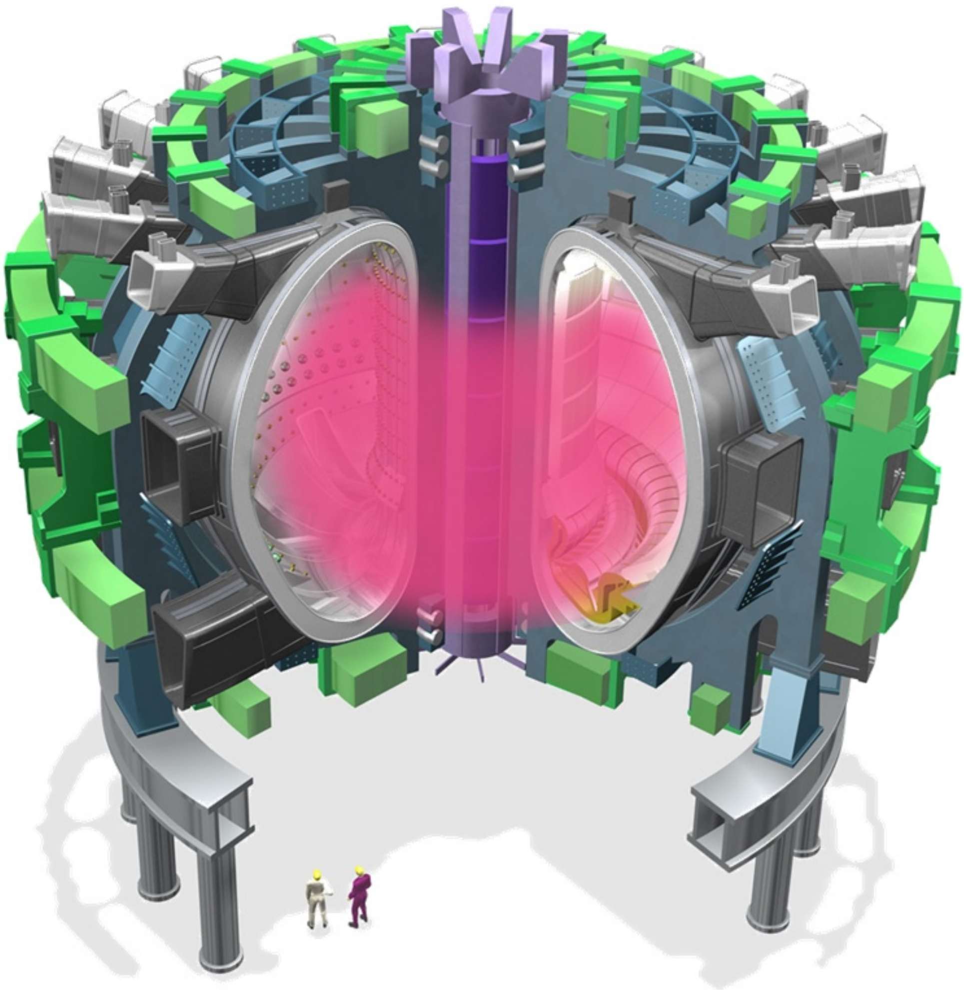
Fusion on the back of an envelope

TU/e 3MF100

2021 - 2022



Niek Lopes Cardozo

About this course; its place in the MSc Fusion curriculum.

Eindhoven University of Technology has developed a full, 2-year Master curriculum on Nuclear Fusion, which went live in September 2012. Please refer to the website to get a full overview of the compulsory and elective courses that can be taken in the Master's Science and Technology of Nuclear Fusion. The present course, 3MF100 'Fusion on the back of an envelope' serves as an introduction and overview. It touches on all the different subjects that are dealt with in more depth in the specialisation modules. Therefore, it is mandatory for students who are enrolled in the Fusion Master programme. But the course is also suited as an introduction to nuclear fusion for a broad audience and is taken by many students from other Master programmes.

This course can be taken by students coming from different Bachelor programmes. It uses elementary math and does not require specific knowledge of plasma physics or so. We do make elementary use of vector calculus, differentials and integrals. It helps if you are experienced with those, but the course is set up in such a way that the principles can be followed also for those with limited mathematical training.

The approach we take is, as the name of the course says, 'back-of-the-envelope physics'. By asking the right questions, using common sense, applying generic physics principles, using dimensionless numbers wherever we can and rounded-off numbers where we need them, and making approximations to derive some basic relations, we find that quite a good understanding can be achieved without having to resort to any advanced mathematical treatment. As the name says, you will not even need a calculator. Try it, it's fun and very instructive!

It is essential that you do make the exercises that are part of this course. A lot of the actual educational material is in fact presented through the exercises. You will do a number of them in the class room, as part of the lectures. But you are also expected to work on the more difficult ones at home. They are part of the examination material; quite often variations on a few of those exercises will be used in the exam!

Finally, the quest for fusion energy brings together a lot of different disciplines. To understand and appreciate the difficulties of fusion and ways in which these are tackled, you need to acquaint yourself with a rather wide variety of topics and be able to connect them. It may seem at first that you have to learn disjunct bits of science and technology, but the aim of this course is to make you see the big picture. In fact, a project like the design and construction of a fusion reactor is a perfect demonstration of the need for a truly interdisciplinary approach in science and technology.
Enjoy!

(cover picture: the international fusion reactor project ITER, presently under construction in Cadarache, France. Source: ITER)

Preface

Nuclear fusion is a reaction in which two light atomic nuclei merge, with a small net loss of mass and a corresponding release of energy. A well-known example is the fusion reaction of the hydrogen isotopes Deuterium (D) and Tritium (T). The reaction product is Helium (He) and a free neutron. This reaction releases a tremendous amount of energy: about 17.6 MeV, while the fuel is light and compact. Compare this to a chemical reaction such as burning oil: there an oxidation reaction typically releases 1 eV, where the mass of the reactants is an order of magnitude larger than in the fusion reaction. As a result, the 'fusion fire' releases about a billion times more energy per gram than fossil fuels! In practical terms: a full-scale fusion power plant would use only 1 kg of fuel per day. Or, a glass of ordinary water contains enough deuterium to power a household for months. Nuclear fusion powers the universe - can we make it power mankind?

There are two good reasons to go into fusion research. First, it is fun! Exciting physics combined with extreme challenges in technology. A field in which you can work alongside the best and brightest. Second, there is a fair probability that fusion will be indispensable if we want to achieve a sustainable energy economy on the planet. There are but few truly sustainable energy sources that can develop into a large scale energy supply, and it is not at all clear that the sum of those will be sufficient. Therefore all options, including fusion, must be developed in highest gear.

The first reason provides a personal motivation. This is the reason why brilliant young researchers seek a career in fusion research. The second reason can of course also contribute to the personal motivation. But very importantly: this provides the motivation for governments to finance fusion research, in a worldwide collaboration.

In these lectures both aspects are discussed. The physics - hot, ionised gases (plasmas), trajectories of charged particles in curved magnetic fields, methods to accurately measure the temperature of a plasma at 100 million K, tricks to deduce the heat conduction of such a medium, experimental ways to tweak and control turbulence in hot plasmas; and technology - materials that can withstand the heat load, neutron-hard steels, superconducting magnets, the fuel cycle...etc. But next to the physics and technology also the societal and political aspects are considered. The pursuit of energy is at the basis of the big political themes: war and peace, wealth and poverty, health (pollution; and, e.g. mining of coal is very dangerous and unhealthy), and of course: climate change, the one problem that rules them all! Security of energy supply is among the main worries of governments, these days. But behind all that are the questions: How much energy do we use, how much do we need, how will this need develop? And, how much can we expect from the various sustainable energy sources. How fast can we develop this potential - and why not faster: what are the economical and physical impediments? And finally: will we be in time to avert global disaster?

Contents

1 Literature and links

The material for this course is collected on Canvas. The texts marked 'lecture notes' define the material that is tested at the examination. Note: the exercises are an integral part of the lecture notes and may contain material that is used in the examination. The material marked as 'background reading' is useful but not required.

Websites

The following websites—and links given there—are a good start in an internet search. You are of course welcome to Google your way to more information on fusion.

www.fusenet.eu
www.fusionforenergy.europa.eu
www.fusie-energie.nl (in Dutch)
www.euro-fusion.org
www.iter.org
www.ccfe.ac.uk
fusioned.gat.com

Or let yourself be inspired by a TED talk on fusion energy. Why not tune in to Steve Cowley or Dennis White:
www.ted.com/talks/steven_cowley_fusion_is_energy_s_future?language=uk
http://video.mit.edu/watch/tedxbeaconstreet-bringing-a-star-to-earth-for-energydennis-whyte-13994/

Or be infotained by the following comic produced at the fusion lab in Princeton: http://phdcomics.com/comics/archive.php?comicid=1716

Books

The following book provides a good general background for this course. Highly recommended:

- McCracken & Stott, **Fusion, the Energy of the Universe**, Elsevier
  Still a few for sale at secretariat Fusion (FLUX 5.105), at strongly reduced price (35 Euro).

A very comprehensive - if somewhat advanced for this course - textbook, which is available as a free download pdf, is:

- Kikuchi, Lackner and Tran (editors), **Fusion Physics**; IAEA 2012; www-pub.iaea.org/books/IAEABooks/8879/Fusion-Physics

An excellent, very accessible textbook, which takes as 'back-of-the-envelope' approach but is rigorous at the same time, and which is also used as compulsory course material in the course 3MF110 Magnetic confinement and MHD of fusion plasmas:

- Freidberg, **Plasma Physics and Fusion Energy**; Cambridge University Press

A good introduction into the plasma physics:

- Chen, **Introduction to Plasma Physics**, Plenum, second edition

A compact and comprehensive text on tokamaks (very expensive)

- Wesson, **Tokamaks**, Clarendon Press, any edition

Of all the books on renewable energy, my favourite is:

- David J.C. MacKay, **Sustainable Energy ... without the hot air**.
  Available for free on the web, but consider ordering the real thing.

And another favourite, which is available in both Dutch and English:

- Jo Hermans, **Energy Survival Guide: Insight and Outlook**.

2 The energy problem and fusion

2.1 The energy problem: WHY?

These days it is hardly necessary to argue the fact that the world has an energy problem, which is already hurting us today and which could develop into a global disaster if we don't do something about it. Energy is high on the political agenda. In 2015, president Obama has announced drastic measures to reduce CO_2 emissions and achieve a transition to sustainable energy, the G7 has declared that the world will need a fully sustainable energy system by the end of the century - which is a huge challenge - and even the Pope has taken a position in this matter. In December 2015 the world agreed, in the Paris agreement, to limit global warming to 1.5 C, which calls for fast and drastic reductions of CO_2 emissions. Still, a very brief, bullet-wise introduction.

1. The world population grows and each of us - world average - is increasing his energy use per capita (Figures 2.1, 2.2). To provide a basic standard of living to 10 Billion people we have to triple or quadruple the power generation. But this must be done in a sustainable fashion, whereas today the energy generation is based on fossil fuel (for 80%).

2. Fossil fuel will run out eventually. For oil and gas the end of cheap production is already coming near. Moreover, long before coal is depleted, the greenhouse gas emissions will make the use of fossil fuel unacceptable.

3. Moreover, fossil fuels, in particular oil and gas, are concentrated in some areas in the world, leading to political tensions.

2.2 Energy problem: WHEN? (a note on the time scale involved in the transition to sustainable energy)

The past decade, years even, have seen a rapid growth of the awareness that climate change is real, that it is happening and having a great impact on the lives of people already today, that it is due to human activity, and that it is the highest time to start acting. Climate change today is seen to lead to extreme weather: floods, unprecedented wildfires, prolonged periods of draught, devastating hurricanes. We are seeing the rapid decline of the ice caps on the poles and glaciers. All of this had been predicted correctly - and very visibly - at least 25 years ago - albeit that at the time the uncertainty margins were much larger than they are now. The latest IPCC-report, of summer 2021, reconfirmed in no uncertain terms these predictions. If anything, climate change is happening faster than predicted and the urgency to act larger than ever. After the Paris agreement, there is a reinforced sense of urgency. That means that we'll have to make the transition to sustainable energy before 2050, an unimaginable task. The gap between energy demand and sustainable supply is opening up right now, and the world will have to find ways to essentially reduce the CO_2 emissions to zero within 30 years. Even assuming that we succeed in that task, at that time the demand is still rising fast (the 'developing' economies are expecting to still be growing fast at that time). That means that also after 2050 an equally challenging task remains, namely to enable the growing energy demand with sustainable means. And at that

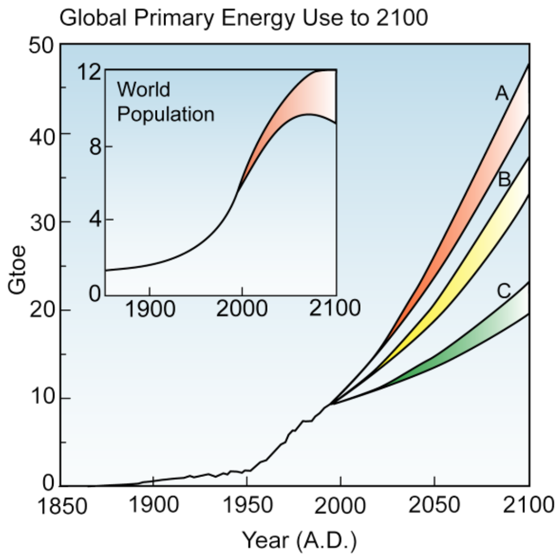


Figure 2.1: Expected growth of the population (inset) and energy demand (expressed in Gtoe, toe = tonnes of oil equivalent) for different scenarios (A,B,C) for the economic growth. Global energy use is expected to double or even quadruple in the coming century as a result of the growth of the world population combined with the growing standard of living. (source: EFDA)

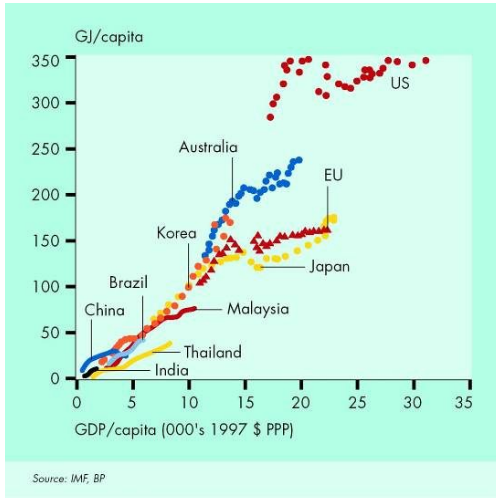


Figure 2.2: The relation between gross domestic product per capita and energy consumption. The strong correlation shows that a country must generate 1 kWh to earn 0.5 \$. This is the unstoppable economical drive behind the growth of mondial energy demand (source: IMF, BP).

time all the proverbial 'low-hanging fruit' will most definitively have been plucked. Therefore, the development of new energy sources, also the new technologies that still need to scale up in 2050, will be essential to meet this extended challenge. (Figure 2.3)
Figure 2.3 shows

1. The fast growth of CO_2 emission in the past 50 years
2. The expected growth of energy demand (i.e. corresponding CO_2 emission if this demand is met using fossil fuels)
3. The allowed CO_2 emission corresponding to the Paris agreement.

The line is practically model independent: we know the starting point and the slope at that point (basically the power plants ordered and planned), we know the end point (the curve has to go to a low level, as the earth absorbs CO_2 only slowly) and we know the surface under the curve: this is the integrated CO_2 emission and this determines the resulting CO_2 concentration in the atmosphere. Thus, with the assumption that the curve is smooth (sharp edges hurt a lot, economically) there is very little variation possible.
The objective of the Kyoto protocol was to keep the world on the line until 2012. The Kyoto protocol was a political milestone that has set a lot - such as the carbon emission trading system - in motion. Yet, in terms of reduction of the CO_2-emission until 2012 it meant very little. The Paris agreement, however, calls for very drastic reductions of CO_2 emissions by mid-century, which implies forceful transitionary measures to be implemented as soon as possible. This could hurt economically - at least that is the view of some politicians (Under president Trump the USA abandoned the Paris agreement, president Biden let the USA return). There

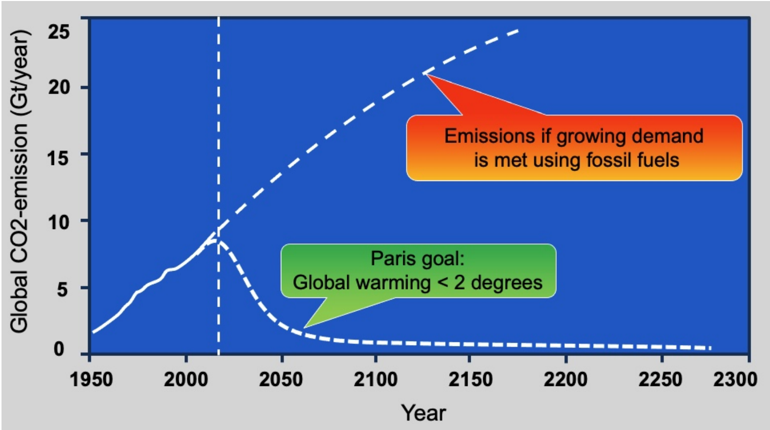


Figure 2.3: Taking the global CO_2 emission as leading parameter, this graphs shows how the emission will develop if no measures are taken, and indicates the reduction of emissions needed to limit the CO_2 concentration in the atmosphere to about 450 ppm. The present CO_2 concentration is around 400 ppm, 450 ppm corresponds to the goals of the Paris agreement. While this implies that drastic changes have to be implemented on the shortest thinkable time scale, note that (due to demographic development) the gap between the two line continues to widen. This will call for the continued expansion of renewable energy, and new energy sources such a fusion will be dearly needed.

are others who point out that the transition to renewable energy could be a driver of the economy. Whoever is right in the short term, it is inevitable that the world economy transitions to a sustainable form. But whether that transition will be smooth, or will fundamentally reform society as we know it, is impossible to say.

So, while the Paris agreement demands that the world CO_2 emission will be substantially reduced by 2050, it is also likely that after that date there still is a very large challenge left. If only because the energy system will keep evolving, and the demand for energy will keep growing. Any new energy technology that we hope to employ then must be researched and developed now. This holds true for nuclear fusion, but equally for other modern developments, such as high efficiency solar panels based on new materials, or large scale conversion technology that is needed to convert electricity to molecules - a form of energy that can be stored and transported.

There is at least 50 years between the R&D of a sustainable energy option and its eventual, large-scale deployment. Failing to develop new sources now - while the oil is still flowing - will deprive future generation from the possibility to solve the problem we have saddled them with. This is irresponsible, unethical, and unacceptable. The present generation will be held accountable.

Therefore we must research and develop all the clean energy sources with a reasonable prospect of large scale energy generation. There are only a few options that can contribute on a global scale: the sun (photovoltaic, solar thermal, and solar fuels), nuclear fission and nuclear fusion. Of these, nuclear fusion is the only of which the physics principle is well understood but the practical application never realised. This course is about nuclear fusion.

2.3 The energy problem: WHAT DO WE DO ABOUT IT?

In view of the arguments above, we'll have to switch from a fossil fuel based energy supply, to a sustainable, CO_2-free energy infrastructure. Presently available options are: renewables and nuclear fission. Fusion, although based on a nuclear reaction, is more like renewables in the sense that the fuel is inexhaustible, the process is clean and safe and produces no long-lived nuclear waste. But the technology and materials used are in many respects more similar to fission.

Fission is still a much-debated form of energy generation. In some countries the social acceptance is high (e.g. France) and a large fraction of the electricity is generated by fission. In other countries the social acceptance is much lower (e.g. Germany, Italy, the Netherlands (roughly 50%)). Note that there is no social protest in the Netherlands against the fact that a significant fraction of the 'Dutch' electricity originates from French nuclear power plants. The recent events with the Fukushima reactors have again confronted the world with the severity of accidents with nuclear plants, and the public acceptance has gone down since then. Germany has decided to bow out of nuclear fission altogether.

Note also that in the free energy market, modern, clean natural gas power plants are turned off in favour of cheaper imported electricity from environmentally bad and high- CO_2 power plants run on coal or lignite.
With the present awareness of the need to reduce CO_2 emissions, the debate about nuclear energy is reopened, also in the Netherlands. At the same time, the down sides of nuclear energy are on display again: international tension about nuclear programmes in Iran, N-Korea ··· Again, the scale of the energy generation is at the root of the problem. Presently some 430 fission plants provide about 5% of the world energy demand. Worldwide 5–10 new fission plants are built per year. That is not even enough for replacement - with an economical lifetime of 50 years 10 per year are needed for that. If we wanted to supply the increment of the energy demand by fission alone, that would require the construction of 1 fission plant per day!

Renewables wind, hydro, geothermal, solar thermal, solar PV, biomass, wave and tidal energy, ··· Figure 2.4 shows the breakdown of energy generation. If you want to find out more about these numbers: try `www.iea.org` (International Energy Agency). It is a sobering statistic, provided by the IEA, showing that there has been only little change in the energy mix since 1973, in particular in the share of renewable energy.

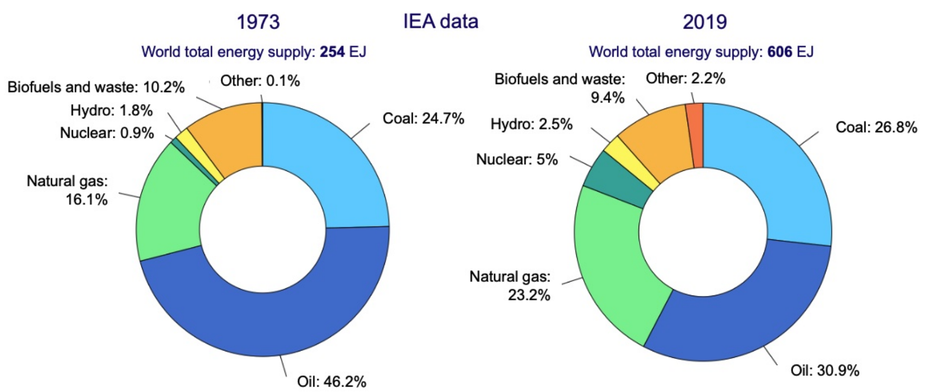


Figure 2.4: The breakdown of the world primary energy shows the dominance of fossil fuel, and the long way the modern renewables still have to go. (Source: IEA website)

The breakdown of the renewable contribution for the USA is given in Figure 2.5. A few comments.

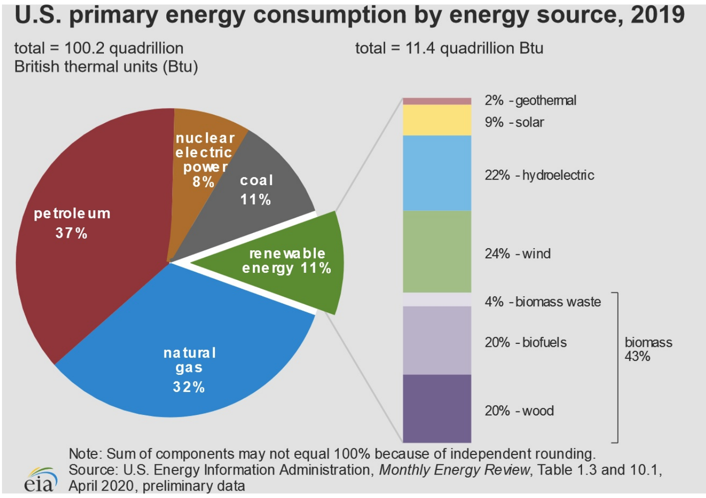


Figure 2.5: The breakdown of the US energy use shows that of the 11% contribution of renewable energy, biomass and hydropower account for 65%. The modern renewables wind and solar still only contribute about 2.4% and 1% to the total energy consumption. (Source: EIA)

First, note that hydropower will not grow much because it has been exploited in most places where this is possible already. Biomass is relatively large. Globally it amounts to about 10% of the energy production, but out of the 10%, 7% is ‘traditional’ energy: people in rural areas using wood fire for cooking etc. This is inefficient and polluting, and causes severe health problems, but it is their only access to energy at present. But certainly we don’t want these 7% to grow. The 3% modern biomass is 1st generation - and it is by now clear that this is not good either: the CO_2 emission reduction is debatable and in many cases the biofuel production replaces food production and leads to massive destruction of tropical rain forest (net effect on CO_2 is negative). Second-generation biomass, in which the entire plant is used, holds a large promise but is in early stage of development. Geothermal can only be applied in specific places - and there it works fine! - and is not expected to become a great fraction of the world energy supply. Yet, we’ll need everything, however modest.

Wind and solar are good options which together could supply up to tens of percents of the world energy demand, and their development should be forcefully pursued. But despite their rapid growth, they are still small and it will take decades for them to really make an impact.

