Lecture : 41

Hidden Markov Model

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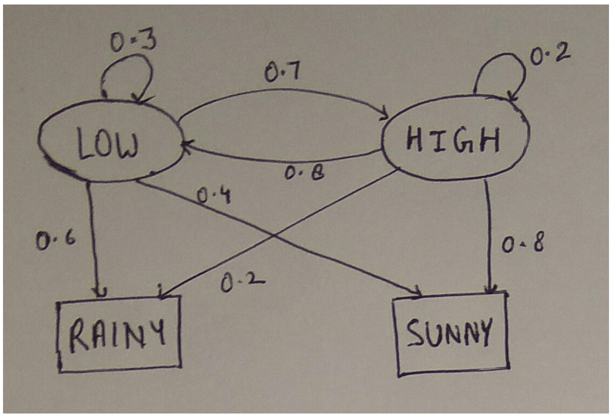
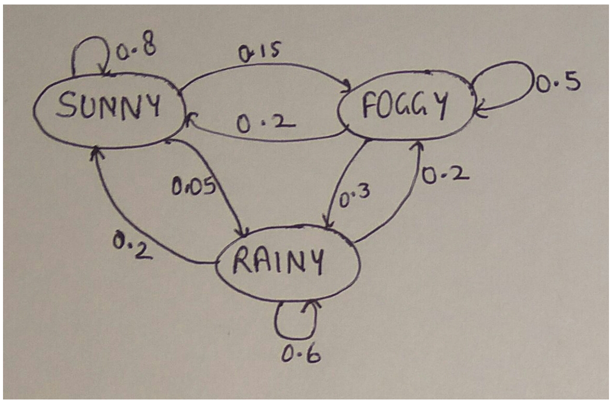
Hidden Markov Model is a statistical probabilistic Markov model in which the system is assumed to be a Markov process with hidden states. HMM has states transition with the considered observations. We will discuss few examples next.

In the next figure we have a simple HMM that has two states that the temperature of the atmosphere is high or low. The considered observations are that the day will be sunny or rainy.



In the next figure we explain one more example. It has three states that whether the day is sunny, foggy or rainy. And the observations are that the person will bring umbrella or not. The probabilities of the observation is explained in the table.



OBSERVATIONS	Bring Umbrella	Not Bring Umbrella
SUNNY	0·1	0·9
FOGGY	0·3	0·7
RAINY	0·8	0·2

In HMM output only depends on the current state not on the history of the path taken by the system. In the above example, if we want to compute the probability of a person bringing umbrella ($u_i = T$, where T stand for True) given that the current state is rainy and the past state was sunny, we can do it as follows:
$P(s_2 = rainy|s_1 = sunny, u_2 = T)$

$\frac{P(s_2=rainy,s_1=sunny|u_2=T)}{P(s_1=sunny|u_2=T)}$

Here s_1 and u_2 are independent variables, So

$\frac{P(u_2=T|s_2=rainy,s_1=sunny)\cdot P(s_2=rainy,s_1=sunny)}{P(s_1=sunny)}$

Now as we know from conditional probability: $\frac{P(s_2=rainy,s_1=sunny)}{P(s_1=sunny)} = P(s_2 = rainy|s_1 = sunny)$. Using this:

$\frac{P(u_2=T|s_2=rainy,s_1=sunny)\cdot P(s_2=rainy,s_1=sunny)}{P(s_1=sunny)} = 0.8 * 0.05 = 0.04$

1 Formal Definition of HMM

In HMM models states are labeled $S_1, S_2, ..., S_N$, where N is the total number of states. Let's assume that for a particular trial there are T number of observations, so the sequence of observations can be written as $O_1O_2...O_T$. "T" also shows the number of states passed through to reach the final destination. The states of the path are denoted as $Q = q_1q_2...q_T$. The specification of HMM is denoted as: $\lambda = \langle N, M, \{\pi_i\} \{a_{ij}\} \{b_i(j))\}\rangle$.

Instructor also discussed that the dynamic programming can be used to find the optimal solution of HMM till time t. For this we can final the solution till time $t-1$, that can be used further to get the solution till time t.

2 Applications

Hidden markov model has various applications. We will discuss few of them to motivate the readers:

1. Speech Processing: Speech processing has various complexities that can be handled using HMM for example: a person can pronounce same word differently at different times, different people will pronounce same word differently, or different work can be pronounced same by a person like if a person is speaking "wait" then it can denote to "wait" or "weight" based on the context. In speech processing markov modeling, text words are the states and we analyze speech components using HMM.
2. Speech synthesis
3. Document Separation
4. Machine translation
5. Time Series Analysis
6. Activity recognition
7. cryptanalysis

3 Further Readings

Interested readers can explore following topics for more details:

- Difference between HMM and MM
- Two-level Bayesian HMM

- Mixture Model
- Poisson hidden Markov model

4 Bibliography

Following links can be explored for the explanation of notations, if it is not clear.

1. *https://en.wikipedia.org/wiki/Hidden_Markov_model#Applications*