin{cases} \sin x = -\frac{\sqrt{2}}{2} \\ x \in \left[-\pi, -\frac{\pi}{2}\right] \end{cases} \Leftrightarrow \boxed{x = \frac{5\pi}{4}}$

$\Leftrightarrow \cos x = -\frac{\sqrt{2}}{2}$

$\Leftrightarrow \operatorname{tg} x = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$

$x = \frac{5\pi}{4}$ ح

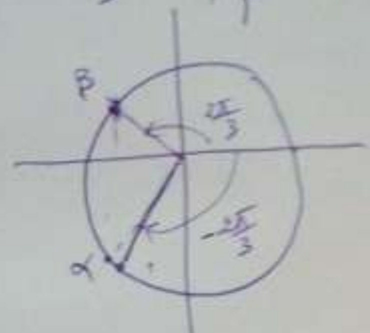


(1)

$\alpha = -\frac{14\pi}{3}$, $\beta = \frac{14\pi}{3}$

$\alpha = \frac{-12\pi - 2\pi}{3}$, $\beta = \frac{12\pi + 2\pi}{3}$

$\alpha = -4\pi - \frac{2\pi}{3}$, $\beta = 4\pi + \frac{2\pi}{3}$



(2)

$\sin\left(-\frac{14\pi}{3}\right) = \sin\left(-4\pi - \frac{2\pi}{3}\right) = \sin\left(-\frac{2\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

$\cos\left(\frac{14\pi}{3}\right) = \cos\left(4\pi + \frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$

(3)

لدينا $\cos x = \frac{1}{5}$ و $x \in [0, \pi]$

لدينا $\cos^2 x + \sin^2 x = 1$ ومنه

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - \left(\frac{1}{5}\right)^2$

$\sin^2 x = \frac{24}{25}$

$\Rightarrow \sin x = \frac{\sqrt{24}}{5}$ أو $\sin x = -\frac{\sqrt{24}}{5}$

بما أن $x \in [0, \pi]$ فإن $\sin x > 0$

إذن $\boxed{\sin x = \frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}}$

(4)

لدينا $\cos x = \sqrt{3} \sin x$ حيث $x \in \left[\pi, \frac{3\pi}{2}\right]$

لدينا $\cos x = \sqrt{3} \sin x$ إذن

$\sin^2 x + \cos^2 x = \sin^2 x + (\sqrt{3} \sin x)^2$

$= \sin^2 x + 3 \sin^2 x$

$\boxed{\sin^2 x + \cos^2 x = 4 \sin^2 x}$

من جهة أخرى لدينا $\boxed{\sin^2 x + \cos^2 x = 1}$

إذن $\sin^2 x + \cos^2 x = 4 \sin^2 x \iff 1 = 4 \sin^2 x$

$\iff \sin^2 x = \frac{1}{4}$

$\iff \sin x = \frac{1}{2}$ أو $\sin x = -\frac{1}{2}$

لدينا $x \in \left[\pi, \frac{3\pi}{2}\right]$ إذن $\sin x < 0$

إذن $\boxed{\sin x = \frac{1}{2}}$

لدينا $\sin^2 x + \cos^2 x = 1$ ومنه

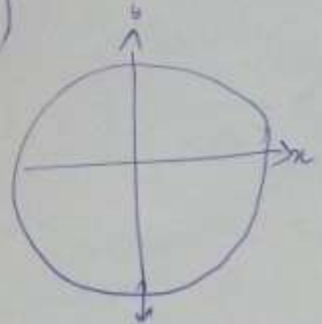
$f(x+\pi) = \sin(2x+2\pi)+2$

$f(x+\pi) = \sin(2x)+2$

$\boxed{f(x+\pi) = f(x)}$ و

منه f دالة دورية دورها $T=\pi$

$\boxed{f'(x) = 2\cos(2x)}$



لما $x\in[0,\frac{\pi}{4}]$ فإن $2x\in[0,\frac{\pi}{2}]$ منه

منه $\boxed{\cos(2x)\geqslant 0}$

لما $x\in[0,\frac{\pi}{4}]$ فإن $f'(x)\geqslant 0$

بالتالي لما $x\in[0,\frac{\pi}{4}]$ فإن f متزايدة تماما

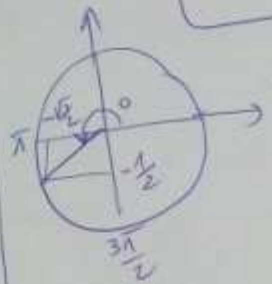
$\cos^2 x = 1-\sin^2 x = 1-\left(\frac{1}{2}\right)^2$

$\boxed{\cos^2 x = \frac{3}{4}} \Rightarrow$

$\cos x = \frac{\sqrt{3}}{2}$ أو $\cos x = -\frac{\sqrt{3}}{2}$

بما أن $x\in[\pi,\frac{3\pi}{2}]$ فإن $\cos x<0$

منه $\boxed{\cos x = -\frac{\sqrt{3}}{2}}$



لدينا

$\begin{cases} \sin x = -\frac{1}{2} \\ \text{و} \\ \cos x = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \boxed{x = \frac{7\pi}{6}}$

$\boxed{x = \frac{7\pi}{6}}$

التمرين 3

$f(x) = \sin(2x)+2$

$D_f = \mathbb{R}$

لدينا $\boxed{\forall x\in\mathbb{R} \text{ فإن } x+\pi\in\mathbb{R}}$

و $f(x+\pi) = \sin(2(x+\pi))+2$

لما $x\in[\frac{\pi}{4},\frac{\pi}{2}]$ فإن $2x\in[\frac{\pi}{2},\pi]$

بالتالي $\cos(2x)\leq 0$ منه لما $x\in[\frac{\pi}{4},\frac{\pi}{2}]$

فإن $f'(x)\leq 0$ أي f متناقصة تماما

لما $x\in[\frac{\pi}{4},\frac{\pi}{2}]$

x	0		$\pi/4$		$\pi/2$
f'		$+$	0	$-$	
f	2	↗	3	↘	2