Why does this take so long? The fundamental problem is that energy infrastructure typically has a long life time, of the order of 50 years. Therefore, an industry that builds this infrastructure will stop growing when it reaches the level that is needed to support the final installed power level. That is when the production capacity matches 2% of that final installed power. The last 50 years of the growth of a new source is therefore linear, and takes 50 years. This is borne out by historic data (see ’Kramer and Haigh, Nature 462, 3 December 2009: `http://www.nature.com/nature/journal/v462/n7273/full/462568a.html`). The linear growth phase is preceded by exponential growth, which is typically quite fast, doubling the capacity every few years, but this entire phase does not produce any net energy of significance. In the paper ’Lopes Cardozo, Lange and Kramer, ’Fusion: Expensive and taking forever?’ JoFE, 2015’ this typical growth template is considered and the consequences for the development of fusion power are addressed (reprint on CANVAS).

Again, it is the sheer dimension of the world energy demand, combined with the fact that the economical lifetime of energy infrastructure is long, that is the core of the problem.

There are signs that things are changing though. The investments in fossil power are decreasing. Yet, we are still investing massive amounts of money in the wrong infrastructure! Any power plant built today will be around for the coming 60 years. And, shockingly, the world still subsidises fossil fuel at a level much higher than the total subsidies for renewables. (see e.g. `https://www.iisd.org`). Which goes to show that severe measures are necessary to bring the required revolution about.

A final graph that is worth some lengthy contemplation and study is given in Figure 2.6. Note where the USA spend most of their energy, the large amount of conversion losses in the production of electricity and, again, the still tiny contribution of renewables.

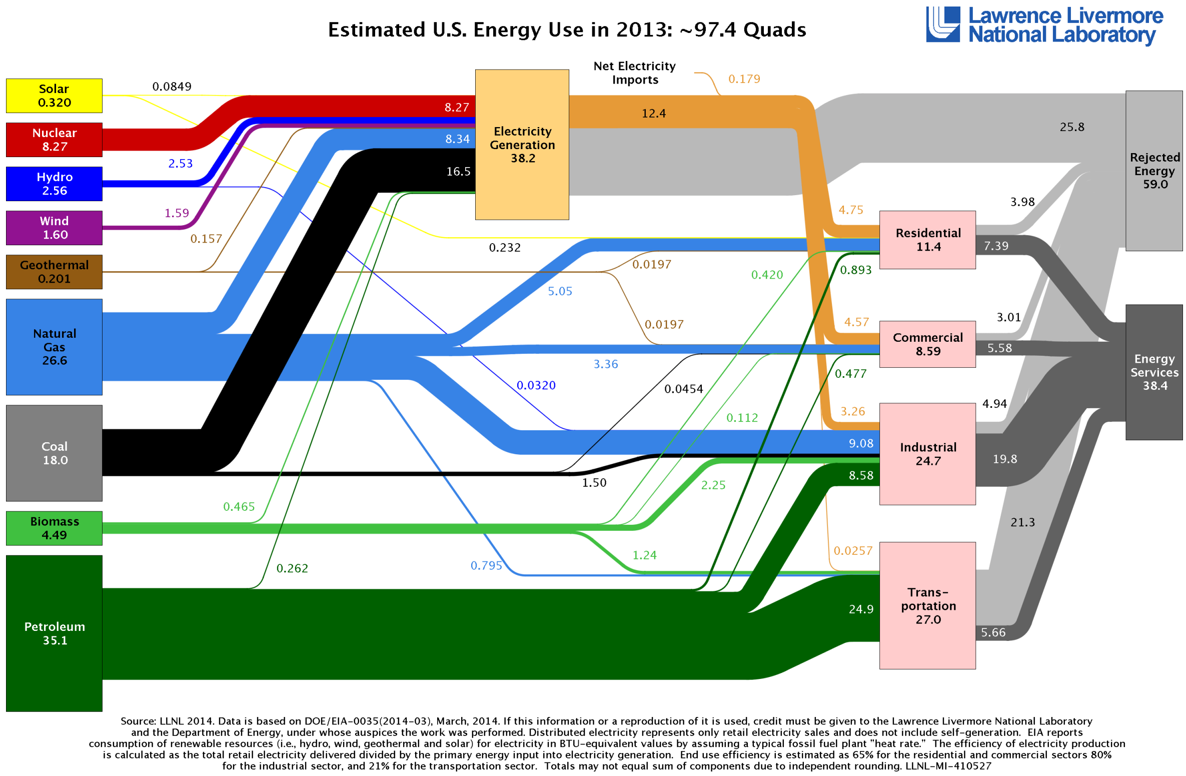


Figure 2.6: Flowchart of US Energy use (source: LLNL)

Hence, replacing the present, fossil fuel-based power infrastructure by a new one based on sustainable technology will take many decades, both for technological and economical reasons.

In this game fusion is among the very few options for a sustainable, large-scale energy generation. We cannot afford to not explore and develop such options. Hence, the development of fusion is a must, not because we think the research is fun (although we do think the research is fun!), not because fusion will provide cheap electricity (fusion power will probably not be cheap), but simply because mankind is faced with a gigantic problem and fusion could provide a part of the solution.

To further place things in perspective, it is important to have a good sense of how we use energy. For instance, few people realise that only about 20% of our energy consumption is in the form of electricity, domestic electricity use is even only 7% of total energy use. We use a lot more energy to heat our homes (volume heating), to move around (transportation) and to make stuff (industry).

There is a lot of confusion about this. Energy and electricity are often used as if they are the same thing,

while there is a factor of 5 between the two. If the power from a wind park is expressed in the number of households it can power, remember that that represents only 1/14th of the (representative) energy use of that family. Also note that many of the 'new' energy sources provide electricity - which is fine, but implies a huge mismatch between present use (and infrastructure) and the sustainable energy infrastructure we must build. This is also why, in parallel to sustainable electricity generation, we need to develop conversion and storage technology.
The book 'Sustainable Energy $\cdots$ without the hot air' by David MacKay puts everything in perspective. Further, it works towards 'a plan on a map'. Not only must we calculate how much of each of the sustainable energy sources we need - where anyone can device his/her own favourite energy mix - but we need to see the implementation projected on the map of the country. MacKay does this for the UK. I copy a few representative pictures from the book below (the book is free downloadable - as is a very useful 10 page summary). The map shows the implementation of a plan that is a sort of average of the various mixes. Note a. that it gets in total some 25% of the power from nuclear fission and 'solar in deserts' (imported somehow); and b. that this plan - as all the other variants - implies that you will see the production of energy whichever window you look out of. I also copied in a useful table giving the power per unit area for various sources.

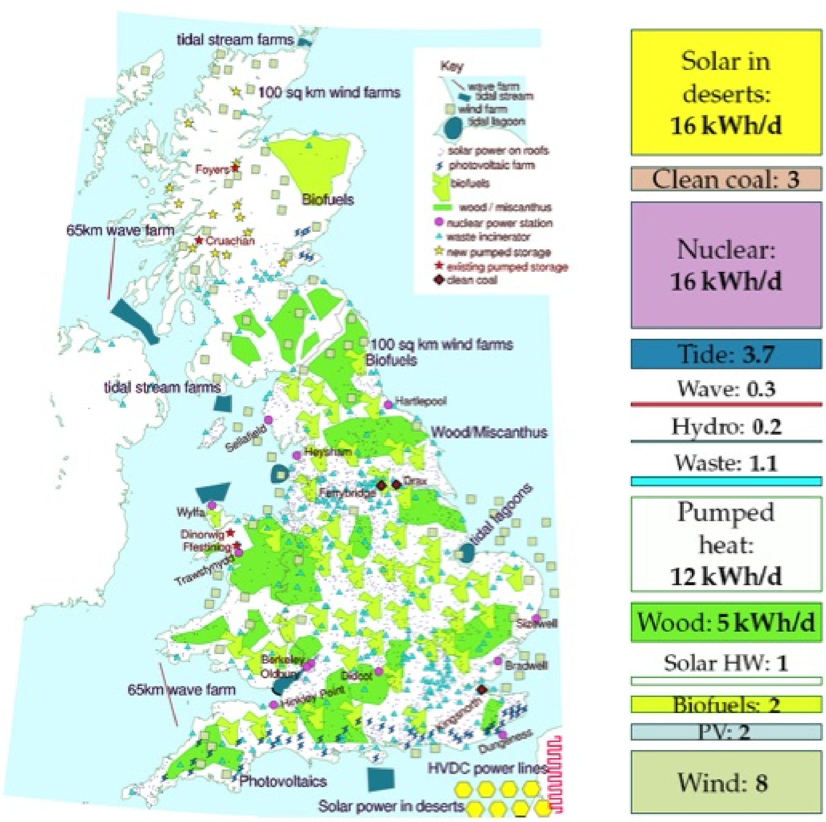


Figure 2.7: The 'plan on a map' of MacKay: one of the energy scenarios mapped on the UK. Note that this scenario, while pretty much covering the UK, still needs to import solar energy from plants in the desert. (from MacKay: Sustainable Energy without the hot air)

Sustainable Energy — without the hot air

David JC MacKay

Power per unit land or water area	
Wind	2 W/m^2
Offshore wind	3 W/m^2
Tidal pools	3 W/m^2
Tidal stream	6 W/m^2
Solar PV panels	5–20 W/m^2
Plants	0.5 W/m^2
Rain-water (highlands)	0.24 W/m^2
Hydroelectric facility	11 W/m^2
Geothermal	0.017 W/m^2

Figure 2.8: If we want to project a renewable energy plan on a map, we need to know the area that is needed. Note that solar PV panels in this list are already by far the best (from MacKay: Sustainable Energy without the hot air)

Problems

2.1 Fusion in the energy mix — the big picture.

(a) The energy demand will grow in the 21st century. Take as starting point that in 2000 the world population was 6 Billion, and that the 2 Billion in the rich countries made up 75% of the total energy demand. Calculate the factor by which the energy demand will increase by the end of this century, making reasonable assumptions on the increase of the population and energy use in the developing countries, and assuming that population and energy demand are stable in the rich countries.

(b) It is often said that there is plenty of coal — at least for 200 years at the present rate of consumption. Now, suppose we run out of oil and gas in the coming decades, and that we have to supply all the power calculated under a) by coal, clearly the 200 years reserve will last much shorter. Estimate how much shorter (or: what is the expected time the coal reserve will last) and make your estimates clear.

(c) Suppose that the rich countries manage to reduce the CO_2 emission to zero by 2100, and that the rest of the world manage a 50% reduction of CO_2-emission (per watt). What would that mean (approximately) for the total CO_2-emission by 2100, compared to now. Is that good enough?

(d) What percentage of the energy generation is based on fossil fuel at present? What percentage of the energy consumption is in the form of electricity? What percentage of the energy demand is due to transportation?

(e) Estimate how many person-hours of physical labour correspond —energy-wise— to 1 barrel of oil.

(f) Estimate how much energy (express in litres of oil) is needed to produce 1 litre of milk and bring it to your table. (Borrowed from 'Energie Survival Gids')

(g) Domestic electricity use is about 7% of total energy consumption. Compare the statements: 'this wind park will power 1/3rd of all Dutch households' to 'a new nuclear power plant will reduce CO_2 emissions of the Netherlands by only 2%'. (Adapted from 'Sustainable energy without the hot air')

(h) Calculate energy content of:

   i. 1 litre of beer; 1 kg of oil; 1 kg of liquid hydrogen; (when burned)
   ii. 1 kg of Li-ion battery (charged).
   iii. 1 kg of uranium (in fission reaction, depending on process);
   iv. 1 kg of D-T mixture (when fused).

(i) Calculate the energy in the oceans based on the d-d reaction. How many years of energy for humankind, if all energy consumption were to depend on it?

(j) Compare initial investment cost for 1 GW (year average output power) installation: wind, PV, solar thermal, fission, fusion. Compute the bottom price of 1 kWh, by assuming a life time and interest rate. Compare this to the price of 1 kWh from the grid (separate the kWh, the network cost and the taxes). And from an AAA-battery.

2.2 Fusion in the energy mix – limits to growth

A new energy source can be said to be *available* when its total installed effective power is 10–100 MW. The total energy demand of the world is estimated to be some 20 TW in 2020. Assume for the moment that a new source goes through an exponential growth until it reaches 10% of its final, potential installed power. As the latter is typically 10–20% of the world demand, this 'materiality' point (as it is called by GJ Kramer and M Haig (Nature, 2009)) corresponds to 1–2% of the world demand.

(a) The exponential growth of new energy sources typically has a doubling time of 2–3 years. How long does it take to go from 'availability' to 'materiality'.

(b) The exponential growth is dominated by investment costs, which for many different technologies are found to be around 5 Euro per Watt of installed power by the time 'materiality' is reached. Estimate the annual investment at the end of the exponential growth. Compare to the world energy cost of approximately 10 thousand billion (!) Euro/year.

(c) The fusion roadmaps of different ITER parties foresee the operation of 3 DEMO plants by 2060, each with a fusion power of 5 GW. Taking that as your calibration and assuming the same doubling time as above, when will fusion reach 2% of the world energy demand (materiality), i.e. when can fusion enter the energy mix?

(d) One difference between fusion and some other technologies is that fusion appears to have more fundamental technological challenges. Give 2 major technological issues that need to be resolved before fusion can become commercial.

(e) After the exponential growth phase the development of a new source is expected to follow a linear growth.

(f) What could be a reason why the growth stops being exponential and becomes linear?

(g) How long does the linear growth take?

(h) Which two factors determine the growth rate of that linear phase?

3 Let's build a fusion reactor

3.1 Design a fusion reactor

So, let's assume that society asks—or will ask—for a fusion reactor. The request to design and develop such a reactor will necessarily end up with us scientists: physicists, chemists, mechanical and electrical engineers, software and control engineers ··· without us no reactor.

So, let's brainstorm: what questions do we have to address? First: which fusion reaction do we select? There are many possibilities, you can find them in books and look up the reaction rates or cross-sections. As we already anticipate that the fusion reactor will be a tough challenge, we pick the one that has the highest reactivity. The plot below shows the reaction rate, i.e. the cross-section of the fusion reaction (which depends on the energy of the fusing particles) multiplied by the velocity, averaged over a Maxwellian velocity distribution with temperature T. To get to power density, this rate has to be multiplied by the densities of the two reacting species, and the energy released in a single fusion reaction. Clearly, the deuterium-tritium reaction is by far the best bet.

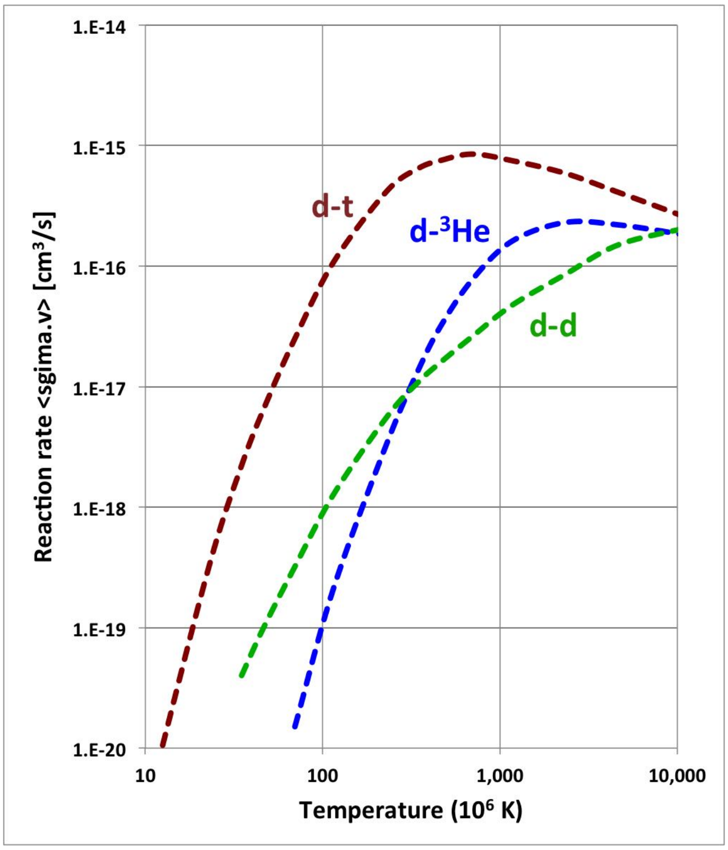


Figure 3.1: The reaction rages of the three most relevant fusion reactions show that the d-t reaction gives the highest rate, and does so at the lowest temperature. Furthermore, the energy release per reaction is much higher than that of the d-d reaction, while being similar to the d-3He reaction

The picture immediately reveals the working temperature of the D-T reactor: at least 10–20 keV. Realise that the power loss from a reactor will also depend on the working temperature—at least linearly—so that the optimum temperature is lower than the temperature at which the reaction rate is at its maximum.

Electronvolt

The energy unit 'electronvolt' corresponds to a temperature (T) of about 10 thousand Kelvin, through the relation $E = k_B T$, where $k_B = 1.38 \cdot 10^{-23}$ Joule/Kelvin is the Boltzmann constant and 1 eV $= 1.6 \cdot 10^{-19}$ Joule. We normally give the temperature in units of keV (kilo-electronvolts), so that the temperature is a measure of the thermal energy of the particles.

This temperature is so high because the nuclei that must fuse repel each other due to their charges (Coulomb interaction). Only when they get very close the 'strong interaction' (nuclear force with a very short range) takes over and makes the particles fuse. This 'Coulomb barrier' that has to be overcome is about 200 keV high, but thanks to the quantum-mechanical effect 'tunnelling' there is a finite probability that particles with a much lower energy fuse. Note that for particles that collide with too high energy the fusion cross-section becomes smaller again: the particles can overcome the Coulomb barrier but do not have enough time to react.
Now you may think that an easy way to achieve fusion reactions is to bombard a solid deuterium target with a beam of high-energy tritium ions. This is correct, in this way fusion reactions can be generated easily and neutrons can be generated. However, this approach can fundamentally not lead to net energy gain. The reason is that the passage of the beam through the target has two effects: energy loss (heating of the target) and fusion reactions. The cross-sections of both processes are so vastly different that the heat released in the fusion process is insufficient to compensate the losses. Hence: this concept can produce neutrons, but no fusion power plants.
A concept that does work is the so-called thermonuclear fusion. Now, we bring the deuterium and tritium nuclei together in a vessel and bring them to a temperature high enough to get a good fusion yield. Hence, the thermal energy of the particles, i.e. the temperature, should reach tens of keV. Where we should keep in mind that the particles have a velocity distribution, and it is sufficient if the fastest particles react. The advantage of this approach is that particles keep colliding with each other as long as they are confined in the vessel. How we achieve that is another matter, we will come to that.
In whichever way we achieve these high temperatures, it is clear that at a temperature of 10–20 keV hydrogen is fully ionised - the ionisation energy being only 13.6 eV. Hence the fusion fuel is a gas of charged particles - ions and electrons: a plasma. Plasma - the fourth state of matter - is something very different from a gas, and in many respects counter-intuitive. The physics of plasmas was developed only in the last century, not least in the frame of fusion research. But also in astrophysical context, as well as for industrial applications of plasmas major contributions to the development of plasma science were made.

3.2 The Lawson criterion

A machine that confines a hot plasma is useful for power generation only if it produces more power than it consumes. This trivial condition directly leads to the well-known Lawson critirion

Lawson criterion

$$n\tau_E T > \text{critical value} \tag{3.1}$$

Here, n is the fuel density (number of particles per cubic metre), T the plasma temperature and τ_E is the so-called energy confinement time. In this criterion it is implicitly assumed that the temperature has a reactor-relevant value, i.e. $T = 10 - 20$ keV. In that range—see the plot of the reaction rates —the D-T reaction

rate is roughly proportional to the temperature squared. There is a reason that we take this temperature: the heat loss from the plasma is function of the temperature, too. So at some point, an increase of the plasma temperature leads to more extra loss than heating by fusion power. That is the temperature at which the plasma will settle for a stable 'burn', also called the 'burn temperature'.
The energy confinement time is a measure for the thermal insulation of the plasma, and hence a measure for the efficiency with which the generated power is used to sustain the (high) burn temperature. Clearly, the less power is needed to keep the plasma at the burn temperature of a few hundred million K, the easier it is to make a net power profit.
Realise that the product of the temperature and the density is nothing else than the kinetic pressure of the plasma, i.e. the kinetic energy density.
Filling in numbers, the criterion states that in order to achieve net power gain, the product of energy confinement time and kinetic pressure must exceed a few tens of seconds $\times$ bars.
In magnetic confinement fusion we aim at a reactor with a confinement time of tens of seconds and a pressure of a few bar. But it should be realised that there is a trade-off between pressure and confinement. There are also schemes that aim at mini-explosions: high pressure, short confinement, and intermediate solutions: medium pressure, medium confinement.

Derivation of the Lawson criterion

The power generated in a fusion reactor is proportional to:

- volume (trivial)
- density squared (because a fusion reaction is a two-particle process!)

and around $T = 10$ keV the reaction rate is approximately proportional to T^2. Hence

$$P_{\text{fusion}} \propto \text{Volume} \cdot n^2 \cdot T^2$$

The heat loss is — by definition — characterised by the energy confinement time, which is defined as:

$$\tau = \frac{\text{stored energy}}{\text{power needed to sustain the plasma}}$$

Hence

$$\tau \propto \frac{\text{Volume} \cdot n \cdot T}{P_{\text{input}}}$$

The simple requirement $P_{\text{fusion}} > P_{\text{input}}$ leads to the Lawson criterion.

3.3 Routes to fusion power

So, we need high density, good energy confinement and in any event bring the fuel to a temperature of a few hundred million Kelvin. There are many ways in which this could possibly be achieved. In short: use gravity for confinement (as the stars do), use inertia for confinement and compensate the short time the fuel stays confined by high density, or use magnetic fields to confine the fuel. And then there are ideas to trick Nature, e.g. by replacing electrons in a deuterium atom by the much heavier muons, and there is a category of approaches often referred to as 'cold fusion' or somewhat more respectably as 'low-temperature nuclear reactions' that rely on yet unknown physics. This course will mostly be concerned with magnetic confinement schemes. But let's have a quick survey of what's on offer.

3.3.1 Gravity

The sun and all other stars are kept together by gravity. It is the sheer size (or rather, mass) of these objects that provides the gravitational force large enough to sustain a very high pressure in the inside, high enough to satisfy the Lawson criterion and thus allow fusion reactions to occur. Even much slower reactions than d-t fusion can be sustained, and that at a temperature of 'only' 10–15 Million K (in the sun). Of course, besides the pressure, also the confinement time is enormous. Gravity: it is stable, inexhaustible and for free. But, unfortunately the mass required to make a gravitational fusion reactor work is far too large to use this concept for power generation on earth.

3.3.2 Inertia: 'Inertial Confinement Fusion' (and the H-bomb)

If we manage to ignite fusion fuel fast enough, inertia will keep the particles together for a finite time – possibly long enough to have significant burn-up of the fuel. In the frame of the Lawson criterion this is the 'short confinement, high pressure' solution. This principle is applied in the hydrogen bomb – uncontrolled fusion is unfortunately much easier to achieve than peaceful applications of fusion power. If the principle is applied in a laboratory, it is called 'inertial confinement fusion' (ICF). Worldwide, the research programme on ICF is as large as that on magnetic fusion. However, much of this research is of a military nature. The primary goal is not to develop a fusion reactor for power generation, but to make an experimental simulator of the hydrogen bomb, in order to test theory and computer codes without having to resort to the banned nuclear tests.

In ICF experiments, a 1 mm radius sphere of solid hydrogen isotopes (D-T mixture) is subjected to intense radiation from all sides. The radiation can be laser light, or beams of heavy ions, or indirectly generated x-rays — it does not matter, as long as the irradiation puts massive energy into the outer shell of the fuel pellet. This outer shell then ablates (i.e. goes from solid to gas phase), and as a result the inside is compressed. In this process, pressure and temperature rise very quickly (adiabatic compression). The process should be organised in such a way that when the highest pressure is reached (i.e. before the compressed pellet starts to expand again) the temperature has just reached the optimised value for the D-T reaction. Thus, ignition may be achieved when the power released in the compressed pellet is sufficient to further raise the pressure and burn up a significant fraction of the fuel.

At present there are several laboratories in which ICF is researched on a large scale, the largest being the National Ignition Facility NIF, in the USA. Visit the website, (`www.llnl.gov/nif/`) and find out the size of this experiment. Figure 3.2 gives a brief pictorial description of NIF with pictures from the website. From the very large to the very small: aerial view of the laser building the size of 3 football pitches; inside of the laser building: the NIF laser can deliver up to 2 MJ of energy in a 1 ns pulse; inside of the 10 m diameter reaction vessel: note the big pencil at the tip of which the hohlraum is mounted; close-up of the hohlraum, inside which the pellet (last picture) is positioned in a light web of plastic.The pellet itself is a little ball with a complex layered structure consisting of various materials, with the fuel — the frozen deuterium-tritium mixture — on the inside. The structure is designed to optimise the implosion, so that the energy of the laser is transferred as efficiently as possible to the fuel.

There is a lot of fascinating physics and technology in this field, from high power lasers to the technology to make the pellets, tricks to achieve fast ignition with a very short extra laser pulse, the plasma physics of the

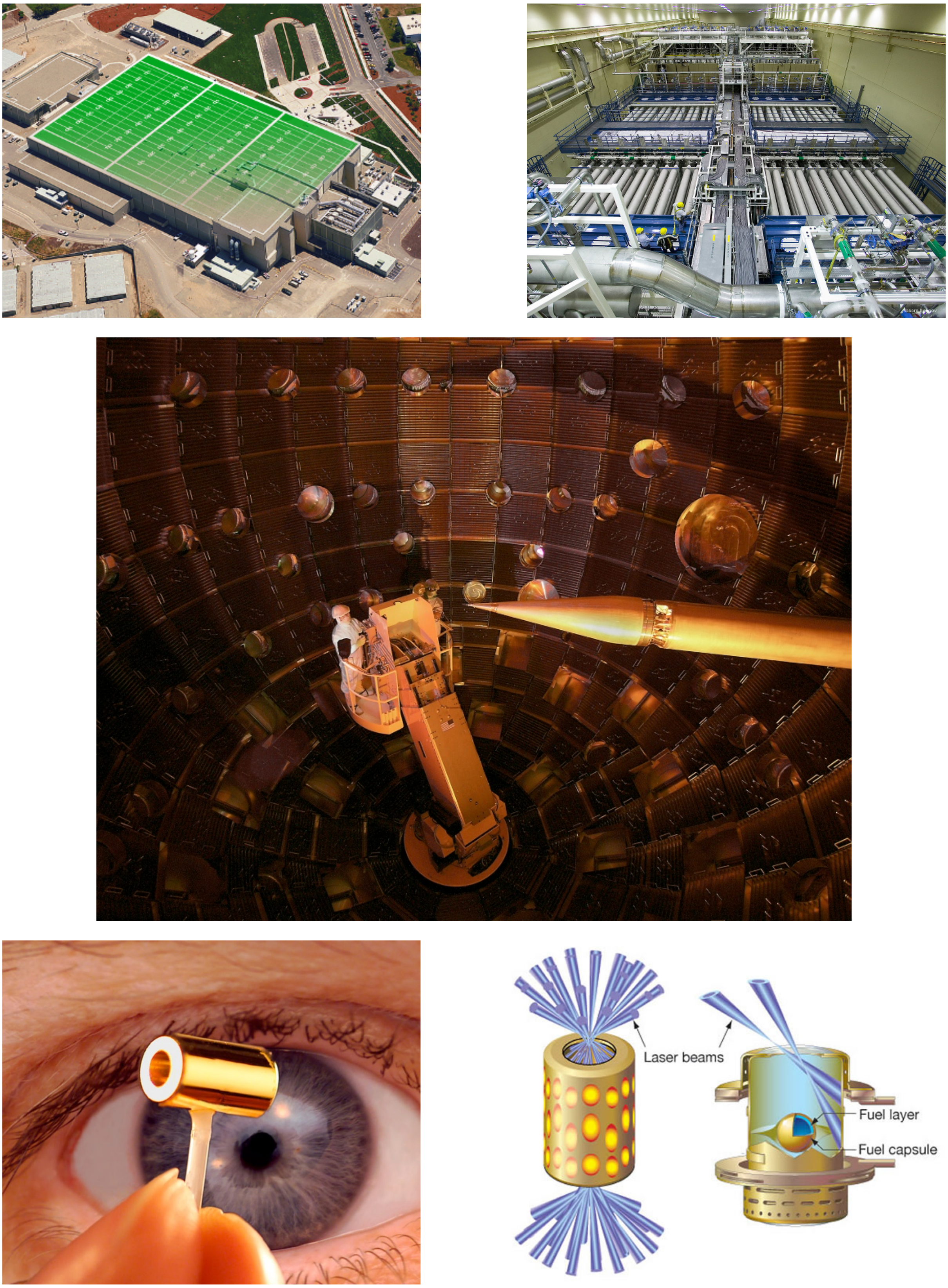


Figure 3.2: NIF facility. Top left: laser building; right: interior of the laser building; centre: reaction vessel; bottom left: hohlraum; right: hohlraum and fuel pellet . All pictures, credit: Lawrence Livermore National Laboratory.

pellet during compression, etc. But the prospects for power generation with this concept are slim. Very big steps have to be made in the efficiency of the laser and its repetition frequency (which should become 10 Hz or so in a power plant, 5 – 6 orders of magnitude faster than the present lasers). Further, you must realise that in ICF, ignition (in the sense that the pellet starts to generate enough power to burn itself) could be reached much before break-even, as the energy efficiency (« 1%) of the laser has to be taken into account. Finally, on a more mundane level: for power applications the pellet should be produced for a fraction of a dollar a piece. The present production price comes out at 10 000 dollar, and even much higher numbers have been quoted.

For a reactor, the rough numbers are that 1 GW is produced by making 10 blasts per second of 100 MJ each. For comparison: a hand grenade (or a chocolate bar, for that matter) is worth about 1 MJ of energy. The explosion demolishes the hohlraum as well as the arm that keeps it in place. In a reactor there wouldn't be an arm—it would be impossible to reach the 10 Hz even if the arm survived: the pellets will have to be hit by the laser pulse during free flight. A technological challenge in itself.

As said, there are many highly interesting physics and technology issues with ICF, it is a really exciting research field. But we do not cover it in any detail in this lecture course because of the lack of perspective as an energy concept.

3.3.3 Muon Catalysed Fusion

A very interesting concept that unfortunately cannot yield net energy, is the catalysis of the fusion reaction with muons. Let us first see how it works. A muon is a heavy electron: rest mass about 200 times that of an electron, same charge, but short-lived (2 μs). The heavy mass gives it a proportionally shorter DeBroglie wavelength, hence a hydrogen atom with a muon instead of an electron has a very small radius. This makes it possible to let such a muonized deuteron approach a tritium ion so closely that, through tunnelling, the fusion reaction cross-section becomes appreciable at room temperature or even at cryogenic temperatures. Cold Fusion! It is a real catalysis, as the muon does not participate in the reaction. It just screens off the charged reactants until they are so close that the strong nuclear interaction prevails. So the muon can be reused as long as it lives. And given the reaction rates, it could catalyse hundreds of reactions during its 2 μs live.

At face value, this is a very clever scheme. The rest mass of a muon, in energy units, is about 100 MeV, i.e. 5 times as much as the energy released in the D-T reaction. So if the muons were produced efficiently, each would have to catalyse only 5–10 reactions to make a net energy gain. That would be easily achieved within the lifetime.

However, the production of muons requires a large accelerator and is energetically not very efficient. This is not a matter of optimising the design, the problem is intrinsic: the process that produces the muon also produces other particles, so that more than 90% of the energy is wasted (from the muon production perspective). In the most optimistic design studies, the energy spent on the production of a single muon is 5–10 GeV, rather than 100 MeV rest mass. So now, a single muon must catalyse some 300 fusion reactions to regain its production energy. Taking into conversion efficiencies in the remainder of the process, in any practical energy generator, the number of fusion reactions that a single muon must catalyse is well in excess of 1000.

Now, that appears to be impossible. The two limitations are 1. the short life of the muon, and 2. the fact that there is a probability of about 0.5% that a muon sticks to the reaction product (helium) after it has catalysed the D-T reaction, and is then lost as a catalyst. The latter effect alone limits the number of catalysed reactions per muon to 200.

Too bad, it is such a neat idea, using our deep knowledge of the building blocks of nature!

3.3.4 Fusor

Another neat idea to produce controlled fusion is to confine a plasma by an electrostatic field. Basically, you organise a strongly negative electric potential in the middle of a vacuum tank, and you inject ions that start to oscillate through the cloud. The ions pick up energy if they approach the central part where the floating potential is negative and are slowed down again when they leave the central zone. Hence, there is a small volume in the centre where the ions meet, the interaction zone. By setting up the experiment properly, the interaction energy could be chosen such that fusion occurs. The negative potential can be created by a grid or by a bunch of electrons that are kept in place by magnetic fields.
This simple experiment, of which there are various variations, is called the Fusor. There are dozens of fusors around the world, mostly built in garages by fusion enthusiasts. It's not that difficult and it works: if you do the experiment very carefully, you can generate fusion reactions, including the tell-tale neutrons. However, an analysis shows that also in this experiment it is not possible to get a net energy gain. Again, it is the principal difficulty is in the collision cross-sections: there is a much larger probability that the ions lose their energy by colliding with either the electrons or the grid, than that they produce fusion energy.
Still, the fusor is such an interesting experiment that we have build one at the TU/e. In fact, ours is one of the best fusors in the world! And it produces iconic pictures: see Figure 3.3 The project is run by students, so if you are interested to join the Fusor@TU/e team, let me know.

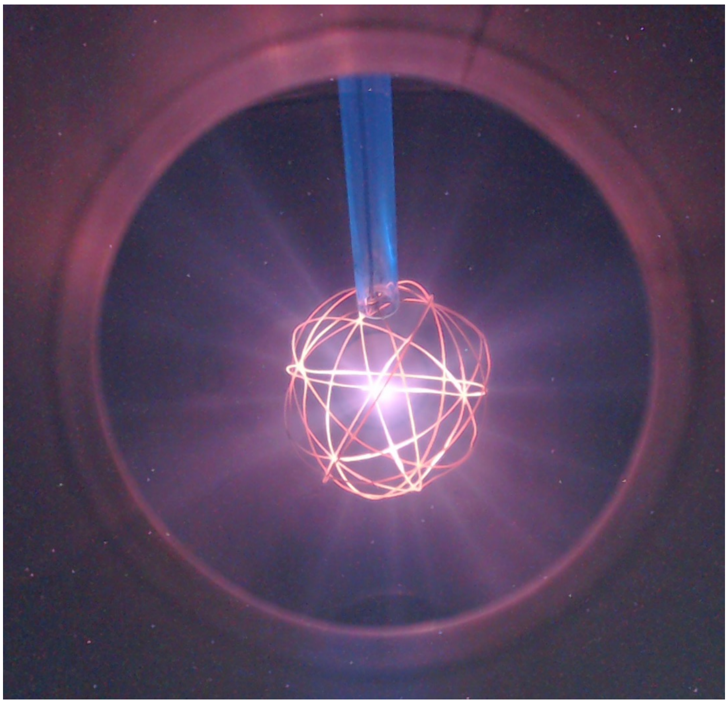


Figure 3.3: The FUSOR@TU/e: the white-hot grid is at a high negative potential so that ions in the vacuum tank are accelerated towards the centre, where they collide. The fusor cannot produce net energy gain from fusion, but it is a very cheap and efficient source of fusion neutrons!

3.3.5 'Cold Fusion'

Fusion by electrolysis?

On Easter day 1989, the American electro-chemists Fleischmann and Pons announced in a press-conference (rather than a scientific paper!) that they had achieved production of heat by nuclear fusion in an electrolysis experiment. They had performed the following experiment. Heavy water (D_2O) was electrolysed in a standard set up, but in which the anode was made of Palladium. Palladium is one the metals that can absorb very much hydrogen in its lattice. Fleischmann and Pons reasoned that the boundary potential created during electrolysis would effectively result in a pressure on the hydrogen inside the palladium lattice so immensely high that fusion reactions would occur at room temperature.

They did the experiment and measured the heat released by calorimetry. From this measurement they concluded that there was 'excess heat', corresponding to an energy released per dissociation of the order of 1 keV. Three orders of magnitude too high for a chemical reaction! And so they announced in a press conference that they had achieved nuclear fusion 'in a jar'.

The claim was so spectacular that many laboratories immediately set up experiments to check the claims. Even during the same Easter weekend experiments were started all over the world, based on copies of copies of copies of the original fax press release. But the results proved to be irreproducible. All sorts of interesting effects were observed, but fusion was not among them.

Cold fusion research has continued at a small scale in several places around the world, but no energy production was ever demonstrated. If you are interested: Wikipedia has extensive information on the subject.

The latest improbable branch on this tree is the Energy Catalyser of Rossi, first in the news in January 2011. He claims to have devised a reaction involving Nickel and hydrogen in a table top set up, producing some 10 kW of net energy gain. Interestingly, his line of reasoning is: "it produces energy, so I don't have to explain how it works, I will just patent the set up and sell energy sources". Since there is no indication of the mechanism or explanation of the energy production, and no one is allowed to look inside the box, it is difficult to make any substantial comment. It is just very, very unlikely. It does not help that Rossi has been convicted for energy scams in the past.

Notwithstanding the fact that cold fusion is largely the domain of charlatans, there is a body of serious literature that claims that under certain circumstances nuclear reactions occur in conditions that according to established theory would preclude their occurrence. The generic term for these explorations is 'low energy nuclear reactions' or LENR. The results are characterised by bad reproducibility, and it is difficult to separate the garbage from the sound science here, but it would not be scientific either to categorically deny the possibility that nature has secrets we that don't exist in the physics textbooks. You may want to google a bit for LENR. But try not to get lost.

3.3.6 Bubble fusion

In the beginning of the century, so-called 'bubble fusion' made it to the newspapers. This is related to the phenomenon of sonoluminescence: very loud sound can cause bubbles in a fluid to vibrate and eventually collapse, whereby the adiabatic heating of the gas (vapour!) inside the bubble can lead to temperatures high enough to cause light emission.

A group at Oak Ridge National lab (lead by dr. Taleyarkhan) made strong claims that in this way fusion conditions could be achieved in ammonia. But it can be shown rigorously that such claims are wrong. If you google 'bubble fusion' you get more to read than anyone needs.

3.4 Magnetic confinement: general considerations

If we want to confine a plasma stationary rather than in short pulses as in inertial confinement, magnetic fields are the ticket. The term 'magnetic confinement' itself is often a source of misunderstanding. That is probably because the term really applies to two different physical phenomena combined.

First: in a fusion reactor the plasma is hot in the centre, but will have to be cold where it touches the walls, so there is a very steep pressure gradient. Something has to balance this gradient, and that something is the magnetic field, through the Lorentz force. So, here the phenomenon is that the magnetic field pushes the plasma inward to balance the pressure gradient that wants the plasma to expand. Second, the effect of the magnetic field is to strongly limit the mobility of the plasma particles in the direction perpendicular to the field. In fact, the mean free path of the electrons is measured in kilometres along the field, whereas across the field it is only 0.1 mm or so. In the diffusion coefficient, this works out (D is proportional to mean free path squared) as a reduction of the losses by 12–14 orders of magnitude.
The magnetic field is therefore extremely important. It is also very expensive: typically 30% of the construction cost of the tokamak.

3.4.1 Magnetic confinement in linear geometry: 'pinches'

The simplest way to confine a plasma is in linear geometry. By running a longitudinal current through a plasma in a straight tube the so-called Z-pinch is created. The Z indicates that the current runs in the Z direction, the word pinch reflects the fact that the magnetic field generated by the current 'pinches' the plasma. If the current is ramped up fast enough the compression of the plasma is adiabatic (no energy loss during the process) and in that case the plasma is heated as well as compressed.
The set-up is really simple, and works. However, the Z-pinch plasma is fundamentally unstable. It is instructive to give this a closer look, because the instabilities that occur in a Z-pinch will return in modified form in the toroidal devices, where a strong toroidal field must be applied to stabilise them. A longitudinal field also works in a linear device: the combination of a longitudinal field and a poloidal field results in helical field lines, which explains the name of such a device: screw pinch. But in linear geometry a longitudinal field comes with enormous end losses, which rules the concept out as reactor. In toroidal geometry that problem is cured.

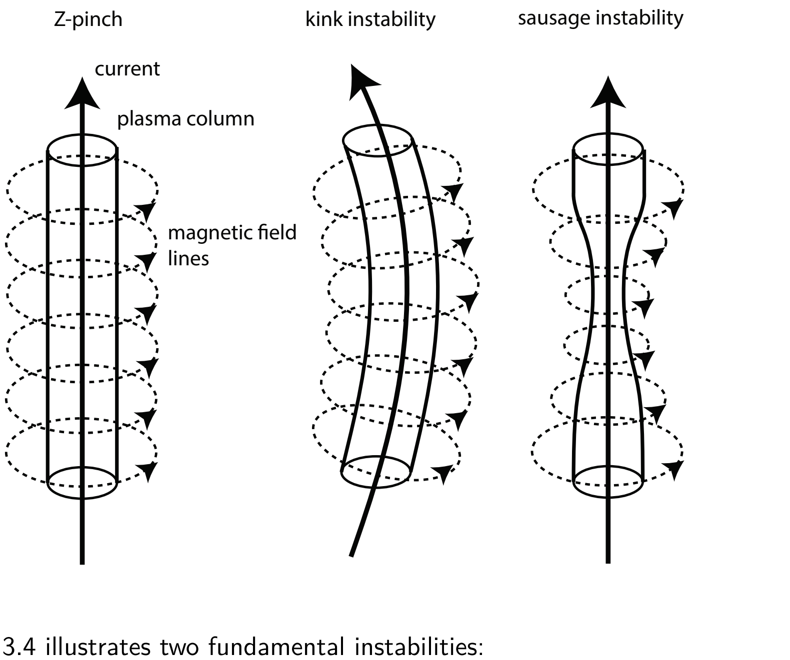


Figure 3.4: Z-pinch. Left: unperturbed cylindrical plasma pinched by a longitudinal current; centre: kink instability, which bends the column; right: sausage instability, which squeezes the column. Both instabilities can be understood from the perturbed field lines: magnetic pressure increases quadratically with field line density.

Figure 3.4 illustrates two fundamental instabilities:

- 'kink instability' : if the plasma column deviates infinitesimally from the perfectly straight cylinder, the magnetic pressure is larger on the inside of the bend than on the outside. This pressure unbalance reinforces the original bend. Result: the bend grows exponentially and the plasma breaks open.

- 'sausage instability'[1]: in those places where the plasma happens to be constricted, albeit by an infinitesimal fraction, the magnetic field increases, hence the magnetic pressure, so that the original effect, the constriction, is reinforced: instability. Also this leads to a break-up of the plasma column.

Note that 'instability' means: the effect reinforces the cause. Instabilities initially grow exponentially. Eventually their growth is limited by other, nonlinear, effects.

Both instabilities can be tamed with a longitudinal magnetic field. The combined field is then helical, and the machine is now appropriately called 'screw pinch'. To get good and stable compression, both field components need to be ramped up simultaneously — not an easy task if it has to be done fast!

The Z-pinch does still play a role in fusion research, in a derived form: the 'dense Z-pinch'. In this variant, the plasma is formed by running a very large current through a wire formed by frozen hydrogen. In this way a very high density can be achieved, the concept being a cross-over between magnetic confinement and inertial fusion. It turns out that even better results are achieved if instead of a single wire an array of wires is used. This concept — perhaps more than being a road to a fusion reactor — provides an extremely intense source of radiation (x-rays). And as such, it can be the driver of an ICF scheme: the Z-pinch takes over the role of the laser. This is an active research field, see Fig. 3.5

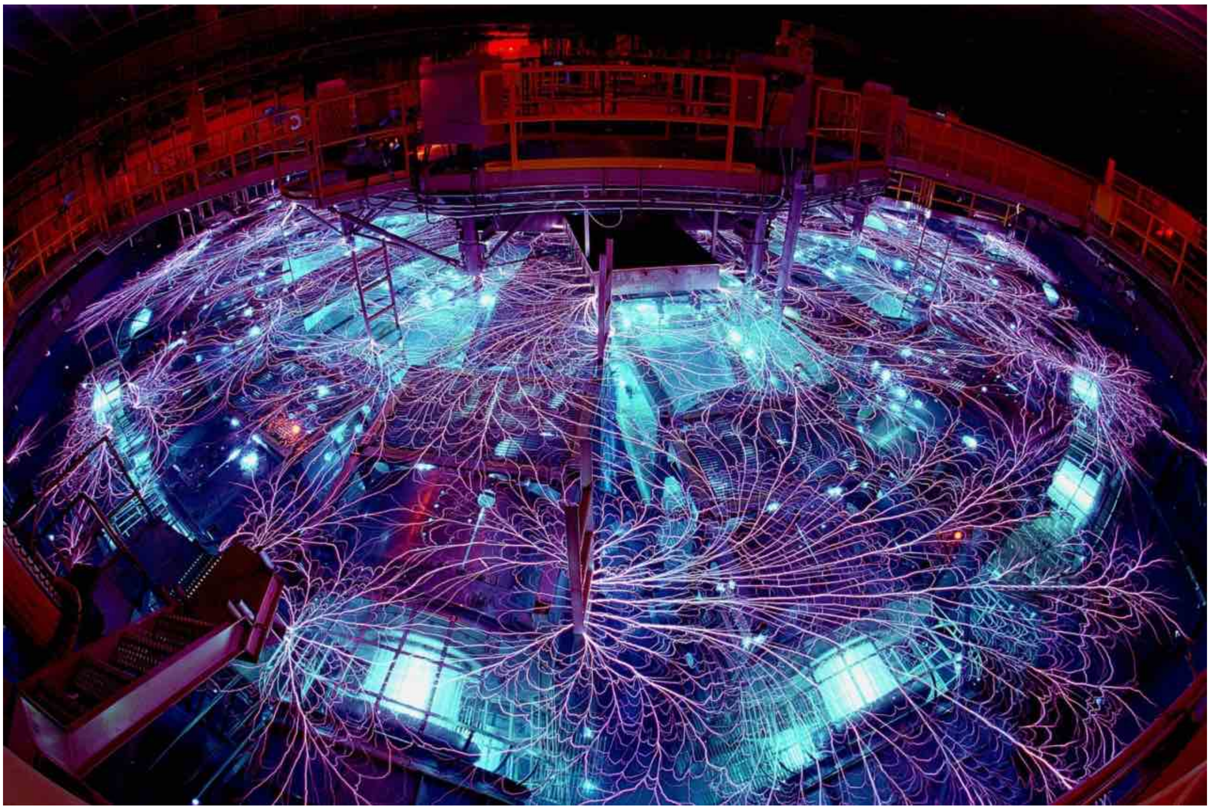


Figure 3.5: The Z-machine in Sandia, `http://zpinch.sandia.gov/`

Theta-pinch

Another possibility is to only apply a longitudinal field. This member of the family is called 'theta-pinch'. It is a pinch, i.e. the field is ramped up quickly to achieve adiabatic compression. It is much more stable than the

[1] I have noticed that some people associate the depicted shapes with sausages and kinkiness rather the other way round. Can't help it. The tube with constrictions does in fact resemble the traditional English sausage.

Z-pinch, but the field lines connect directly to the outside world, so the end losses make this configuration unsuitable for fusion applications.

3.4.2 Magnetic confinement in linear geometry; mirror machines

The end losses in a linear system can be counteracted in two ways. The first is to bend the straight cylinder into a torus. This is trying to cheat nature, and you have to pay for that with the particle drifts caused by the curvature and gradient of the field. But it works: see Section 3.4.3 and further.

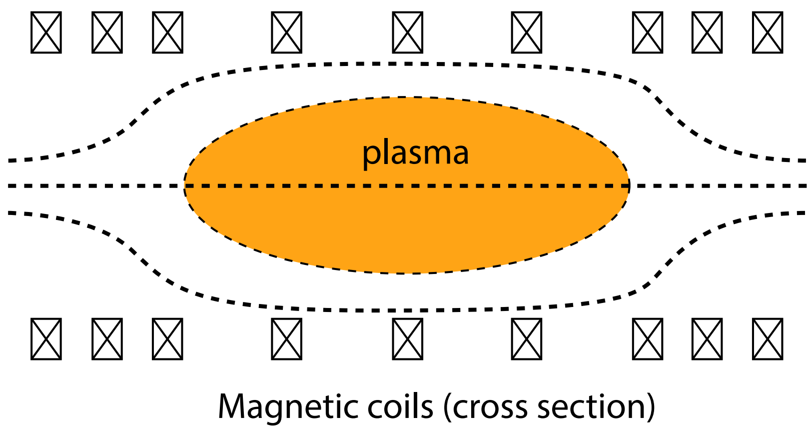


Figure 3.6: Schematic of a magnetic mirror device.

Another trick to reduce end losses is to make use of the mirror effect. Figure 3.6 shows how an increase of the longitudinal field (which is steady state in this case, unlike in the pinches) causes the field lines to close in on each other. Hence the mirror effect will reflect particles that try to escape, provided their pitch angle is sufficiently large (see Chapter 5).

The principal problem of the mirror machine is that particles that do not have sufficiently large pitch angle, i.e. those that live in the 'loss cone', are lost from the system. Thus, a hole in the velocity distribution is created, and this leads to instabilities. All sorts of advanced tricks have been invented to counteract this problem: double mirrors, using RF radiation to readjust the velocity distribution, etc. But the problem persisted and at present the research on mirror machines is very limited worldwide.

3.4.3 Magnetic confinement in toroidal geometry; general aspects

For this reason, fusion research presently concentrates on toroidal machines. Here, the end losses are avoided altogether. This comes at the price of curvature and gradients of the magnetic field, which cause a range of very interesting phenomena, most of which we could do without! But altogether, these side effects of the toroidal geometry can be dealt with and do not cause insurmountable difficulties. Let us briefly review them.

First, the gradient-B drift causes charge separation: the electrons drift in a direction opposite to the ions. This leads to a vertical electric field, which in turn causes an $E \times B$ drift which makes the plasma as a whole move outward at a speed of 1 km/s or more. The drifts themselves cannot be suppressed, neither can the E-field be counteracted by and external field. So the only remedy against the ExB drift is: short-circuiting the top and bottom half of plasma ring, by means of a poloidal field. This will allow the electrons and ions to cancel out the vertical electric field by flowing freely along the field lines. The sum of a toroidal field and the indispensable poloidal field makes a helical field: the field line spiral through the torus.

This spiralling field is a common element of all toroidal confinement concepts. It can be created in two different ways:

1. using external coils: this concept is called 'stellarator', or sometimes torsatron or heliotron.
2. by generating a purely toroidal field component with external coils, and in addition a poloidal field by running a current in the plasma ring. This is the 'tokamak'.

3.4.4 Magnetic confinement in toroidal geometry: the Tokamak

The tokamak is technically the least complex, and there is no doubt that the tokamak is the simplest and cheapest way to make a stable and clean plasma with a temperature of tens or even hundreds of million Kelvin. The concept is furthest developed of all. JET is a tokamak, and so is ITER. Being the workhorse of fusion research, the tokamak is treated in more detail in Chapter 7 and following. For here you'll have to make do with a picture. (See Figure 3.7)

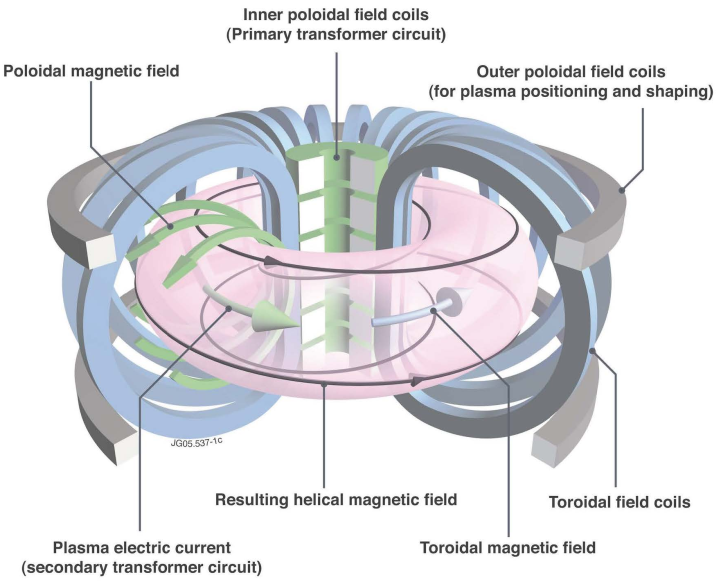


Figure 3.7: The work horse of fusion research: the TOKAMAK. Helical magnetic field in a torus, induced by external coils (the toroidal component) and an electric current that runs in the plasma itself inducing a poloidal B-field. (Source: EUROfusion)

3.4.5 Magnetic confinement in toroidal geometry: the Stellarator

The stellarator is about one generation behind in development (effectively some 15–20 years), but does have some important advantages that may be decisive in the long run:

- The stellarator does not have a plasma current and therefore no potentially damaging disruptions. The plasma current in a tokamak is associated with an amount of magnetic energy comparable to the total kinetic energy in the plasma, and in a sudden loss of confinement ('disruption') this energy is released and can damage the machine.
- The plasma current in a tokamak also implies that a tokamak is a pulsed machine (unless one resorts to technically complex and expensive current drive methods). The stellarator does not need a plasma current because the helical field is induced by external coils only. It is therefore an intrinsically steady state machine.

Hence, the stellarator solves two of the open issues: disruptions and continuous operation. But the technology of the coils and the very complex geometry make the stellarator a technically an even more challenging device than a tokamak.

The most straightforward method by which a helical field can be generated with external coils is by··· helical coils. Sounds obvious, but this implies that the coils must be wound in situ, because the wires have to pass through the central hole of the torus. Coils and vacuum vessel become intertwined, which is a big disadvantage when it comes to maintenance or repair, and is practically ruled out for a full-scale reactor. Nonetheless, almost all experiments of the stellarator family use this method.
However, the advent of supercomputers has allowed the design of modular coils that produce the desired helical field. This concept, the 'optimised stellarator', is now applied to a large experiment under construction in Greifswald (Germany), Wendelstein 7X (see figure 3.8). `http://www.ipp.mpg.de/ippcms/eng/for/projekte/w7x/index.html`
The Fusion group at TU/e has a strong link with W7X, there are always TU/e students in Greifswald. And this is a particularly exciting period, as the machine has become operational in 2015.

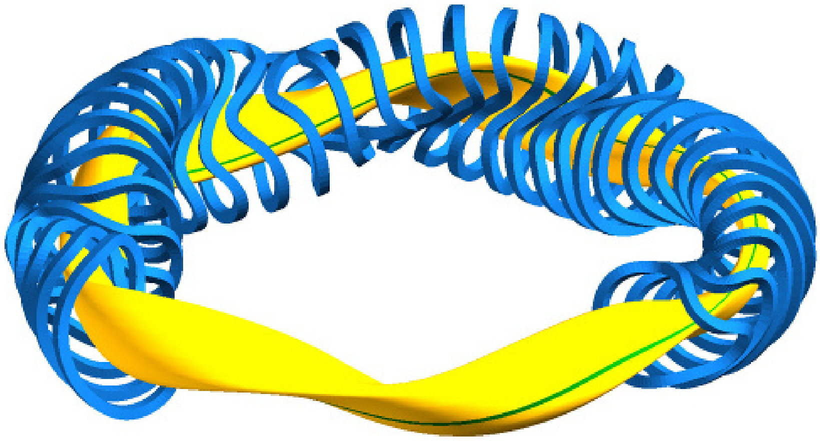


Figure 3.8: Superconducting coils and shape of the plasma in the Wendelstein 7X stellarator. Note the 5-fold symmetry and strong changes of plasma cross section. (Source: www.ipp.mpg.de)

Because all magnetic fields in the stellarator are generated by external coils, it is an intrinsically steady state concept. Of course, the plasma must be heated by external means until the fusion reactions produce enough power to sustain the burn temperature. The absence of a plasma current also means that disruptions do not occur in the stellarator.

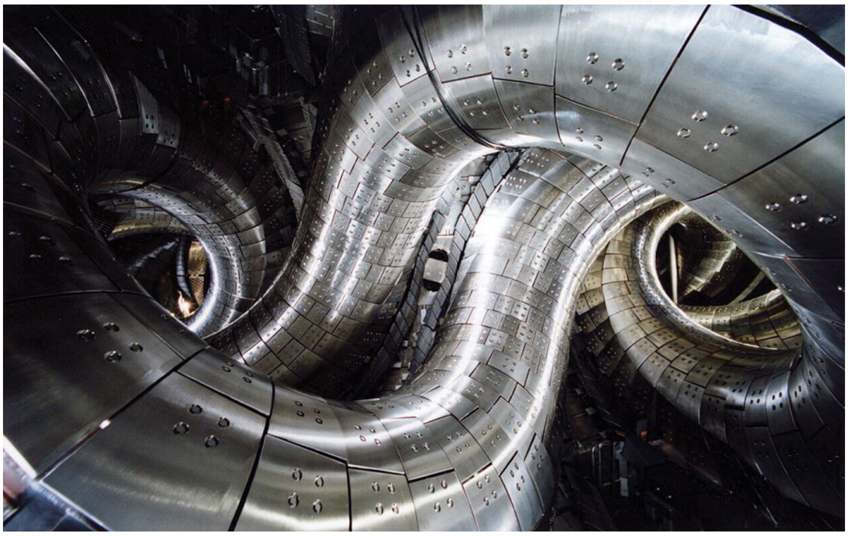


Figure 3.9: Interior of the Large Helical Device (LHD) in Japan. (source: NIFS, Japan)

The Japanese 'Large Helical Device' does have locally wound, superconducting helical coils. The photograph shows the inside of this fantastic machine. This is not an 'optimised stellarator', but its performance is nonetheless very good. The drawback with respect to the optimised stellarator is that LHD can achieve fairly impressive parameters: high density, high temperature, good confinement—but it cannot have them at the same time. `http://www.lhd.nifs.ac.jp/en/`

3.5 When can we have fusion power?

3.5.1 A short history

1950s development of the concepts of magnetic confinement

1960s experiments with a great many different magnetic confinement concepts. This was an evolutionary phase, in which by 1968 the Russian invention 'tokamak' came up trumps and demonstrated achieved plasma temperatures in excess of 1 keV. In a tokamak, the plasma is confined in a torus. The confining field has two components: a strong component in toroidal direction (i.e. the long way around the torus) is generated by external coils. The much weaker field perpendicular to this - the poloidal field - is the result of a strong electrical current in the plasma. The resulting field is helical, and this is needed to ensure stable confinement. We'll come back to this extensively (See Ch. 5).

1970s many laboratories around the world build a tokamak. Initially the only heating mechanism was the Ohmic dissipation, due to the electrical current in the plasma. But as this current also generates the confining magnetic field, there was a coupling between heating and confinement, which made it difficult to assess the physics of the confinement. In this period, also the measurements were very global: average density and temperature could be determined, but very little resolution in time or space.

1980s the era of the large tokamaks JET (Europe), TFTR (USA) and JT60 (Japan). The plasma is heated by external heating methods, decoupling heating from confinement. The plasma temperature reaches the working range of the fusion reactor (>10 keV). Measuring techniques now allow the determination of density and temperature with ever better spatial and temporal resolution, as well as more sophisticated investigations of instabilities of the plasma and turbulence. The insight in the plasma physics is growing fast, also because the advent of fast computers brings theoretical models closer to the experiments through numerical modelling.

1990s till now After the development of external heating sources, now also active control methods are improving fast: local heating, local current drive, local fuelling, control of plasma rotation, etc. Computational models are becoming better and better and start to reach a level of predictive power. Very importantly, in the tokamaks JET and TFTR experiments with the Deuterium-Tritium mixtures were carried out (before, and in all other tokamaks, tritium could not be used, so that all experiments had to be done in deuterium only - with much lower fusion yield). Fusion power of 10 and 16 MW is achieved, in TFTR and JET, respectively. 'Scientific breakeven' is reached: the externally applied power needed to sustain the plasma temperature is equal (or very close) to the fusion power generated. Note that this is net input power, no conversion losses are counted in. And to be precise here: the best performance was achieved in deuterium-only plasmas, and 'fusion power' is defined as the DT-equivalent power. The record performance experiments were done in 1997. Since then, the programme moved to the next critical item, i.e. active control of turbulence (advanced tokamak operation) and the interaction of the plasma with the wall.
In the past decade important progress has been made in the understanding of turbulent transport, the instabilities that occur in the edge of a reactor plasma, and in plasma-wall interaction. As to the latter: whereas until around 2000 Carbon was the material of choice for the plasma-facing components, the present consensus is

that it is best to avoid carbon and stick with tungsten. Besides all of that, a big effort went into the definition and start of the ITER-project.

3.5.2 Where are we today, and where do we need to go?

JET is the largest tokamak and has generated 16 MW of fusion power during a few seconds. To do that, some 25 MW of input power were needed. So, JET does not provide net power multiplication, but this was never the aim. We know that we can increase the power multiplication of the reactor by scaling it up: fusion power scales as the volume, the power losses as the linear dimension. Hence we need to build a larger tokamak, and this is ITER. The figures show the very fast historic improvement of the performance of fusion reactors, and the road map towards fusion electricity.

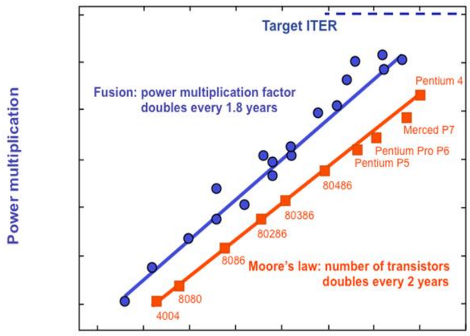


Figure 3.10: The net power multiplication, i.e. fusion power divided by the power needed to run the reactor, is a good measure of the performance of a fusion reactor. This has shown an historic growth faster than Moore's law! However, the next point on the graph has to come from ITER, and will only be realised in 2035-2040.

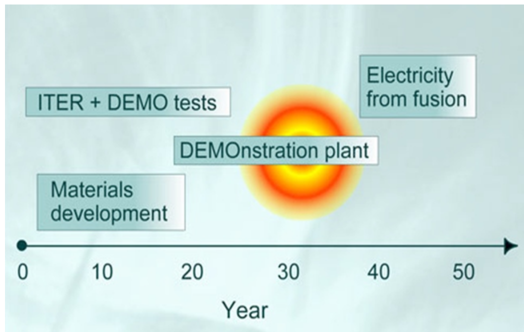


Figure 3.11: The roadmap of fusion energy: ITER is the proof of principle, materials and other technology must be developed in parallel to ITER, and the two lines feed into the design and construction of DEMO. It is thought that 3 or 4 countries will build DEMO reactors independently. After the DEMOs come the first and subsequent generations of commercial plants. (source: EUROfusion)

3.5.3 What's the programme?

First, we need to build a test reactor that produces much more power than it consumes. This is ITER, the worldwide joint experiment in fusion. ITER must demonstrate 10-fold power multiplication at the 0.5 GW level. ITER is under construction in Cadarache, in the south of France. It is roughly twice as big (linear) as JET and the construction is estimated at 20 Billion Euro (price level 2018). The construction takes 15 years, ITER should be operational in 2026. The real science programme lies in the period 2030–2045, with the first deuterium-tritium experiments presently planned for 2037.
Is that it? No, ITER is a scientific proof-of-principle, demonstrating the soundness of the scientific basis of the design, and a test bed for reactor technologies. After ITER we need to build DEMO, a real demonstration power plant. DEMO should be somewhat larger than ITER, but in many respects simpler (because it is not an experiment any more). The expectation is that if ITER is successful, several countries will build a DEMO:

the DEMO generation could be fully operational (after the d-t results of ITER, the DEMO designs can be finalised, the construction can be started, and after completion of construction a few years of commissioning are needed) by 2055-2060 at the earliest. And shortly after DEMO, the construction of the first generation of commercial reactors can be launched.

If all is planned well and no obstacles of technological or political nature are encountered, DEMO can deliver electricity to the grid in 40 years. It is the expectation that if ITER lives up to its promise, there will be several DEMO's built in parallel. In fact, the road maps of China, S-Korea and Europe all aim at electricity generation with fusion by 2060, with a first generation of commercial fusion plants operational in 2070.

Is that enough, then? No, one generation of commercial plants is not enough. Remember that by that time the world power demand will have grown to some 30 to 40 TW. Counting with a unit size of 1 GW (electric) for a fusion plant, one must think of 1000 fusion reactors to even contribute a few percent of the total demand, and ten times that to cover a substantial fraction. To build these will take tens of years: as with other sources, after the initial exponential growth - which takes several decades - the real contribution to power generation comes during the linear growth phase - which takes another 50 years.

3.5.4 The European fusion R&D strategy

Fusion research in Europe is carried out in national laboratories, coordinated by Euratom (Brussels). The strategy in highly condensed form is as follows:

1. build ITER (top priority)
2. but keep an activity in the development of concept improvements, and
3. develop in parallel to ITER the technology and materials needed for DEMO.

The development of materials for DEMO and reactors beyond DEMO is particularly important, as these devices will generate unprecedented neutron fluence. Therefore materials must be tested on their ability to survive in the neutron environment. For these tests the International Fusion Materials Irradiation Facility (IFMIF) is planned, which will provide a high flux of 14.1 MeV neutrons, similar to the fusion reactor. The planning is to build IFMIF in parallel to ITER. IFMIF itself is a large device, too, as is illustrated in Figure 3.12. Check out ifmif.org if you are interested in the latest developments.

IFMIF will provide the ultimate tests, but the material development is an ongoing activity. In the Netherlands, the NRG-group in PETTEN (`www.nrg-nl.com`) is involved in these studies, using the High Flux Reactor as a neutron source (with lower energy).

In fact, Europe has a much more detailed fusion roadmap, called 'Fusion Electricity, A roadmap to the realisation of fusion energy'. This document consists of a highly readable 60-page summary, supplemented by technical annexes. The first version was published in 2013, while an updated edition saw the light in 2018, with the more accurate title 'European Research Roadmap to the Realisation of Fusion Energy'. I strongly recommend you to read the summary document, which can be downloaded on the euro-fusion.org site. The pdf is also in the 3MF100 folder at CANVAS. After you have studied the course material, you should be in an excellent position to understand and appreciate this document. I find the updated title more apt because it indeed a 'research roadmap': it describes the road to the first demonstrator. For the next phase, the deployment of fusion power, there is no roadmap.

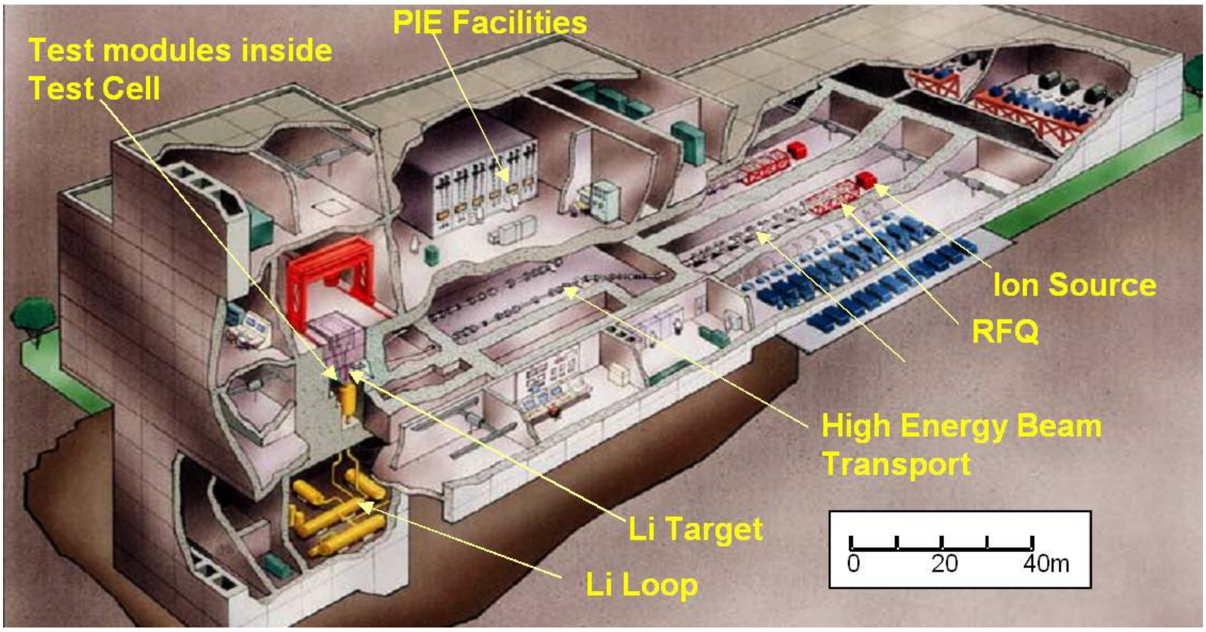


Figure 3.12: Artists impression of the International Fusion Materials Irradiation Facility IFMIF. The building is filled mainly with the deuterium beams, that hit the liquid lithium jet on the left-hand side of the picture. There you'll find the test cell where samples can be irradiated. Not very well visible in this picture: the volume in which the high neutron fluxes are reached is only about 0.5 liter! (source: IFMIF Design Report 2003)

Problems

3.1 Lawson criterion and basics of confinement

(a) Derive the Lawson criterion. (only the proportionalities, no constants asked in the derivation). Comment on the role of the efficiency of the conversion of fusion power to electricity.

(b) The constant in the Lawson criterion has the dimension [Bar.s]. What is the typical value for the pressure (nT) and τ_E aimed for in a tokamak reactor like ITER.

(c) The 'magnetic pressure' is expressed by $p_{\text{magnetic}} = B^2/2\mu_0$, where B is the magnetic field in Tesla and $\mu_0 = 4\pi 10^{-7} NA^{-2}$ is the magnetic permeability in vacuum, giving p in Nm^{-2}. How large is the magnetic pressure of a 5 Tesla magnetic field?

(d) Which structure of the tokamak reactor contains this pressure?

(e) Estimate the pressure in the vessel of a fusion reactor before we even start filling it with the deuterium and tritium gas out of which the plasma is formed. Give a value (order of magnitude) + a reasoning. (yes, you can work this out with very little prior knowledge)

(f) Calculate the central pressure in a fusion reactor of the size of ITER for central $T = 10keV$ and $n = 10^{20} m^{-3}$. Then estimate energy content of the plasma in a fusion reactor for the above conditions. Compare to: a hot bath or a Mars bar (look up the energy content on the wrapper). Make some reasonable assumptions on the dimensions of the reactor.

(g) Calculate the energy confinement time for ITER, if it is operated with 70 MW input power and has plasma conditions as above.

(h) As we shall see later, there is a limit to the electron density in fusion reactor. Suppose (this is very reasonable) that a reactor runs close to that limit. Compare the generated fusion power for a plasma with pure d-t mixture, and one with 3% carbon impurity, in otherwise identical conditions.

(i) Further to this: the reaction produces helium, so it is unavoidable that the ideal d-t fuel mixture will be diluted by helium. How much reduction of the fusion power would result from a 10% helium content in the plasma? What does this mean for the burn-up fraction of the exhaust gas? And what are the consequences for the fuel cycle?

3.2 Historical context: the reactions that power the sun

The sun does not in fact use the deuterium-tritium reaction that we envisage for a fusion reactor, but a rather complex series of reactions known as the 'proton-proton cycle'.

(a) Write down the 5 steps of the proton-proton cycle, or draw a diagram

(b) The names of Arthur Eddington and Hans Bethe are associated with the discovery of fusion as the energy source of the stars. What were their contributions

(c) Before fusion was proposed as the energy source of the stars, what could people possibly have thought of as the stellar power source?

(d) Lord Kelvin, in particular, put forward hypotheses concerning the power source of the sun. It brought him into a long and bitter conflict with Charles Darwin. What was the conflict about?

(e) Yet another famous name that is often associated with the energy released in fusion reactions: Albert Einstein. His famous formula $E = mc^2$ would explain how the mass difference before and after the reaction is converted into energy. Fair enough, but how about a chemical reaction, such as the oxidation of methane? Is there a mass deficit, too? In other words, is the mass of the reactants larger than that of the reaction products in an exothermic reaction?

3.3 A fusion reactor fuelled by Helium-3 from the moon?

Refer to the graph of the reaction rate as function of the plasma temperature to answer the following questions (and if you are not familiar with reaction cross-sections, make sure you read up on this concept!):

(a) Express the fusion power density in terms of the reaction rate $<\sigma v>$ and the density of the reacting particles n (assume the reactants have the same density).

(b) At which burn temperature is P_{fusion} maximal, for the d-t reaction, for given pressure?

(c) Now we use the same reactor to burn ^{3}He: d + ^{3}He $\rightarrow ^4$He + proton + 18.4 MeV. At which temperature is P_{fusion} maximal now?

(d) What is the ratio of the maximum P_{fusion} for d-t and d-^{3}He (same reactor, same pressure, but each burning at its optimum temperature?)

(e) What do you tell the government: go get ^{3}He from the moon?

3.4 Alternative fusion concepts: Beam on target concepts

You can make fusion reactions by shooting a beam of deuterium ions on a tritium target. The problem is that the probability of an elastic scattering event is much larger than that of a fusion reaction. Scattering leads to heating of the target, energy that is mostly lost. The graph below shows the cross sections for scattering and fusion, for deuterium incident on a tritium target.

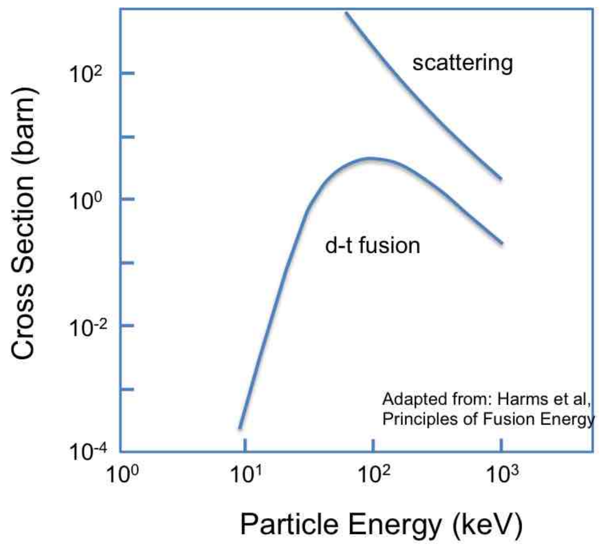


Figure 3.13: The cross section for the scattering of d on t is always much larger than that for fusion

(a) Estimate fusion-power versus losses as function of beam energy.

(b) This estimate is still very optimistic. Why? (which other process(es) were neglected?)

(c) What does this mean for a viable fusion energy-generating scheme?

3.5 Alternative fusion concepts: The fission-fusion hybrid scheme

In the so-called Fission-Fusion hybrid scheme, a neutron from a d-t fusion reaction is used to generate fuel (^{239}Pu) for a fission reactor.

(a) How much energy gain does this process yield? (this needs some googling)

(b) Fusion makes many neutrons per MJ of produced energy (compared to fission). What would make the better business case: selling neutrons (e.g. to the owner of a fission reactor, or for the production of isotopes for medical purposes, or for material research) or selling energy? (in your considerations, take into account — apart from an obvious estimate of the market price of neutrons and joules — the different demands on operation of the device.)

(c) So what do you conclude: does it make sense to have a commercial neutron production programme based on fusion, and let the energy application grow out of that?

3.6 Alternative fusion concepts: Muon-catalysed fusion

The muon-catalysed fusion scheme fails because it costs so much energy (5 GeV or more) to produce a muon. But we can get muons for free from the sky!

(a) if we could use the cosmic muon flux that reaches the earth, how much power per m^2 could we get from that if we use the muons to catalyse d-t fusion? (Assume a realistic number of catalysed reactions per muon.)

(b) Compare this to the $power/m^2$ from a solar plant.

(c) Conclusion: is the muon flux that impacts on the earth a viable potential energy source?

3.7 Alternative fusion concepts: Inertial Confinement fusion

In inertial confinement fusion an energetic laser pulse is used to compress a small amount of fusion fuel. The Lawson criterion is satisfied by the combination of a very high pressure and a very short confinement time. Check out on the internet (national ignition facility):

(a) what are typical numbers for the pressure and confinement time in an inertial fusion experiment? Compare to magnetic confinement fusion (MCF).

(b) how much energy does a single laser pulse of NIF carry (maximum)?

(c) What is the energy efficiency of the NIF-laser: how much energy into the laser building for 1 Joule of laser power at the exit?

(d) What is the coupling efficiency of the laser energy to the pellet?

(e) If ignition is achieved in a NIF inertial fusion experiment, how much fusion energy is released? Compare to a hand grenade (to get an impression of the sort of explosion we are dealing with). And to the energy in a Mars bar (any piece of chocolate).

(f) Explain how the 'hohlraum' is used to improve the homogeneity of the energy distribution over the surface of the target

3.8 Break-even, ignition and commercial operation

n a steady state fusion reactor, such as ITER, 'break-even' is achieved when the total fusion power generated in the reactor equals the power fed into the plasma in order to sustain it. The ratio (fusion power/external power) is called Q ('capital Q'). So for break-even Q=1.

(a) Explain why Q must be significant larger than 1 to achieve 'ignition'. (i.e. the plasma can sustain itself)

(b) In ITER the target is to achieve Q=10. Explain why Q must be another factor larger in a commercial power plant. And if this extra factor is needed, why is ITER still a meaningful experiment?

(c) In inertial fusion, because of the pulsed nature of the experiment, 'ignition' has a somewhat different meaning than in a quasi steady-state experiment. Comment on the relation between 'break even' and 'ignition' in the tokamak (steady state) versus inertial fusion. (also consider the efficiency of the laser)

4 Outstanding questions and challenges in fusion research

4.1 Can we make an energy-producing, ‘burning’ plasma?

As we have seen, in the magnetic confinement programme we have mastered the stable confinement of a hot plasma in a torus, either a tokamak or a stellarator. The required burn temperature of 15 keV is achieved routinely in the large tokamaks, even a much higher temperature is no problem. Also the density can be brought to the required level. Hence, with the Lawson criterion in mind, you might say that the only step that remains to be made is the achievement of a sufficiently long confinement time. But here, the good news is that scaling up always helps. And therefore there is little scientific doubt that we can create a self-sustaining ’burning’ plasma.

So what are the challenges? In fact, at this point you may want to read the European Fusion Roadmap document, as it spells out rather nicely the eight ’missions’ of the fusion programme: issues that stand between us and a power producing fusion reactor. But below you find a slightly different take on the same material.

4.2 Can we improve the efficiency of the reactor? Understanding and control of the turbulent transport processes in the plasma

In the scaling of the reactor - what is the minimum size at which the plasma can burn - heat conduction plays a determining role. The better the thermal insulation of the plasma, the less power is needed to keep it at the high burn temperature and the smaller the reactor can be.

What determines the heat conductivity? The theoretical limit is given by the random particle motion and collisions. This limit is easily calculated. Experimentally, the thermal transport is found to be much higher than this theoretical minimum, the additional transport being due to turbulence.

In the 1990’s experimental methods were developed that reduce the turbulent transport to a level close to the theoretical minimum, at least for the ions. The lighter electrons, that are much more effectively confined by the magnetic field, remain ’anomalous’ — just the technical term for ’we haven’t quite figured out what is going on here’. But even so their contribution to heat loss is only comparable to that of the ions. The turbulent transport is an active field of interesting physics research. The plasma is strongly nonlinear, it shows bifurcations and self-organisation, etc. It is the challenge for the scientist to understand the nonlinearities and use them to steer the plasma to optimal performance.

4.3 Exhaust of particles and energy: plasma wall interaction

The envisaged fusion reactor will typically produce 1–2 GW electrical power, i.e. 3–6 GW thermal power. Of this, 80% is produced in the form of energetic neutrons that need to be slowed down in a ’blanket’, where their energy is transferred to heat and finally to electricity. The blanket technology is under development and will be tested for the first time in ITER. The technology is very challenging — we’ll get to that.

However, 20% of the fusion power is released in the form of energetic alpha particles, and these are confined by the magnetic field. Just as well, because their energy is needed to sustain the high plasma temperature. The power carried by the alpha particles must be transferred to the thermal plasma by collisions, but eventually also this power flux will reach the wall, where it is removed through cooling and finally converted into electricity. This heat flux is channelled to a relatively small area of the reactor, the 'divertor', by the magnetic field structure, and here the heat flux will be rather large. In ITER, the exposed surfaces are typically expected to get a 10 MW/m^2 heat load. On top of that, the heat flux is carried by the extremely corrosive hydrogen radicals, and all of this combined with the large neutron flux. In those circumstances the wall material must not melt or erode: quite a challenge. Of the entire periodic table of elements, only Be (beryllium) and W (tungsten) appear to be usable. These are the materials that will be employed in ITER.

The good news for the researcher is that in the plasma surface interaction zone, just in front of the surface, very interesting physics and chemistry is found. The interaction of the plasma with the wall, etching and erosion, redeposition of eroded material, formation of a variety of unusual molecules and radicals: fascinating science, that moreover cross-links to industrial plasma physics research (industrial spin-off). Also in astrophysical objects the conditions can be similar to those in the fusion reactor. Especially the formation of dust, as a precursor to planet formation, is an example.

4.4 Disruptions and 'ELMs' (Edge Localised Modes)

Of the various types of fusion reactors, the tokamak is furthest developed. There are promising alternatives, such as the stellarator, but these lag behind by 1–2 decades. However, the tokamak has a principal drawback: it needs a large electrical current in the plasma. Two problems arise: first, the current must be sustained (see below) and second, the current represents a significant amount of free, magnetic, energy. To be more specific on the latter problem: if an event occurs that cools the plasma (loss of confinement, enhanced turbulence,$\cdots$), the plasma conductivity drops and through Lenz's law high electric fields are generated. These, in turn, will drive currents in unwanted spots, such as the vacuum vessel, or beams of relativistic electrons, which can lead to structural damage to the tokamak. The short of it is that we cannot tolerate disruptions in a reactor.

The physics of such a disruption is rather fascinating. It involves exponential growth of instabilities in the plasma, formation of chaotic magnetic field with strongly enhanced thermal conduction, etc. But we need to understand and predict the occurrence of disruptions, and learn to avoid or at least mitigate them. Fast measuring systems, real-time control systems, and mitigation techniques such as the rapid injection of gas are the key technologies in this field.

A second persisting difficulty of the tokamak is the Edge Localised Mode (ELM). The best performance of a tokamak is achieved when it is operated in the so-called H-mode regime (H for High confinement). This is characterised by a very steep gradient of the plasma pressure in a thin layer - a few cm - close to the edge of the plasma. But this pressure gradient is not stable. It builds up gradually and then collapses in a very short time (0.1 ms or so). This leads to a thermal energy release to the wall that is a severe problem, because it causes melting and/or erosion of the wall material. Like disruptions, ELMs cannot be tolerated and must therefore be suppressed. Recently it has been shown that magnetic perturbation fields can achieve this, but it has to be demonstrated in ITER that such methods also work in reactor-sized machines.

4.5 Steady-state operation: requires non-inductive current drive

As said, the magnetic confinement in a tokamak is based on an electrical current that runs in toroidal sense in the plasma ring. In a full-scale reactor, this current is typically 25 MA. This current must be driven, and clearly electrodes cannot be used to do that. The most obvious way to drive this current is induction: use the plasma ring as the secondary winding of a transformer. However, transformers work for AC (alternating current) only, as the principle they use is Lenz's law: the time derivative of the magnetic flux induces an electric field in the ring. Steady-state operation means: no time derivatives, hence no induction. Machines like JET or ITER are extremely large transformers, so although not steady state, their plasma pulses have a duration of 1 minute and more than 15 minutes, respectively.
What other methods for current drive do we have? First: as we will see later, the torus geometry leads to a special, rather non-trivial effect called 'bootstrap current'. This is a current in the toroidal direction that results from a pressure gradient in radial direction. This is not a small exotic effect, if set up properly, the bootstrap current can make out tens of percents of the total current required. For free!
Nonetheless, we need to be able to drive the missing fraction. There are several very smart physics tricks to do this, either by using electromagnetic waves to manipulate the velocity space of the electrons, or by shining in high-energy neutral hydrogen beams. Interesting physics, advanced technology and a field of expertise of the Dutch fusion physics programme. It should be said, though, that these techniques are rather costly in terms of Amperes per Watt of power, and that is a serious drawback for a power plant. With the efficiencies achieved today the plant could consume half the electric power it produces for the current drive alone. Clearly an area that needs further development.
Alternatively, it could be considered to use the reactor in a quasi steady-state mode, i.e. long pulses (think hours) interrupted by short recharging breaks (minutes). The generated power, which results from a thermal process with a long time constant, could perhaps still be continuous.

4.6 Optimisation and control of the 'burning' plasma

ITER will demonstrate that a fusion reactor can be built that can generate 10-fold power multiplication at the 500 MW level. Such a plasma basically heats itself and is therefore called a burning plasma. Just what we want. The flip-side of this coin is that without external heating, most of the means of the operator to influence what goes on inside the reactor are lost. Yet we still need to manipulate the plasma. If not for safety (the general tendency of the plasma is to extinguish, the aim of the operator is rather to make the fusion reaction go as fast as is possible), then in any case to optimise the burn. This requires e.g. control of the turbulence that governs the heat and particle transport. Among the few tools that are available for this task is the use of narrow, focused beams of high power microwaves that are locally (resonantly) absorbed in the plasma. These can locally drive current, at precisely controlled positions, and in this way turbulence can be influenced. It so happens that on the topic of 'control', a very fruitful collaboration between the DIFFER institute and the 'control systems technology' group of the TU/e has grown.

4.7 Technology

Of course, the fusion reactor calls for the development of new technologies. Key examples are:

Materials for the 'first wall'. The first wall, as seen from the plasma, has to deal with the neutron flux that is produced in the fusion process. (After the first wall the neutrons are stopped in the so-called blanket, their energy carried off by cooling water, but something has to separate the blanket from the plasma). This neutron flux causes swelling and defects of the material. Steels are being developed that can handle the extreme neutron flux, but this is still a very large challenge. Apart from damaging the materials, the neutrons will also create radioactive isotopes of the wall material. By choosing the right materials and avoiding even traces of elements that form long-lived radioactive waste, the first wall material can lose its radioactivity in 50 - 100 years after activation.

Remote handling. Inside the reactor the activation levels are so high that installation and repair work must be performed with remote handling. Of course, the whole device is constructed to allow this. Already in JET most work can be carried out by robots, and ITER will be fully remote handleable. But this is modern technology that needs further development. A lot of expertise can be borrowed from industrial applications of robots, but this must be carried over and adapted to the very special conditions in a fusion reactor. At the same time, it is fair to say that fusion has been a driver of the development of this technology.

Superconducting magnetic field coils. In a device like ITER, with a permanent magnetic field of $>$ 5 T, the magnetic coils have to be superconducting, there is no alternative. The ITER coils are among the largest superconducting coils ever made, and there are many technological issues associated with them. Mostly connected with the application of superconductors in a very high ambient field, which leads to high stresses on the material. You should also realise that, because of the superconducting coils, ITER is like an extremely large cryostat, at 4 K or so. But inside this huge cryostat is a 500 MW heat source! For ITER this technological challenge is well in hand. But it will certainly be a breakthrough for fusion if high-T_c superconductors become available for the high field applications in fusion reactors. So fast is this technology developing at the moment, that whereas a decade ago high-T_c superconductors were hardly commercially available, as of 2021 several privately funded fusion companies base their business case upon them. The MIT spin-off CFT has demonstrated their prototype 20T high-T_c magnet in summer 2021 - an important milestone - and the UK-based company Tokamak Energy explores the application of such magnet technologies to 'spherical' tokamaks. So, this development is really taking off.

Fuel Cycle. The fusion reactor uses tritium, but this is not stable (half-life 12.3 year) and therefore not found in nature. The raw fuel is lithium. In special modules placed directly around the hot plasma in the reactor, the 'blanket', lithium reacts with the neutrons produced in the d-t fusion reaction, and this reaction produces a new tritium. In principle every neutron produced in the fusion reactions should therefore 'breed' a new tritium, and obviously one needs a breeding ratio above 1 because some neutrons will be lost without reacting with lithium. The solution of this puzzle lies in the multiplication of neutrons, e.g. with beryllium. But the technology to achieve an effective breeding ratio above 1 and thus realise a reliable fuel cycle is still young and has never been tested in realistic conditions, simply because these were not to be found on earth. ITER will be the first real test ground for the 'test blanket modules'.

4.8 Complexity and reliability

Supposing that we manage to solve all of the above issues, will that clear the road to commercial fusion power? Consider this. ITER has been touted as the 'most complex device ever built by man'. More than 10 million parts, more than an aircraft carrier. That sounds cool, it's an engineer's dream. But it is not good for the viability of fusion as an energy source. We should have much preferred ITER to be the 'simplest device ever constructed by man'! Because complexity is the enemy of reliability. And a fusion power plant needs to be available 24/7, 365 days of the year. To compute the availability, you have to multiply the probability of failure and the time to repair. I invite you to give some thought to this.

4.9 Politics

An international project like ITER represents a unique, unprecedented, international collaboration. In fact, the budget of ITER ought to be too small for the project to appear in the agendas of high-level politicians. But ITER has a high prestige factor. Therefore it does feature high on political agendas, it does have a very high visibility. And this is good, because it creates the indispensable support at the political level. But for a technical project there are also drawbacks associated with this. In fact, the ITER project was delayed by at least 10 years for non-technical reasons, having to do with site location, negotiations (first who had to pay how much, later who was allowed to contribute how much). And even now that the project has been started, the more important decisions - especially those that impact the budget of the project - have to go to the political level which can mean months of delay in the decision process.
The parties in the ITER International Organisation are Europe, Japan, the USA, China, South-Korea, India and the Russian Federation. More than half of the world population is associated with this scientific experiment! Europe is the leading party, hosting the device and signing for almost half of the cost. The ITER team is very international, as you might expect. Cultural differences, differences in management style and ways of coming to decisions are sometimes difficult and need to be handled very carefully. But at the same time, the huge variety of cultural backgrounds in team joined by a common goal, creates a very special, stimulating atmosphere and is a real enrichment that working in fusion offers. And the fact that these countries work together in this huge and prestigious project does in fact have great political significance. ITER is often rightfully called a 'peace project'.

Problems

4.1 Challenges
Before fusion can be rolled out as energy source several scientific and/or technical issues need to be resolved.

(a) List at least 4, essentially different, major issues that need to be resolved before we can have commercial fusion power

(b) Of each of these, explain why it is an issue.

(c) Of each problem, sketch the possible solution routes that are being considered and/or pursued (if there are any). And possibly your own solution, too.

(d) Discuss, for each issue, if it is generic for fusion devices or specific to e.g. the tokamak. Consider Stellarators, Inertial fusion and any other scheme you can find on the internet.

(e) Which of the issues must be addressed in ITER, which can be dealt with in other fusion devices, which don't need a fusion device such as ITER at all to be sorted out, and which need very specific, presently not existing facilities?

(f) Apart from the technical and scientific difficulties, there may be other issues that could slow down or hinder the development of fusion power. Mention at least 2.

(g) Of each of those, explain what the problem is, and a possible way to deal with it.

(h) Which, of this multitude of important and urgent problems, would you like to work on if you could do research in fusion?

4.2 Spin-off

Of the outstanding issues, some are not specific to fusion. Conversely, some of the solutions of issues encountered in fusion research may find application elsewhere, either as a scientific or technological advance or as commercial application.

(a) Discuss issues which the development of fusion shares with other fields.

(b) Discuss examples of scientific advance or technological development that was spurred by fusion but found application elsewhere in science or technology

(c) Discuss examples of commercial spin-outs of fusion research, and how they relate to fusion research and development.

5 Basics 1: Trajectories of charged particles in a magnetic field.

5.1 Preface

In these lectures we will mostly be concerned with fusion concepts that use 'magnetic confinement' to achieve the Lawson criterion. And since the fusion fuel will be heated to 100 Million K or more, all particles will be charged, i.e. ions or electrons. It is therefore fundamental to have a good understanding of the behaviour of charged particles in a magnetic field. Now, it makes a big difference whether we consider individual particles or lots of particles together. In the latter case, we enter the realm of plasma physics, which is characterised by all sorts of collective behaviour of particles and the dynamics coming forth from that. This is basically the result of the interaction between the particles having a long range - the Coulomb interaction. But before we enter into that adventure, let us look at the wondrous world of individual charged particles in curved, inhomogeneous magnetic fields. This we can easily calculate. And whatever collective effects the plasma physics is going to add, the fundamental motion of charged particles in a magnetic field will always be there. So, in this chapter there is only one force, the Lorentz force $\mathbf{F}_{\text{Lorentz}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, with q the charge of the particle, $\mathbf{v}$ its velocity and $\mathbf{E}$ and $\mathbf{B}$ the electric and magnetic fields, respectively.
Before we continue: anyone dealing with plasmas will need a basic ability to handle vectors, vector products and vector relations. You'll need to work out in which direction the Lorentz force works and Maxwell's equations are fundamental to any calculation involving electric and magnetic fields. For some this is well-known territory, other may need to fresh up their vectorial talents. In the text box we summarised the basic formulas - those are really all you need. Just make sure you are able to do the formal manipulations. The problems will ask you to figure out how Maxwell's laws in combination with Gauss's or Stokes' theorems are used to devise instruments that measure electric and magnetic fields.

Toolbox: vectors calculus and Maxwell's laws

5.1.1 vector relations

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) \tag{5.1}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C} \tag{5.2}$$

$$\nabla\cdot(\phi\mathbf{A}) = \phi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla\phi) \tag{5.3}$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \tag{5.4}$$

$$\nabla\times(\nabla\phi) = 0 \tag{5.5}$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0 \tag{5.6}$$

(The bold face symbols are vectors, ∇ is the nabla operator and the italic unbold symbol ϕ is a scalar).

5.1.2 Gauss's and Stokes' theorems

$$\text{Stokes}'\text{ theorem} : \int(\nabla\times\mathbf{A})\cdot dS = \int\mathbf{A}\cdot d\mathbf{l} \tag{5.7}$$

In words: the integral of the curl of vector **A** over a surface equals the integral of **A** itself along the contour of that surface.

$$\text{Gauss}'\text{s theorem} : \int\nabla\cdot\mathbf{A}\,d\tau = \int\mathbf{A}\cdot d\mathbf{S} \tag{5.8}$$

In words: the integral of the divergence of vector **A** over a volume equals the integral of **A** itself over the surface of that volume. Both theorems are extremely handy when you are working with Maxwell's equations (see below).

5.1.3 Maxwell's equations in vacuum

$$\epsilon_0\nabla\cdot\mathbf{E} = \sigma \tag{5.9}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{5.10}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{5.11}$$

$$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{j} + \epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right) \tag{5.12}$$

in which σ is the charge density, and **j** denotes the current density. Equation 5.10 implies that **B** is divergence free: no magnetic monopoles.

5.2 Gyration

A charged particle moving in a magnetic field gyrates. If the field is homogeneous (field lines straight and parallel and equally spaced) the particle gyrates around a field line due to the Lorentz force, while the motion along the field line is not affected at all by the field. The gyration is also called the cyclotron motion - hence the subscript c in the following. The frequency (ω_{ce}) and radius (ρ_{ce}) follow from the Lorentz force, the velocity and mass of the particle according to Newton's laws (classical mechanics). So we equate the centripetal force $F = mv^2/r$ to the Lorentz force. For an electron (subscript e) this works out as follows, where we introduce the notation $\mathbf{v}_\perp$ for the velocity perpendicular to $\mathbf{B}$:

$$F = \frac{m_e v_\perp^2}{\rho_{ce}} = e v_\perp B \tag{5.13}$$

Combine with the trivial relation between angular velocity and orbit velocity to find

Electron cyclotron frequency

$$\omega_{ce} = \frac{eB}{m_e} \tag{5.14}$$

Electron cyclotron radius

$$\rho_{ce} = \frac{m_e v_\perp}{eB} \tag{5.15}$$

In practical units, this results in

$$f_{ce} = \frac{\omega_{ce}}{2\pi} = 28 \times B[T] \text{ GHz} \tag{5.16}$$

or '28 GHz per Tesla', i.e. typically 50–150 GHz in a fusion plasma. These are mm-waves, with typical wavelength of a few mm. For our typical fusion plasma, the electron cyclotron frequency lies in the same range as the plasma frequency (see Chapter 6). An electron with $v_\perp = 10^7$ m/s has a gyration radius of about 0.1 mm.

The electron gyration is very fundamental to magnetically confined plasmas. Since the frequency depends only on the magnetic field, it is a strong resonance. The plasma emits radiation at the electron cyclotron frequency (the so-called Electron Cyclotron Emission — ECE) which provides an excellent and much-used possibility to perform spectroscopic measurements and obtain information on the inside to the hot plasma. Likewise, it is possible to heat the plasma by injecting high power mm-waves at a frequency that is resonant with the electron gyration or its harmonics at some point in the plasma.

5.3 E $\times$ B-drift ('E cross B drift')

A plasma placed in crossed **E** and **B** fields will drift in the direction perpendicular to both fields. This is visualised in figure 5.1. The **E**-field accelerates the particle during one half of its gyration and decelerates it on the way back. In the picture, the velocity is highest at the lowest point and lowest at the top. The gyro-radius is proportional to the velocity, hence varies during a gyration period, resulting in the drift as shown.

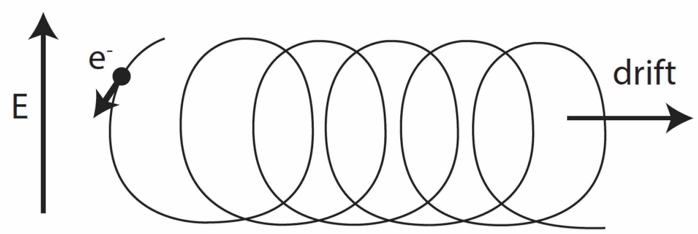


Figure 5.1: Electron orbit in crossed **E** *and* **B** *fields. The magnetic field points towards the viewer. Note: the motion is in the plane of the paper; this is not a perspective drawing of a helical orbit.*

The drift velocity is proportional to the velocity difference between top and bottom of the gyro motion, which must be proportional to the acceleration times the time of acceleration:

$$v_{E\times\mathbf{B}} \propto \Delta t \cdot a \propto \frac{1}{\omega_{ce}} \frac{eE}{m} = \frac{E}{B} \tag{5.17}$$

Here Δt denotes the time difference between top and bottom of the rotation, and a is the acceleration in the $\mathbf{E}$-field. Note that neither the mass nor the (sign of the) charge appears in this expression. This is easily understood. The picture is for a negative particle, but a positive particle drifts in the same direction: both the sense of the gyration and the direction of the acceleration change sign. Likewise, the mass drops out: a higher mass is accelerated more slowly, but there is more time because the gyration frequency is lower.
A mathematically more formal derivation is simple if you realise that the Lorentz force, averaged over a gyration period, cannot lead to an acceleration perpendicular to $\mathbf{B}$. Hence

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \tag{5.18}$$

Taking the cross product with $\mathbf{B}$ gives (use the vector relations given in the text box):

$$\mathbf{E} \times \mathbf{B} + (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} = \mathbf{E} \times \mathbf{B} - \mathbf{v}_{E\times B} B^2 = 0 \tag{5.19}$$

from which we find the expression for the 'E-cross-B' drift:

$E \times B$ **drift**

$$\mathbf{v}_{E\times B} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{5.20}$$

5.4 Grad-B drift

Magnetic fields are rarely homogeneous, and the magnetic fields in the toroidal devices in which fusion plasmas live are by definition not homogeneous. Such B-fields have gradients and curvature. Both lead to drift movements of the charged particles. Let us start by considering a gradient in the B-field, in the direction perpendicular to the B-field itself. Figure 5.2 sketches how in such a field—because the gyro-radius depends on B—the gyro radius of a particle varies over a gyro period, being smaller where the field is higher. The result is a drift perpendicular to the gradient and the B-field, quite similar to the $E \times B$ drift.

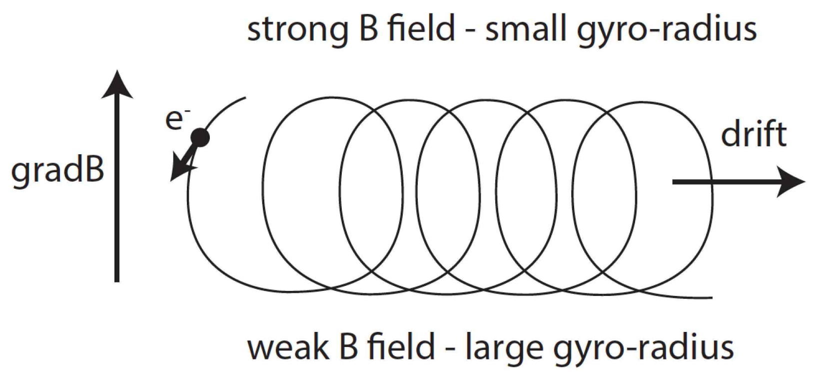


Figure 5.2: Electron orbit in a magnetic field gradient perpendicular to the field, which directed towards the viewer.

The magnitude of this drift movement can be estimated by a dimensional consideration. First, the drift velocity must be proportional to the only velocity in the problem, $\rho_{ce}\omega_{ce} = v_\perp$ (subscript e for the electrons, similar for other species). The dimensionless scaling parameter must be: the ratio between the gyro radius and the typical length over which the B-field changes, the so-called gradient length L_B. This ratio is a measure of the variation of the field seen by the electron during its gyration. From these consideration we find

$$v_{\nabla B} \propto \rho_{ce}\omega_{ce}\frac{\rho_{ce}}{L_B} = v_\perp \frac{\rho_{ce}}{L_B} \tag{5.21}$$

with

$$L_B = \frac{B}{\nabla B} \tag{5.22}$$

The rigorous derivation yields practically the same result:

Gradient-B drift

$$\mathbf{v}_{\nabla B} = \frac{1}{2}\rho_{ce} v_\perp \frac{\mathbf{B} \times \nabla B}{B^2} \tag{5.23}$$

Note that in this case the drift is in opposite direction for ions and electrons. Thus, the grad-B drift leads to a charge separation, which leads to an **E**-field, which in turn leads to the $E \times B$ drift discussed above.
Note also that the $E \times B$ drift velocity depends on the velocity of the particle in the plane of the gradient of B, which in this case is perpendicular to **B**: $v_\perp$. This is obvious for the grad-B drift, but there is another drift—the curvature-B drift, discussed below—which depends on the parallel velocity rather than the perpendicular.

5.5 Curvature-B drift

If we put an electron on a field line and give it a velocity exactly parallel to that field line, clearly there is no Lorentz force. Now what happens if the field line makes a curve. What does the electron do? Will it follow the field line? But it cannot! Because, if it were to follow the field line, its velocity would be parallel to the field line at all times, therefore the Lorentz force should be zero at all times. But without force, the electron would go straight and therefore could not follow the field line!
Let us do the thought experiment and consider what happens when the electron arrives at the bend in the field line in some detail.

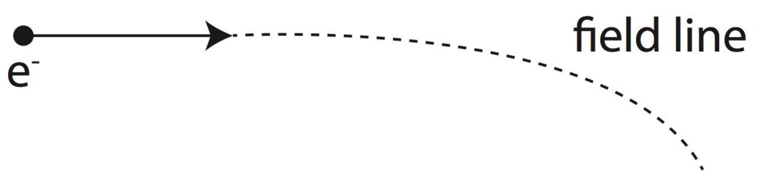


Figure 5.3: An electron following a straight section of a field line approaches a curved section.

1. Just where the field line starts to bend: there is still no Lorentz force, so the electron goes straight ahead. There simply is no force to pull it around the corner.

2. But as soon as the direction of the electron - going straight - and the field line start to deviate, there is a Lorentz force. However, this will NOT make the electron follow the field line. It is directed perpendicular to both the field and the instantaneous velocity of the electron. So it results in a motion perpendicular to the plane in which the curved field line lies. In the picture above that is: perpendicular to the page.

3. So, this velocity builds up. But this new velocity does produce a Lorentz force perpendicular to the field line along the radius of curvature. And this Lorentz force will make the particle follow the curvature of the field. It follows the curvature but it does not follow the field line! To follow the curvature of the field the particle must drift in the direction perpendicular to the curvature.

The value of this drift velocity v_{curvB} is easy to derive by equating the centripetal force (needed to let the electron make the same curve as the field):

$$F_{\text{centripetal}} = \frac{mv_{\parallel}^2}{R} \tag{5.24}$$

to the Lorentz force, realizing that v_{curvB} is the only velocity component perpendicular to $\mathbf{B}$:

$$F_{\text{Lorentz}} = ev_{\text{curvB}}B. \tag{5.25}$$

The result is:

$$v_{\text{curvB}} = \frac{mv_{\parallel}^2}{eRB}. \tag{5.26}$$

Expressed in a form that maintains the vector relations, this works out as follows:

Curvature-B drift

$$\mathbf{v}_{\text{curvB}} = \frac{mv_{\parallel}^2}{eRB}\frac{\mathbf{R}\times\mathbf{B}}{RB} \tag{5.27}$$

5.6 The Mirror-effect

5.6.1 Imagine you are a particle that moves into a region with increasing B field

A charged particle that gyrates around a field line while it is moving along the field line in the direction of increasing magnetic field, feels a Lorentz force that has a component in the direction opposite to its average velocity (i.e. its velocity along the field, or, its velocity averaged over a gyration period.). You may want to give this a moment's thought, because the Lorentz force is by definition always perpendicular to the instantaneous velocity. As a result, the parallel velocity of the particle becomes smaller, until that particle comes to a standstill and turns back, towards the lower field. All the time the gyration continues, always with the same rotation sense (obviously). This is the magnetic mirror effect, which occurs when there is a gradient of B parallel to $\mathbf{B}$ itself. Figure 5.4 shows how in such a configuration field lines are not parallel, so that there is a net Lorentz force pointing towards the lower field.

The mirror effect occurs thanks to the gyration. A particle that does not gyrate, i.e. with a velocity directed exactly along the field, does not feel any Lorentz force and hence cannot be reflected by the magnetic mirror. Clearly, how far a particle can penetrate in a high field region must depend on the ratio between its perpendicular and parallel field components. To calculate the point of reflection, you might think that you have to integrate the Lorentz force along the path of the particle, which would be a tough job. Fortunately, this is not necessary. There is a much simpler method, which makes use of a constant of motion that we will derive below: the magnetic moment.

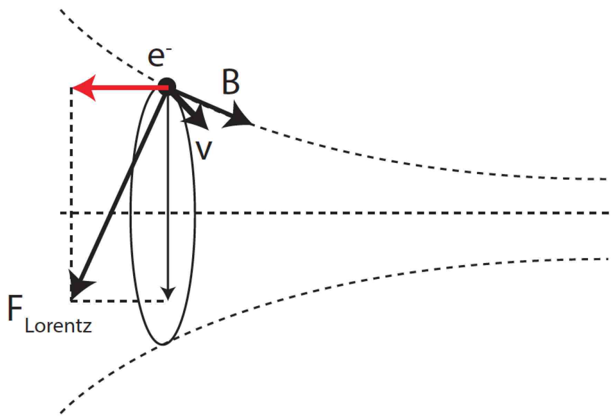


Figure 5.4: Mirror force due to a magnetic field gradient parallel to the field direction. The field is stronger to the right than to the left. Due to $\nabla \cdot \mathbf{B} = 0$, the field strength B is proportional to the density of field lines per unit area. Hence the field lines are not parallel, but converge to the right.

5.6.2 A very useful constant of motion: the magnetic moment.

The magnetic moment μ of a particle in a magnetic field is defined by

Magnetic moment (definition)

$$\mu = \frac{mv_\perp^2}{2B} \tag{5.28}$$

In an inhomogeneous magnetic field, the force exerted by the field on the magnetic moment is given by:

$$\mathbf{F} = -\mu \nabla B \tag{5.29}$$

Now, we combine the equation of motion ($F = ma$) with the conservation of energy of the particle (remember: the Lorentz force is perpendicular to the instantaneous velocity of the particle and therefore cannot change its kinetic energy).
Equation of motion:

$$F = ma \Rightarrow -\mu \nabla_{||} B = m\dot{v}_{||} \tag{5.30}$$

where $\nabla_{||} B$ is the gradient of the field along the field.
Multiply both sides of the equation by $v_{||}$ and notice that in the frame of the particle the gradient of B appears as a time derivative: $v_{||} \nabla_{||} B = dB/dt$.

$$\text{Equation of motion:} \quad \frac{1}{2} m \frac{d}{dt} v_{||}^2 = -\mu \frac{dB}{dt} \tag{5.31}$$

$$\text{Conservation of energy:} \quad \frac{1}{2} m \frac{d}{dt} v_\perp^2 + \frac{1}{2} m \frac{d}{dt} v_{||}^2 = 0 \tag{5.32}$$

$$\text{Combined:} \quad B \frac{d\mu}{dt} = 0 \tag{5.33}$$

(work this out for yourself!)
We see that μ does not vary if a charged particle moves in a **B** field with a gradient along the field. Hence μ is a constant of motion, also called 'adiabatic invariant'.

5.6.3 trapped and untrapped particles; the loss cone; the trapped particle fraction

Now it is easy to calculate which particles will be reflected by a magnetic mirror and which won't. Consider a field that varies between B_{low} and B_{high}. Particles that have parallel velocity $v_{\perp,low}$ in the region with B_{low}, will exactly come to a standstill at B_{high} if the following condition applies (check this!):

The Mirror ratio

$$\frac{v_{\parallel,\mathrm{low}}^2}{v_{\perp,\mathrm{low}}^2} = \frac{B_{\mathrm{high}} - B_{\mathrm{low}}}{B_{\mathrm{low}}} \tag{5.34}$$

In words: the velocity space is separated in two regions according to the mirror ratio. Particles in a cone (the 'loss cone') along the $v_\parallel$-axis can escape.

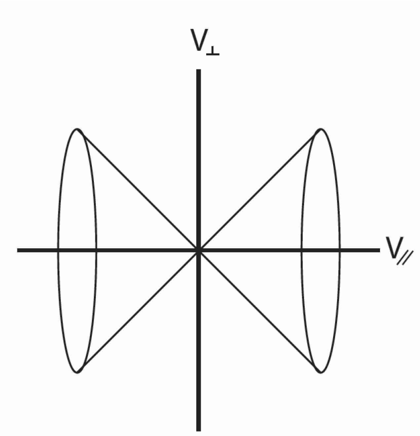


*Figure 5.5: Since charged particles gyrate $\perp$ **B**, the criterion for a particle to be trapped in a magnetic mirror, $v_\parallel/v_\perp <$ constant implies that in 3-dimensional velocity space (2 dimensions for $v_\perp$) the escaping particles are in a conical volume called 'loss cone'.*

The fraction of the particles that are trapped in the mirror is found by calculating the relative volume of the two loss cones, compared to the full 4π steradians. This calculation is fairly straightforward if we use a standard result from goniometry, which relates the solid angle Ω of a cone to its angle θ in the cross-section in a plane:

$$\Omega = 2\pi(1 - \cos\theta) \tag{5.35}$$

(see e.g. `http://en.wikipedia.org/wiki/Solid_angle`). In our case, the angle θ of the cone is given by

$$\frac{1}{\tan\theta} = \frac{v_{\parallel,\mathrm{low}}}{v_{\perp,\mathrm{low}}} = \sqrt{\frac{B_{\mathrm{high}} - B_{\mathrm{low}}}{B_{\mathrm{low}}}} \tag{5.36}$$

from which follows that $\sin\theta^2 = 1 - \cos\theta^2 = B_{\mathrm{low}}/B_{\mathrm{high}}$.
The fraction of untrapped particles then is $\Omega/(2\pi) = 1 - \cos\theta$, and hence the fraction of trapped particles $f_t = \cos\theta$. With the above expression for $\cos\theta$ this yields the trapped particle fraction in the mirror:

Trapped particle fraction

$$\text{trapped particle fraction:} \quad f_t = \sqrt{1 - \frac{B_{\mathrm{low}}}{B_{\mathrm{high}}}} \tag{5.37}$$

When we discuss toroidal systems, we'll see that the toroidal geometry introduces gradients of B parallel to **B**, and that particles trapped in that mirror play a very important role. And, to anticipate the confusion that may arise: in a toroidal system the particles in the 'loss cone' are not lost, they circulate in the torus. The trapped particles remain in the torus too, but they bounce between local mirrors in the field and for that reason cannot complete their tour around the torus.

5.7 Take-home messages

Charged particles in a magnetic field

1. gyrate (gyroradius $mv_\perp/eB$, gyrofrequency $\omega = eB/m$ or $f = 28$ GHz per Tesla)

2. drift (due to curvature B, gradient B and if there is an E-field across B ($v_{E\times B} = E/B$)

3. get reflected (if they move towards increasing B: the mirror effect). This means that in a toroidal geometry a large fraction of the particles can be 'trapped'.

Important: the magnetic moment $\mu = mv_\perp^2/2B$ of a particle is conserved during its motion.

Problems

5.1 Magnetic coils: Maxwell's laws in practice

(a) Give an expression for the voltage between the open ends (i.e. no current) of a single wire loop in a time varying magnetic field. And what is the voltage if instead of a single loop, the coil has N windings? (Don't look up expressions for this, derive the result starting from Maxwell's laws).

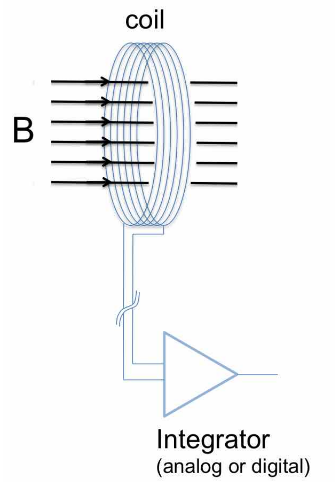


Figure 5.6: Magnetic loop

(b) How is such a loop used to measure the magnetic field? What is the complication? Why is the signal fed into an integrator? Would this integrator have a high or low impedance?

(c) Show that a long solenoid that itself forms a loop can be used to measure the enclosed current (this is called a Rogowski coil). Derive a formula by making use of Maxwell's laws and the theorems of Gauss and/or Stokes.

(d) The Rogowski coil in the sketch is wound in such a way that the return wire follows the coil again (rather than going out the short way). Why is that?

(e) If you wanted to measure the total toroidal current in a tokamak, you could use such a Rogowski coil. Where would you place it? Why is this measurement insensitive for the position of the plasma? (this is what you want: you want the total plasma current, irrespective of its position or spatial distribution)

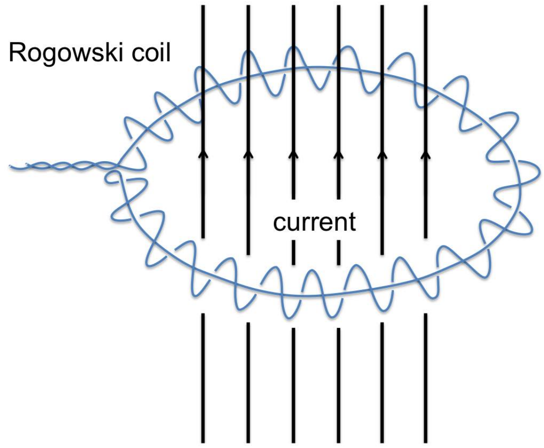


Figure 5.7: Rogowski coil

(f) If you also wanted to have information on the position of the plasma current (and thereby of the plasma), what could you do (still using magnetic coils).

5.2 Gyration and drifts.

We have seen how charged particles gyrate in a B-field, drift when the field has a gradient and/or curvature and how an electric field adds another drift effect. Now let us apply this in a physical situation. Consider a straight and infinitely long wire, that is surrounded by a plasma. The wire carries a current I, the magnetic field at a distance r from the wire is given by $B = \mu_0 I/(2\pi r)$. There are no currents in the plasma.

(a) Give the gyro frequency of the electrons at a distance of 10 cm of the wire, which carries a current of 1 MA.

(b) Describe (a drawing will do) the orbit of an electron that only has a velocity component perpendicular to the magnetic field.

(c) Describe the orbit of an electron with velocity predominantly parallel to the magnetic field direction.

(d) Now, answer b) and c) again if, in addition to carrying the current, the wire is electrically charged to a potential ϕ with respect to infinity. Assume that the electric force is small compared to the Lorentz force.

(e) Now, we replace the wire by a plasma column with radius a, for which we assume a uniform current density $j = I/\pi a^2$. (So this would be a 'Z-pinch'). Answer the questions b) and c) again, for an electron at a distance a from the wire. Consider the difference for an electron that is just 1 gyroradius inside a and one at exactly a.

5.3 The magnetic mirror effect

In a magnetic mirror, particle trajectories are affected by the gradient of B parallel to $\mathbf{B}$. The constant of motion you want to consider in this case is the magnetic moment (μ), as well as the kinetic energy.

(a) The charged particle is subject to the Lorentz force, which accelerates it. Why is its kinetic energy conserved?

(b) Show that the magnetic moment is a constant of motion, by combining the equation of motion of a magnetic dipole in magnetic field with a parallel gradient, and the conservation of kinetic energy.

(c) Consider a magnetic mirror in which the magnetic field is varied from B_{low} to B_{high}. Whether a particle is trapped or not depends on its pitch angle $(v_\perp/v_\parallel)$ at $B = B_{\text{low}}$ and the mirror ratio $B_{\text{high}}/B_{\text{low}}$. Derive this dependence (using your result under b).

(d) Calculate the fraction of trapped particles as function of the mirror ratio, assuming an isotropic velocity distribution.

(e) Calculate the mirror ratio of a toroidal magnetic field in a torus as function of the inverse aspect ratio $\epsilon =$ (minor radius/major radius).

The inverse aspect ratio ϵ is a 'small parameter': effects due to the curvature of the torus with respect to a cylinder can often be expressed as a correction on the cylindrical result with ϵ as the leading parameter.

(f) Which fraction of particles can in principle be trapped in a tokamak, as function of the inverse aspect ratio? Would you reckon this as a small correction on the cylindrical case, despite the fact that ϵ is much smaller than unity?

(g) What do you think might be the effects of trapped particles in a tokamak?

6 Basics 2: Elementary concepts of plasma physics

As is clear, the burn temperature of a fusion reactor is some 10–20 keV, and this obviously means that the fuel is in the fourth state of matter: plasma. A very significant part of fusion-related physics therefore is plasma physics.

6.1 Plasma — general aspects

A plasma is a partially or fully ionised gas. However, this expression may be misleading, because the plasma, by virtue of its charged constituents, behaves totally different from a gas. The two simple reasons are:

1. the interaction between the particles is the Coulomb force, and this has a long range. (whereas in a gas the short range interactions dominate). A consequence of the long range of the Coulomb force is that a plasma particle feels the influence of very many—typically 10^8 in a fusion plasma—neighbours. In a plasma, collective phenomena are important.

2. the charged particles in a plasma feel magnetic and electric fields through the Lorentz force. And since moving charges represent currents, which in turn generate magnetic fields, a plasma can easily display a staggeringly rich phenomenology with no more ingredients than a bunch of charged particles and some energy. Look at the cosmos to see this on display. Therefore, a description of a plasma needs to take into account the influence of the (self-generated) magnetic and electric fields.

With the exception of lightning, plasma does not occur naturally on earth: it is far too cold here! But the universe is filled with plasma, more than 99.99% of the visible mass is plasma: the sun, the other stars, and the almost empty space between the stars, it is all plasma. But it is very easy to make plasmas, and the world is filled with applications. TL-lights, processing plasmas in the fabrication of computer chips, deposition of thin layers, $\cdots$, there are only few products that do not have a plasma-involving step in their production. Plasmas do exist in an enormously wide parameter range, temperature and density can each vary over more than 6 orders of magnitude. We can distinguish plasma by these two parameters, as in figure 6.1

Important

In these lecture notes, we shall take for the typical 'fusion' plasma:
temperature $k_B T = 10^4$ eV (100 million K), density $n = 10^{20}$ m^{-3}.

6.2 The screening effect: Debije length

A fundamental example of collective behaviour is the Debije length or screening length. Consider a neutral cloud of charged particles, e.g. electrons and protons. Now, bring a positive perturbing test charge into this system. What is the effect of the electric field of this charge? Clearly, from very close by the field is simply determined by the charge itself, all other particles are far away. But, the charge will attract the electrons in

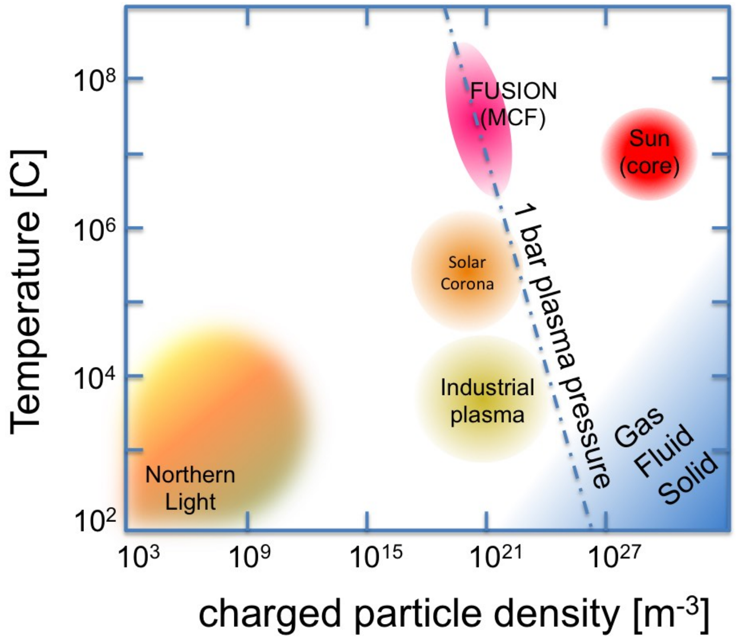


Figure 6.1: Plasmas come in shapes and sizes, from the pinhead to the galaxy sized, and from the extremely rarefied and fairly cold to the dense and hot. The fusion plasmas are among the hottest, but at 1-10 Bar are much lower in pressure than the plasmas that make up interior of the stars

the cloud, thereby restoring the overall charge neutrality around it. In other words: the charge is screened by the plasma. At sufficiently large distance no electric field of the test charge can be measured. The typical distance over which the screening effect is realised is the Debije-length, λ_D (after Debije, who derived the theory for colloids).
The Debije-length depends on:

- the density of the electrons: higher density gives more effective screening
- the temperature of the electrons: the collective movement of electrons needed to achieve the screening is in competition with the random thermal motion.

Debije length (practical units)

$$\lambda_D[\mathrm{m}] \approx 10^{-4}\sqrt{\frac{T_e[\mathrm{keV}]}{n_e[10^{19}\mathrm{m}^{-3}]}} \tag{6.1}$$

So this expression gives λ_D in metre, where T_e is the electron temperature expressed in keV and n_e is the electron density in units of $10^{19}\mathrm{m}^{-3}$. These are so-called practical units, chosen such that in the typical fusion plasma the argument of the square root is of order unity. For other types of plasma this can be very different, of course.

Derivation of the Debije length

Assume that the ions are fixed (they are very slow compared to the electrons), that the electrons do all the work and that their distribution follows Boltzmann: $n_e = n_e(0)\exp(e\phi/kT)$, with ϕ the electric potential and $n_e(0)$ the unperturbed electron density.

The test charge perturbs ϕ, and we look for a self-consistent solution of ϕ and the electron distribution. Maxwell relates ϕ with the charge density: $\nabla^2\phi = -\rho/\epsilon_0$. The charge density ρ is simply the local difference between electron and ion density: $\rho = e(n_i - n_e)$. For simplicity assume that the ion density n_i is fixed, everywhere equal to the unperturbed electron density, while the electron density follows Boltzmann as above. Combining this you'll find

$$\epsilon_0\nabla^2\phi = en_i\left(\exp(e\phi/k_BT) - 1\right) \tag{6.2}$$

Taylor-expand the exponential (you want the far-field solution, where the argument is small) to find

$$\epsilon_0\nabla^2\phi = en_i e\phi/k_BT \tag{6.3}$$

Since we want an approximate solution, you can solve this in 1 dimension to make life easy (i.e. instead of a point charge, we have introduced a charged plane in the plasma). Try a solution of the form $\phi \propto \exp(-x/\lambda_D)$ and you find the expression for the Debije-length.

Important: from this derivation you see that it is the Boltzmann factor that introduces the T-dependence. This factor exactly describes the competition between the random motion (temperature) and the potential that has an ordering effect.

Conditions for a plasma to behave like a plasma

In the typical fusion plasma $\lambda_D = 0.1$ mm. This is the distance the plasma needs to maintain quasi-neutrality. Therefore, a very important condition for a plasma to behave as a plasma is that λ_D must be much smaller than the dimension of the plasma. A fusion plasma easily satisfies this condition.
A second condition for the concept of charge screening to work: the number of electrons in a sphere with radius λ_D must be very large. Our fusion plasma again passes the test easily, with typically 10^8 electrons in the 'Debije-sphere'.
This also shows that each electron or ion in the plasma feels the influence, through the Coulomb force, of some 10^8 other electrons and ions. A collision in a plasma therefore is seldom a two-particle interaction. Rather, the trajectory of an electron is influenced by the tiny ripples in the field due to those 10^8 neighbours (see section 6.4 on the collision frequency).

6.3 Plasma frequency

So far we have considered the plasma as a static medium. If we perturb the plasma, there is a wealth of frequencies to distinguish. The most fundamental is the plasma frequency.
Consider, again, a cloud of charged particles (e.g. electrons and protons) and assume that the ions are fixed in position. Now, displace the electron cloud with respect to the ions. Where the electrons stick out we have a negative surface charge, and on the opposite side of the cloud a positive surface charge, see figure 6.2. As a result, there is an electric field inside the cloud. This value of the E-field is simply derived from Maxwell's laws, and is proportional to the surface charge density σ: $E = \sigma/\epsilon_0$. The surface charge, in turn, is the product of the displacement x, the electron density n_e and the electron charge e.
This electric field exerts a force on the electrons (and on the ions, too, but they accelerate much slower because of their mass)). This force is

1. directed opposite to the displacement of the electron cloud, and

2. proportional to the surface charge, therefore to the displacement.

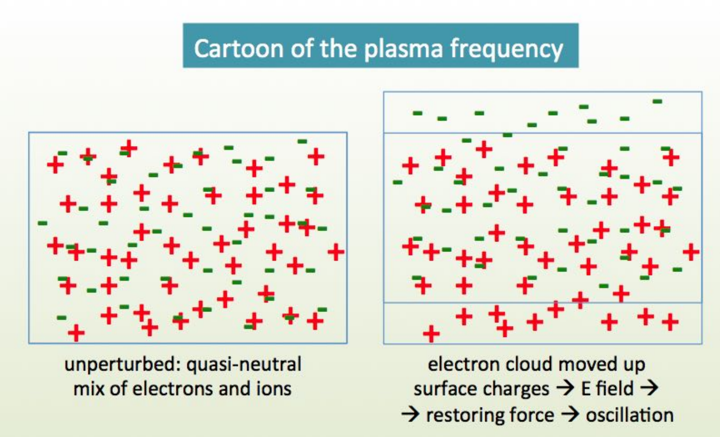


Figure 6.2: Left: neutral plasma; right: small vertical displacement of the electron and ion clouds, leading to a charge separation and an electric field proportional to the displacement.

A restoring force proportional to the displacement implies a harmonic oscillation, the frequency of which is easily derived by quantifying the electrical field and inserting this in the equation of motion of the electron cloud. This is the plasma frequency.

Derivation of the plasma frequency

$$\text{Force on electron cloud (per unit volume):} \quad F = -n_e eE = -n_e e(xn_e e)/\epsilon_0 \tag{6.4}$$

$$\text{Equation of motion (per unit volume):} \quad \frac{-n_e e(xn_e e)}{\epsilon_0} = n_e m_e \ddot{x} \quad \Rightarrow \quad \ddot{x} = -x\frac{e^2 n_e}{\epsilon_0 m_e} \tag{6.5}$$

$$\text{Result: oscillation with plasma frequency} \quad \omega_p = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}} \tag{6.6}$$

Filling in the constants, we get an easy–to–remember formula for the plasma frequency f_p, expressed in Hertz when the plasma density is entered in m^{-3}:

Plasma frequency

$$f_p[s^{-1}] \approx 10\sqrt{n[m^{-3}]} \tag{6.7}$$

⇒ Note that the plasma frequency depends only on the electron density, nothing else.

The plasma frequency is the basic frequency with which the plasma can respond to perturbations. For example, transverse electro-magnetic waves with a frequency below the plasma frequency cannot propagate in the plasma. If such waves are launched towards the plasma from outside, they will be reflected as soon as they reach a place where the plasma density is high enough to make the plasma frequency match the frequency of the injected wave.

6.4 Collisions, collision frequency, electrical resistivity

The electrical resistivity of a hot plasma can be very low, comparable to that of copper or lower. More interestingly, the resistivity decreases with increasing plasma temperature. This remarkable property has far-reaching consequences. To mention but one: in a tokamak the principal plasma heating is due to the current in the plasma (which is there to provide the poloidal magnetic field in the first place). However, the higher the temperature becomes, the lower the resistivity, hence the less effective the 'Ohmic' heating. This is the reason that we need other heating methods: Ohmic heating loses its effectiveness at a temperature of a few keV, far too low for a fusion reactor.

Let us first sketch how the frequency of the electron-ion collisions—the ones that cause the electrical resistance and Ohmic dissipation—depend on the electron temperature. We already saw that one-on-one collision are the exception, an electron feels the Coulomb field of 10^8 neighbouring ions (and fellow electrons).

Therefore, we need to define the concept of 'collision time' as the time it takes an electron to be deflected, as a result a great many interactions, over 90 degrees (i.e. when its initial momentum is fully lost). Realise that the total deflection angle is the results of many small angle deviation, of arbitrary direction. Hence, the velocity of the electron undergoes a random walk in phase space, and as for any random walk, the total distance (so in this case: angle ϕ) covered is proportional to the square root of the number N of small steps (small angle deviations, θ):

$$\phi(N) = \sqrt{N}\,\theta \tag{6.8}$$

and because N is proportional to the electron velocity v, the ion density n_i and the time t, we get:

$$\phi(N(t)) = \sqrt{N(t)}\theta \quad \Rightarrow \quad \phi(N(t)) \equiv 90^\circ = \text{constant} \times \sqrt{n_i v \tau} \times \theta \tag{6.9}$$

Now the big question is how the elementary deflection angle depends on the velocity of the particle. To start: θ is by definition the ratio of the velocity $v_{||}$ before the interaction, and the velocity $v_\perp$ perpendicular to this picked up during the interaction: $\theta = v_\perp / v_{||}$

The perpendicular velocity $v_\perp$ is the result of the momentum transfer, which is proportional to the duration of the interaction. (The force itself, i.e. the Coulomb force, does not depend on the velocity). Therefore, $v_\perp$ is inversely proportional to $v_{||}$. As a result we get: $\theta \propto v_{||}^{-2}$, and inserting this in the expression for ϕ we finally get $\tau \propto v^3/n_i$.

This is a very important property of plasmas. Note that the mean free path of an electron grows with the fourth (!) power of its velocity. Hence, an electron loses its resistance when it accelerates.

Finally, the average velocity squared is proportional to the temperature, hence in a thermal plasma the collision frequency and as we shall see, the resistivity, go as $T^{-3/2}$.

By taking into consideration that the Coulomb force is also proportional to the ion charge Z, we find

Collision time

$$\tau \propto \frac{v^3}{n_i Z^2} \tag{6.10}$$

An interesting consequence of this relation is that electrons that happen to live in the tail of the velocity distribution function collide far less frequently than the 'thermal' electrons. For this reason, it is relatively easy to change this part of the distribution function. This can be done e.g. by absorbing waves on those electrons, or even simpler, with an electric field. An electric field will lead to a current, i.e. a flow of the electrons with respect to the ions. But two interesting effects now become apparent:

1. Electrons in the high velocity tail of the distribution function feel less drag by the ions than the slower ones. Therefore, the current will be carried predominantly by the faster electrons. (It can be calculated how the current is distributed over the electron velocities).

2. In low density plasmas, it is even possible that electrons that happen to have a very high velocity are accelerated so much between collisions, that their collision frequency effectively goes to zero. These electrons—so-called runaway electrons—undergo a free fall in the electric field, a phenomenon common to tokamak and astrophysical plasmas. In a tokamak these runaway electrons can reach tens of MeV, despite the fact that the accelerating field is only a fraction of a V/m. Their mean free path can be as long as hundreds of thousands of km, in a machine with a circumference of a few tens of metres!

6.5 Electrical resistivity of a plasma

From the collision frequency we can derive the resistivity of the plasma. After a collision the electron has lost its forward (parallel) velocity. In an applied electric field E, it will be accelerated until the next collision. Hence its average velocity due to the field, the drift velocity v_d is proportional to $E \times \tau$. The resulting current density is, by definition, $j = -n_e e v_d$. The resistivity η follows from Ohm's law $E = \eta j$. In a pure plasma (electrons and a single species of ions with ion charge Z) this yields

$$\eta \propto \frac{n_i Z^2}{n_e T_e^{3/2}} = \frac{Z}{T_e^{3/2}} \tag{6.11}$$

using the trivial relation:

$$n_e = n_i Z \tag{6.12}$$

In the more general case of a multi-species plasma, we must add the collision frequencies of the electrons with each of the ion species, taking their respective densities and charges into account. This then yields:

Resistivity

$$\eta \propto \frac{Z_{\text{eff}}}{T_e^{3/2}} \quad \text{with} \quad Z_{\text{eff}} \equiv \frac{1}{n_e} \sum n_i Z_i^2 \tag{6.13}$$

Here we have introduced the 'effective ion charge' Z_{eff}. It only appears in its pure form in the resistivity, but it is often used as a measure of the purity of a plasma. As we shall see later, the radiative power losses from the plasma, as well as the dilution of the fuel, depend critically on the plasma purity. Hence Z_{eff} is a very important qualifier.
Finally: note that η, somewhat surprisingly perhaps, does not depend on the density: a higher density does increase the collision frequency, but this is exactly compensated by the fact that there are correspondingly more electrons that contribute to the current.

6.6 Take-home messages

- Debije shielding length: distance over which the plasma maintains quasi-neutrality. Scales as $n^{-0.5}$ (higher particle density is more effective at screening); and as $T^{0.5}$ (thermal motion competes with r the ordered motion needed for shielding. The Debije-length is e.g. important where a plasma is close to a material wall: a sheath will form with a width of several Debije-length.
  A plasma needs to be much larger than its Debije-length to behave like a plasma at all. And it must have many particles in the 'Debije sphere'. In a fusion plasma both conditions are easily met.

- A plasma has a natural oscillation frequency: the plasma frequency (oscillation of the electrons with respect to the ions). It is proportional to $n^{0.5}$. No other dependences.

It is a cut-off frequency for wave propagation: waves below the plasma frequency don't propagate. A pre-warning: if there is a magnetic field, too, this picture gets more complex.

- Collisions: when two charged particles pass each other, the deflection angle is proportional to v^{-2}, due to the shorter interaction time for higher velocity. As a result, the collision time is proportional to v^3 and the mean free path to v^4. In other words: the faster the particle, the less is collides. One consequence is the phenomenon of 'electron runaway'.
- Resistivity: from the collision time follows the plasma resistivity, which is proportional to $T^{-3/2}$. The hotter the plasma, the better it conducts electricity. One consequence: heating a plasma by running a current through it becomes less effective when the plasma gets hotter. 'Ohmic' heating therefore is not sufficient to reach the ignition temperature. The resistivity does not depend on the plasma density, but is proportional to the effective ion charge Z_{eff}.

Problems

6.1 Plasma physics: getting a feel for numbers

Much of this exercise is straightforward calculation. Its purpose is to give you a feeling for orders of magnitude of important quantities in a fusion plasma. It should help you develop a coherent picture, in which different effects have a place.

In the hot core of a fusion reactor the typical parameters are: $T_e = T_i = 10keV; n_e = 10^{20}m^{-3}$; $B = 5T$. Assume a pure hydrogen plasma (the ion charge $Z = 1$)

(a) Calculate the electron collision time and the ion collision time (use formulas below)

(b) Calculate the electron and ion mean free paths.

(c) Calculate the ion and electron Larmor radii, and their gyro-frequencies.

(d) Compare the inverse gyro-frequency to the collision time for both species. What do you conclude for the 'magnetisation' of both species? Also compare the electron gyro-frequency to the plasma frequency.

(e) Below also a formula is given for the ion-electron energy exchange time τ_{ex}. Give a useful definition of this quantity and give an expression for the energy density that is transferred from ions to electrons when their temperatures are not equal.

(f) The energy exchange time is a factor $(m_i/2m_e)$ longer than the electron collision time (τ_e is the collision time of electrons with ions). Why would this be so?

(g) Calculate τ_{ex} for the fusion plasma and compare this to the energy confinement time of ITER. What does this mean for the difference of T_e and T_i in ITER?

(h) Show – functional dependencies only – that the resistivity of a plasma does not depend on the density. Give a simple explanation, in words, not equations, why this is so.

(i) Now, in a real fusion plasma there is always a finite amount of impurity present, if only the helium that was produced by the fusion reactions! So we need to consider how things change if Z is not equal to 1. Derive the collision frequency — dependencies only — while retaining the Z dependence.

(j) Work out the Z-dependent resistivity. Find a logical definition of the 'effective ion charge' Zeff and show that this can be measured by measuring the resistivity of the plasma.

(k) Calculate Z_{eff} for a plasma with a 3% content of carbon (i.e. $n_C = 0.03n_{ions}$, where n_{ions} is the density of all ion species together), assuming that all carbon atoms are completely stripped.

Formulas:
Electron collision time for singly charged ions: $\tau_e \approx 6.4x10^{14} T_e^{3/2} n^{-1}$ s (T_e in keV)
Ion collision time for singly charged ions : $\tau_i \approx 0.9(2m_i/m_e)^{0.5}\tau_e$ ($\approx 55\tau_e$ for protons) taking $T_i = T_e$.
Ion-electron energy exchange time: $\tau_{ex} \approx (m_i/2m_e)\tau_e$

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7 Tokamak 1: physicists view. The magnetic topology

7.1 The principle of the tokamak

To build a tokamak, in principle all you need is a toroidal vacuum vessel, coils to generate the toroidal field and—as we shall see—the vertical field, and a transformer setup that allows you to drive a plasma current. In the tokamak, the helical field is generated by the combination of

- external coils, which generate a toroidal field (typically a few Tesla), which is constant in time, and
- a toroidal current in the plasma (the 'plasma current'), which generates the poloidal field. The poloidal field is normally roughly an order of magnitude smaller than the toroidal field.

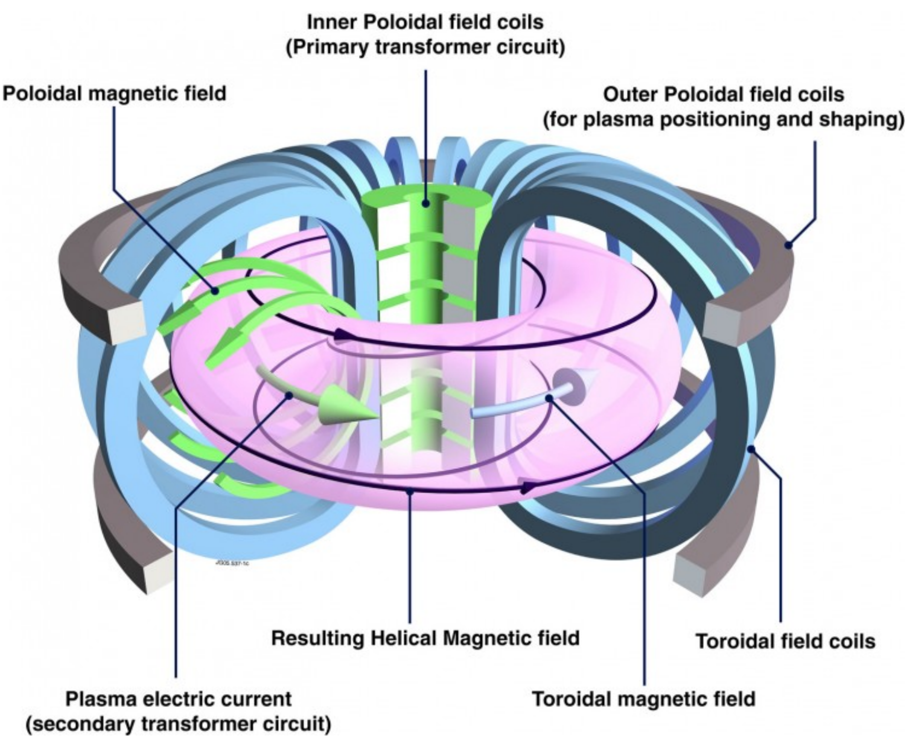

Figure 7.1: Tokamak coil system and fields (source: EUROfusion)

The plasma current is generated inductively, by using the plasma ring as the secondary winding of transformer. Directly driving the current by sticking electrodes into the plasma is obviously not possible. But inductive current drive has its drawbacks, too. Most notably, inductive current drive is based on a time derivative (a rate of change) of the magnetic flux through the central hole in the torus. And this flux cannot continue to grow indefinitely, at some point the transformer saturates. A transformer works for AC, not DC. Therefore, inductive current drive in a tokamak is limited in time. One could say that we use only half a period of the alternating current, but of course we can shape this half-period any way we want: it does not have to be sinusoidal. Large tokamaks are big transformers with a very large 'flux swing', and the voltage required to drive the plasma current, owing to the very low electrical resistivity of hot plasmas, and the so-called bootstrap

current that we shall meet later on, is very low, only a fraction of a Volt. Thus, in reactors like ITER, the plasma current – of some 15 MA! – can be driven by induction for 10 minutes or more.
There is a third indispensable magnetic field component, the 'vertical field'. This is again an order of magnitude smaller than the poloidal field. Its function is to exert an inward force on the plasma (through the cross-product with the plasma current), which is needed to balance the 'hoop force'. The hoop force is the result of the pressure in the plasma ring. Analogous to a bicycle tube, which becomes not only thicker but also bigger when inflated, the plasma ring has a tendency to increase its radius when the pressure increases. Here, the pressure is the sum of the kinetic pressure (like the bicycle tube) and the magnetic pressure. The latter can also be understood as the Lorentz force: the plasma current exerts a repelling force on itself on the other side of the plasma ring (anti-parallel currents). The vertical field is generated by large coils above and under the plasma. O the entire collection of coils in the tokamak setup, these are generally the coils with the largest diameter. In ITER they are even so large that they cannot be transported. As a consequence, there is an on-site coil winding facility at ITER, just for these coils.
The toroidal field (primary function: reduction of heat loss), the poloidal field (primary function: pressure balance, and introducing rotational transform $\longrightarrow$ helical field $\longrightarrow$ suppression of charge separation) and the vertical field (sole function: balancing the hoop force), are the three essential fields that make up the tokamak field configuration necessary for magnetic confinement. In addition to that, there are usually several more coils, which serve to shape the plasma and control the vertical position. For the simplest DIY build-a-tokamak-in-your-shed tokamak, these coils are not required. But every real tokamak has them, and as the fields required for controlling the plasma are not so large, they are technically relatively easy to take on board.

7.2 Magnetic topology; Flux surfaces; Magnetic islands

In a toroidal system (such as the Stellarator and the Tokamak) the aim is to create a magnetic field with a very special topology: field lines lie on nested, toroidal surfaces (see Figure 7.2). It is very important to have a clear picture in mind of this topology: nested toroidal surfaces.

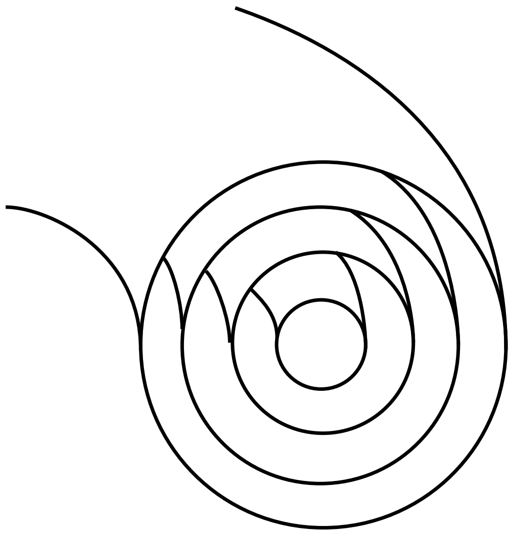

Figure 7.2: Nested toroidal flux surfaces

As transport is very fast along the field lines (parallel to the field transport is unimpeded), pressure and temperature are constant on these nested toroidal surfaces. They are isobars and isothermals. Moreover, also the magnetic flux (to be precise, the poloidal magnetic flux, i.e. the integral of the magnetic field that passes through the central hole of a toroidal surface) is also constant on these surfaces. For this reason they are called 'flux surfaces'.

Note that this implies, for instance, that also the time derivative of that flux is constant over the surface. Hence, the loop voltage is also a flux surface quantity.

The equations that describe the magnetic field lines in a torus form a so-called Hamiltonian system, a standard form in classical mechanics. In fact, the magnetic field in a torus is the exact analog of the phase space of two weakly coupled ideal (undamped, dissipation-free) oscillators. (A single oscillator describes a circle in the $(\mathbf{x}, \mathbf{v})$ phase space, combining the motion of two – provided they are uncoupled – results in a helical trajectory on a toroidal surface: the analog of the toroidal flux surface).

The mathematics of Hamiltonian systems has been the subject of extensive study for many decades, and all those results apply to the magnetic field in a torus, too. An important property concerning the topology of the phase space is the following.

Consider two weakly coupled oscillators. If they are resonant, and in particular if their frequencies are equal, they can exchange energy very easily. Even a very weak coupling is sufficient to channel energy from one oscillator to the other, and back. This has a dramatic effect on the trajectory of the system in phase space: rather than lie on a toroidal surface, the trajectory now makes excursions away from this surface. In the case of the magnetic field in a torus the analog is: if there is a perturbing field with a spectrum of spatial frequencies that are resonant with a field line, then this resonant field line will pick up the perturbation. (Field lines that are not resonant with the perturbing field are not affected). The resonant field line will make excursions in radial direction. The flux surface degenerates and becomes a volume.

Figure 7.3 illustrates how the toroidal surfaces in a torus degenerate under the influence of a perturbing field. We see the formation of so-called magnetic islands: closed flux tubes that wind the torus with the same helicity as the unperturbed field lines. Hence, if the unperturbed field line needed n toroidal turns to realise m poloidal turns, this condition now applies to the island. This winding ratio is called $q = m/n$. (It will come back later under the name 'safety factor'). A single magnetic island with $q = m/n$ is seen as m islands in a poloidal cross-section. But it is still a single flux tube in which temperature and density are constant.

Thus, in poloidal cross-section, a flux surface changes under perturbation into a chain of islands.

Things become even more complex if different flux surfaces – with different q – are perturbed. Now, there are several chains of islands. And if we let these islands grow in width – by increasing the amplitude of the perturbation – at some point the islands touch and then overlap. Where islands overlap the field becomes chaotic. Here, single field lines can fill a volume.

We don't need very larger perturbing fields to seriously affect the ideal topology of nested toroidal surfaces. A perturbation of 0.01% of the toroidal field will generate significant magnetic islands. This is easy to see. In a torus with a major radius of 1 m, a field with a deviation in pitch angle of only 10^{-4} will pick up a deviation from its unperturbed path of about 1 mm after a single turn. A few tens of toroidal turns are sufficient to let the field line make an excursion of centimetres!

Now, you have to realise that the gyro-radius of the electron is only of the order 0.1 mm and that the mean free path of an electron in a fusion plasma is of order km, so that it does trace the field line (apart from the drift) for tens of toroidal turns. Hence, the radial displacement of an electron due to a field perturbation at the 10^{-4} level is already more than enough to lead to enhanced transport. Magnetic islands with a width of a cm, however insignificant compared to the size of the tokamak, are very real structures as seen by the electrons.

Figure 7.4 shows so-called Poincaré plots of the magnetic topology, in polar coordinates. In this representation

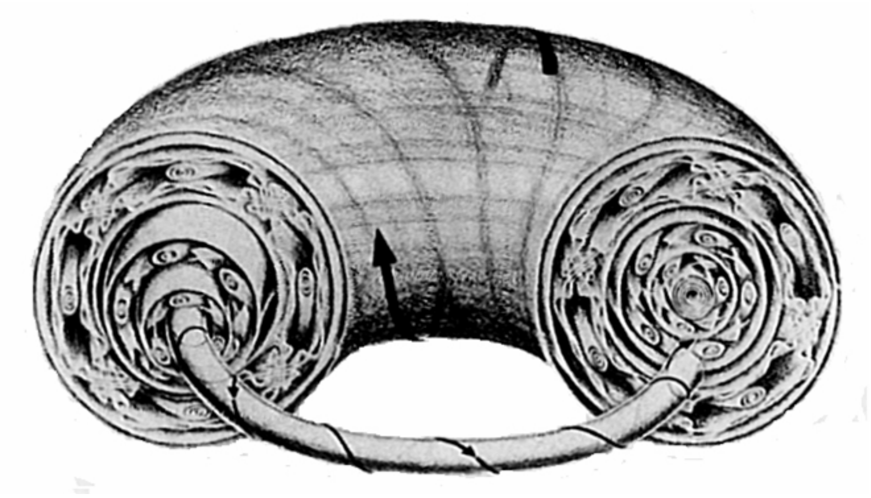

Figure 7.3: Perturbations can change the topology of nested toroidal surfaces. The surfaces degenerate and form so-called magnetic islands, helical flux tubes with the same q-value as the original flux surface. Inside the island the field lines form nested surfaces again. (Source: unknown)

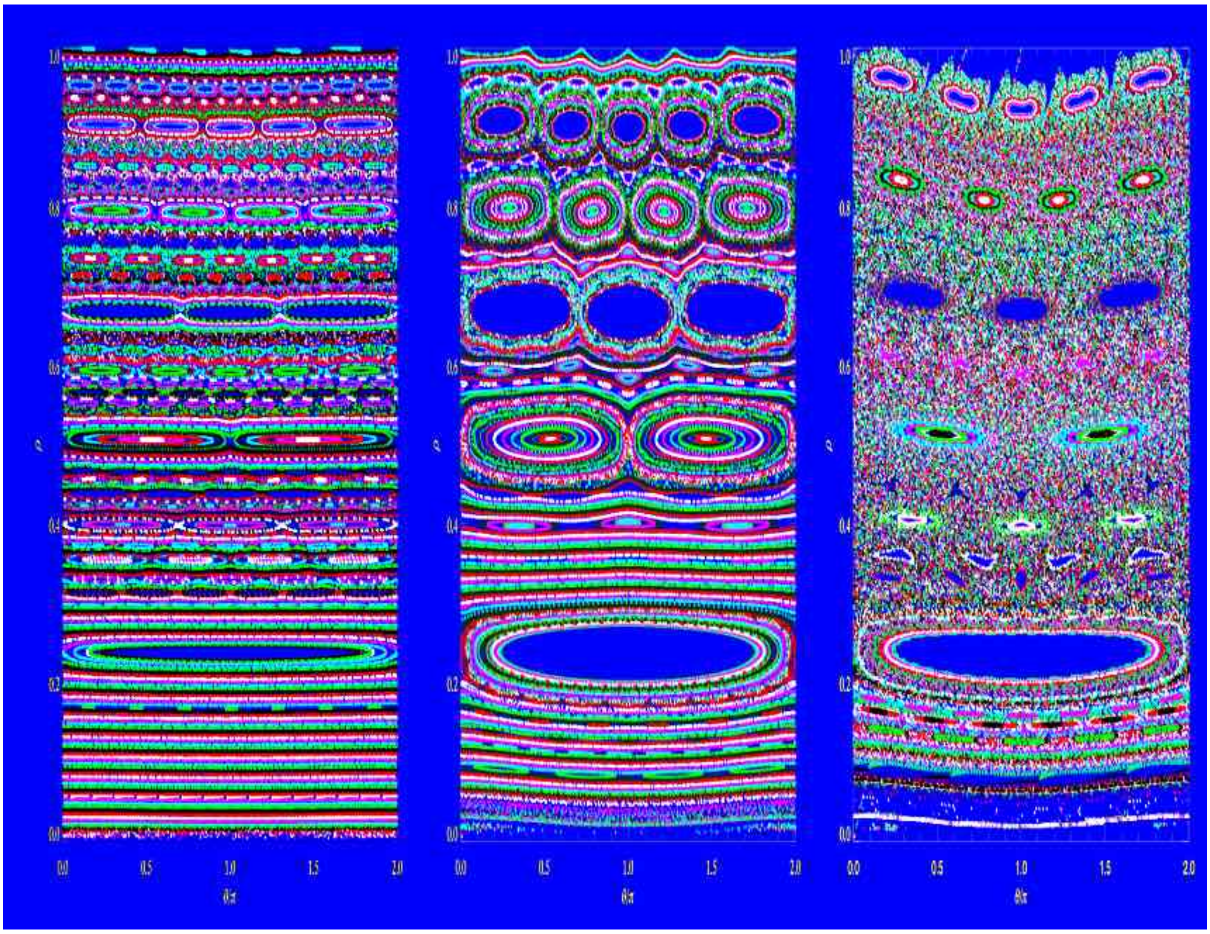

Figure 7.4: Three Poincaré plots for increasing perturbation fields (left: 10^{-4}, right: 10^{-3}). Each plot is a poloidal cross-section of the torus (horizontal coordinate: poloidal angle; vertical coordinate: radius in the plasma, with (top=edge, bottom=centre). (This picture was computed and rendered by Arnold Schilham in the frame of his PhD research)

the islands clearly manifest themselves as chains. In the left hand picture the perturbing magnetic field, 10^{-4}, is not too disastrous for confinement: intact surfaces can be discerned between the island chains. In the right-most picture the perturbation is so large, 10^{-3}, that chaotic areas are filling most of the volume (although there are still remnants of magnetic islands): there are field lines that traverse the entire chaotic region. At this level of perturbation confinement is seriously deteriorated.

A Poincaré plot is constructed by following one are several field lines (following meaning, integrating the field line equations) and marking the spot where they cross the poloidal plane by a dot. In an unperturbed field, this procedure will trace out the flux surfaces – where it should be remarked that on surfaces with rational $q = m/n$ a single field line only results in m dots. Hence, the rational surfaces are barely visible.

By assigning a different colours to field lines that start at different radii, one gets an impression of the radial excursions of field lines. Chaotic field regions are recognised by the mixing of colours. It is observed immediately that - in agreement with theory- the topology inside magnetic islands is again a set of nested toroidal surfaces. Chaos is observed where islands overlap

It is very important to understand that magnetic islands are a topological property of the field. If you were able to see field lines, you would not see any difference between an unperturbed field and a field with islands. After all, the perturbation is at the 10^{-4} level. The islands only become apparent if you follow a field line some 1000 times around the torus. But then$\cdots$ this is exactly what electrons do!

One more word on the topology: when a magnetic island is produced, the topology of the field changes. The unperturbed magnetic field had a single magnetic axis: the innermost flux surface is only a line. But when the field is perturbed and islands are present, each island, too, has a magnetic axis, and these are wound around the original magnetic axis. The double helix!

7.3 Reconnection

To change the topology of the magnetic field one must 'break' field lines and 'reconnect' them in the new topology. This process is known as 'reconnection'. Normally—there are exceptions—reconnection requires resistivity in the plasma. In a superconductor one cannot change the field, as any change of field would induce an electric field (Lenz's law / Maxwell's law — time derivative of magnetic flux generates E-field), and in a superconductor this would immediately result in currents that compensate the change of magnetic flux. Figure 7.5 shows the reaction of a plasma to an external perturbation: without resistivity the field lines bend but the topology does not change; with resistivity the plasma moves to a state with lower magnetic energy by forming magnetic islands through the process of reconnection (η denotes resistivity).

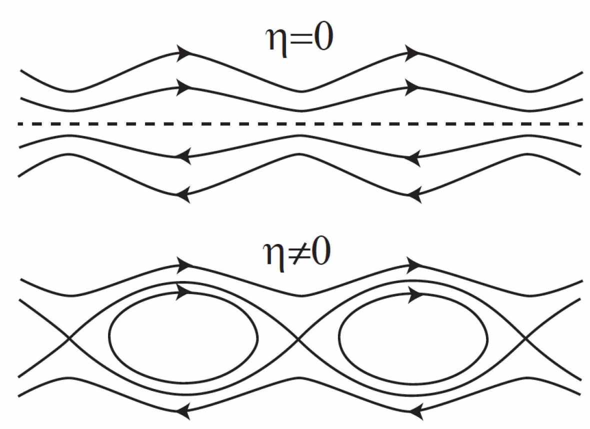

Figure 7.5: Top: magnetic perturbations at zero resistivity: no topology change; bottom: in the presence of resistivity, the same magnetic perturbation will lead to reconnection and island formation.

7.4 Resonant values of q. Mode numbers.

From the above it is clear that geometrical resonances in the torus, i.e. those places where field lines close back on themselves after a finite number of toroidal turns, play a special role. The resonance condition is $q = m/n$, where q is the safety factor. The natural numbers n and m are called the toroidal and poloidal mode number, respectively. A field line closes after m toroidal turns, having completed n poloidal turns at that time. (This is confusing perhaps, but a structure which goes m times round in toroidal direction before closing appears m times in a poloidal cross-section. Hence m is the poloidal mode number). The lower n and m, the stronger the resonance.

7.5 The q-profile

So, q is important. Let us consider it in some more detail. To start, the value of q is not constant in the plasma. Let us consider a plasma with a circular cross-section and an ideal, unperturbed topology of nested toroidal surfaces. A surface is defined by the field lines that lie on it, and these have by definition the same q (else they would cross, something field lines don't do). Therefore, q is a property of the flux surface:

Definition of q(r): (this is a must-remember)

$$\text{number of toroidal turns a field line makes while completing one poloidal turn} \tag{7.1}$$

If we approximate the torus by a periodic cylinder, with period $2\pi R$, the value of q at radius r is easily shown to be given by (this is a must-remember, too):

Cylindrical q

$$q(r) = \frac{rB_T}{RB_p} \tag{7.2}$$

where B_T and B_p are the toroidal and poloidal magnetic field strengths.
The toroidal field is externally applied and uniform (in the cylindrical approximation; in a torus there is the $1/R$ dependence, but for concentrical flux surfaces the averaged field is still the same). The poloidal field, generated by the plasma current, is a function of the radius of the flux surface r:

Poloidal field

$$B_p = \frac{\mu_0 I(r)}{2\pi r} \tag{7.3}$$

where $I(r) = 2\pi \int j(r)r\,dr$ is the toroidal current in the plasma that is enclosed by the flux surface with radius r. Hence, to compute $q(r)$, one has to integrate the plasma current density out to radius r, and this presumes knowledge of the function $j(r)$.

Intermezzo

Here is a problem: j is not determined externally. The only thing the operator of a tokamak can control is the total current, where it runs is the result of a complex process. The current density concentrates where the resistivity is lowest, which is where the temperature is highest, i.e. in the centre of the plasma. But the Ohmic heating is also highest where the current density is highest, which enhances the peaking of the current. Heat transport, however, limits the peaking of the temperature, and therefore of the current density. If these were the only processes at play (which they are not, as we shall see), an equilibrium would establish in which the current density is inversely proportional to the resistivity, hence to $T^{3/2}$.

Clearly, there are only two radii where it is straightforward to determine q. In the limit $r \downarrow 0$, q is fully determined by the current density at the magnetic axis, $j(0)$ (check this!). Conversely, at the edge of the plasma q is determined by the total plasma current I_p. In between those to boundaries, the function $q(r)$ is determined by the integral of $j(r)$. This, again, is not externally defined. But since it is an integral, it is usually a very smooth function.
To get a feeling for the freedom a plasma has to play with q (and thereby with the magnetic resonances), let us consider two limiting cases:

1. The entire plasma current is concentrated in a delta-function a the magnetic axis,
   $I(r) = I_p \rightarrow q(r) = 2\pi r^2 B_T/\mu_0 R I_p$.
   Hence, the q-profile is parabolic with $q(0) = 0$, increasing to $q(a)$ at the edge $r = a$.

2. the plasma current is distributed uniformly over the plasma cross-section,
   $j(r) = \text{constant} \rightarrow I(r) = (r/a)^2 I_p$.
   Hence, the q-profile is flat, $q(0) = q(r) = q(a)$.

Somewhat more generically, we may consider a current density profile given by $j(r) = j_0(1 - (r/a)^2)^\alpha$
This describes a profile that goes from flat for $\alpha = 0$ to strongly peaked for large values of α. Figure 7.6 shows a few examples.
In a healthy tokamak plasma the temperature, and therefore the current density, is peaked in the centre. But apart from the heat transport limiting the peaking of T, there is another mechanism at play. As soon as $q(0)$ drops below 1, there is a surface with $q = m/n = 1$ in the plasma, and this is the strongest resonance possible. At this surface, a universal instability develops, which through the creation of a magnetic island redistributes the energy, particles and current in the centre of the plasma. After the occurrence of this instability $q(0)$ is again at or above 1. Now the game starts over, the T-profile peaks, the j-profile follows suit until $q(0) < 1$ and the instability resets the profiles again. This leads to a periodic gradual increase of the temperature followed by a sharp decrease. This instability is therefore called the 'sawtooth' instability. It is practically universal in tokamaks and is always associated with $q = 1$. The effect of the sawtooth instability is that $q(0)$ in a normal tokamak is always close to unity, say between 0.8 and 1. We'll return to the sawtooth instability in section 7.6.
So, now we know a lot more about the q-profile. Its shape is roughly parabolic, going from $q(0) = 0.8$–1 to $q(a)$. The radial derivative of q, the magnetic shear, is positive everywhere in this standard case. It turns out that the stability of the plasma (against a number of instabilities) is much better if the magnetic shear is negative rather than positive. However, negative shear is not a natural state of the plasma, the temperature and current will always tend to peak in the centre. Nonetheless, it is possible to run a tokamak discharge with negative shear. To achieve this, the current can be chased from the centre, e.g. by non-inductively

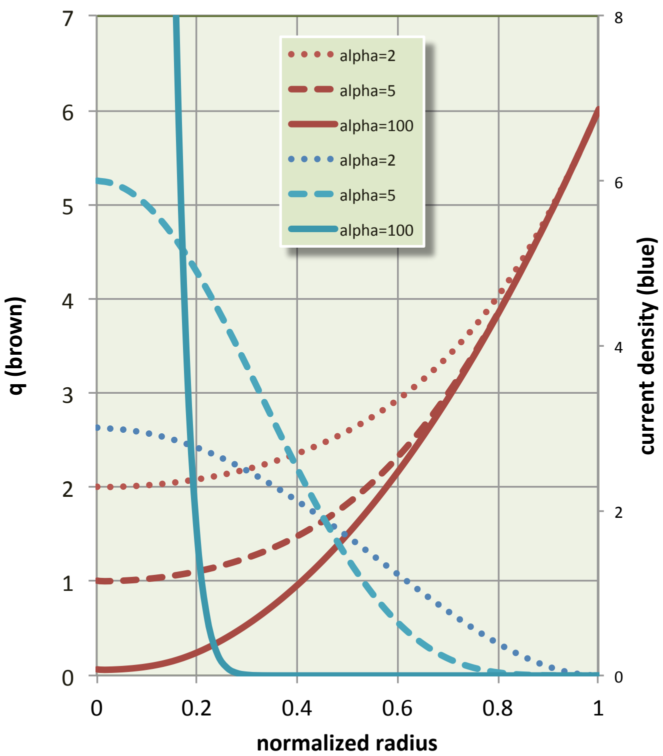

Figure 7.6: A few examples of current density profiles of various peakedness, and the corresponding q-profile (see text)

driving a current in counter direction, or by driving an additional current in co-direction away from the centre, typically at half-radius. If the total current is driven non-inductively, the relation between the current and the temperature is completely broken.

Such experiments are carried out very often. The most common way to achieve a negative shear, however, is a trick. During the ramp-up of the plasma current, the j-profile is hollow due to the skin-effect (the driving E-field needs a skin-time ($\tau = a^2/\eta$) to penetrate the plasma). Interestingly, the skin time is longer for hotter plasmas. Hence, by strongly heating the plasma during the current ramp-up, the operator can create a hollow current density profile that last seconds (in JET). This trick, to stall the current penetration by rapidly heating the plasma is sometimes called 'freezing in' the current density profile. That is: freezing by heating!.

Plasmas in which the stability and confinement are optimised by engineering the q-profile are called 'Advanced Tokamak Scenarios'. They are still limited in their applicability, they do not yet combine high density, high performance with steady state. But they do show that with good understanding of the plasma physics and clever experimental manipulation of the plasma, superior performance can be achieved.

7.6 The sawtooth instability

Above we briefly described the sawtooth instability, which limits the peaking of temperature and current density. It was first observed in 1974. Measurements of soft X-ray emission from a tokamak, which reflect changes in temperature and density, showed a characteristic behaviour: a gradual rise followed by a steep fall. The resulting signal looks like the teeth of a saw, hence the name. The puzzling observation was explained almost immediately by the famous theoretician B. Kadomtsev. This classical model, the Kadomtsev-model, explains the essentials of the instability – even if the model is not complete and a lot of research has been devoted to the sawtooth instability later, when more detailed measurements became available. Figure 7.7 shows 3 stages of the instability. The sketches show a few flux surfaces, where the dotted surface has $q = 1$

(field lines closing after single turn). Because of the positive magnetic shear, $q < 1$ inside the $q = 1$ surface, and $q > 1$ outside. Hence, with respect to the field lines on the $q = 1$ surface, the field lines on either side of it move in opposite poloidal direction.

The $q = 1$ surface is sensitive to perturbations with an $m/n = 1/1$ mode structure. This perturbation results in a tilt of the flux tube, bringing it closer to its neighbours at specific places. Figure 7.7 shows how this can lead to reconnection, forming a magnetic island. The island now starts to grow and pushes the original central plasma, the hot core, out, where it redistributes in poloidal direction. The island takes over the role of the centre, its magnetic axis becoming the magnetic axis of the torus. The sketched temperature profile shows how this instability chops off the peak of the temperature and produces a flat top.

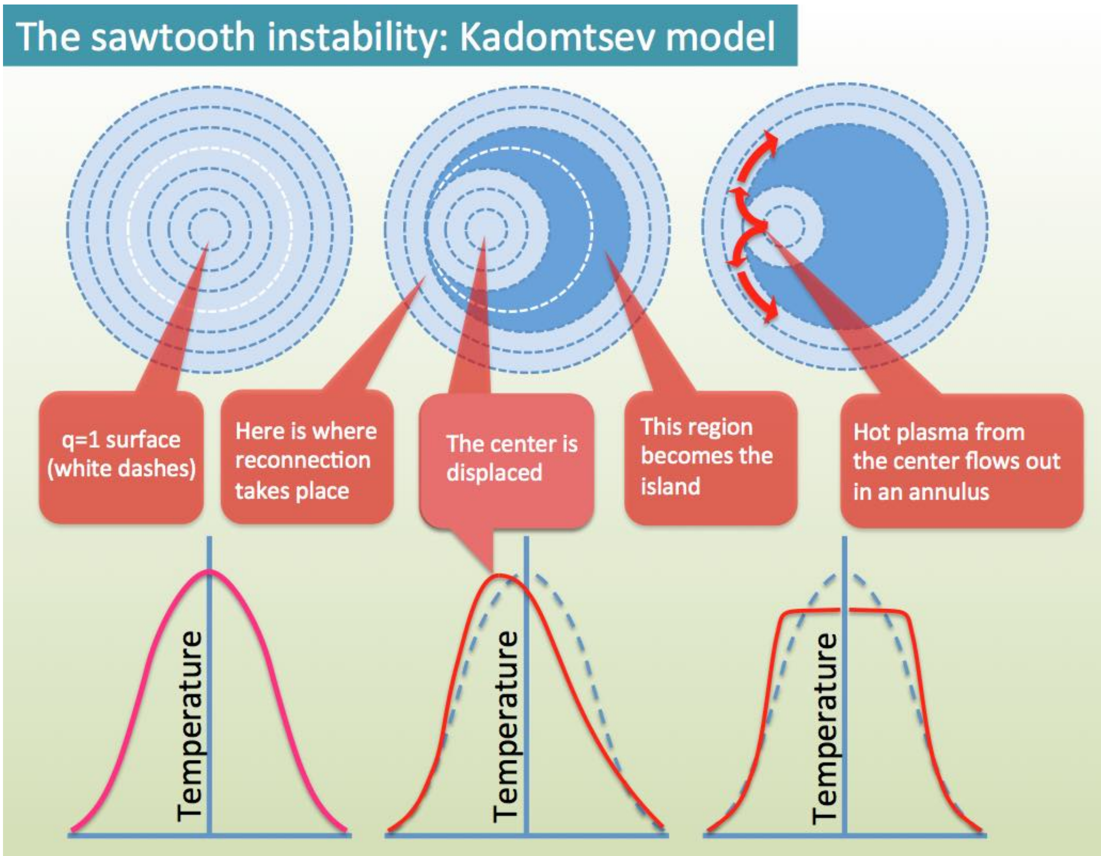

Figure 7.7: Stages of the sawtooth instability. The unperturbed case has concentric flux surfaces, the temperature profile is symmetric. When the centre is displaced, reconnection occurs around the $q = 1$ surface. The displacement is also see on the temperature profile. A large island now forms. Through the X-point of the island the hot center flows out, depositing heat and particles in an annulus just outside the former $q = 1$ surface.

The sawtooth instability has a number of important effects on the plasma.

1. As discussed above, it limits the minimum value of q in the centre to a value close to 1.

2. It limits the peaking of temperature and density.

3. It redistributes particles. This can be a good thing, e.g. to prevent the build-up of a high concentration of He (the fusion 'ash') or other impurities in the centre of the plasma, where they would quench the fusion reaction.

4. The sawtooth instability can be the trigger for the formation of islands at other flux surfaces with different mode number. In particular, the so-called Neoclassical Tearing mode, which could be a limiting factor for the achievable pressure in a fusion reactor, starts from a 'seed-island', and the sawtooth instability could provide this.

So, the sawtooth instability can have good and bad effects on the plasma. For this reason we want to control it in a reactor: by manipulating the frequency and amplitude of the instability, we can use it to control the temperature and pressure peaking, to flush the helium ash, while avoiding the bad effects associated with too large amplitude of the sawtooth.

7.7 Disruptions

The sawtooth instability is beneficial for some phenomena and unwanted for others, but if controlled and kept at small amplitude it is not a problem. In contrast, islands at the $q = 2$ surface—close to the wall—are generally unwanted as they threaten the existence of the plasma. These islands can become very large, up to some 30% of the plasma radius! The magnetic perturbation is then so large that it induces mirror currents in the vessel wall. As a result the mode locks to the wall and can grow even bigger. By this time, the mode is so large that it touches and overlaps with other modes, such as the $1/1$ at the $q = 1$ surface, and the $3/2$ mode between the $q = 1$ and $q = 2$ surfaces. The interaction of the islands leads to formation of chaotic field, which results in a strongly enhanced heat conduction from centre to wall. The plasma cools down in a fraction of a millisecond. This has dramatic consequences:

- the high heat flux could end up in localised spots, where it can lead to melting of the plasma facing components.
- the plasma stops being a good electrical conductor because of the temperature drop. Therefore, the plasma current will try to find another path (Lenz' law, dI/dt is large and therefore generates a high loop voltage) in toroidal direction, and the vacuum vessel is the best possibility. These currents in the vessel wall will have components that are not perfectly aligned to the toroidal magnetic field, with strong forces between the vessel and the coils as a result.
- the high toroidal electric field induced during the disruption can also accelerate electrons, that can reach energies of tens of MeV. Such high-energy electrons can penetrate many centimetres into a metal, so when they lose confinement, they could do damage to e.g. the cooling water pipes in a reactor, causing water leaks. Not desirable in a reactor.

Disruptions can have different causes (see 'operational limits'), but in all cases it is the growth of a large $m = 2$ island close to the wall that kills the plasma. The energy released and the associated forces on vessel and coils scales with volume and plasma current squared, hence disruptions are much more problematic in large than in small tokamaks. In experiments like JET and ITER, disruptions must be avoided as much as possible, and mitigation techniques are put in place to avoid damage in case a disruption happens any way. So, if we cannot avoid a disruption, at least we must detect the initiation in an early phase, so that appropriate measures can be taken.

The disruption is one of the fundamental problems of the tokamak. Ultimately, it can be attributed to the fact that the plasma current represents such a large bath of free energy, comparable to the kinetic energy of the plasma. This is where the stellarator, which does not have a plasma current, has a fundamental advantage over the tokamak. But it comes at the cost of a much more complex device, which moreover still has to demonstrate its capabilities as a reactor candidate.

7.8 Take-home messages

The field components A toroidal magnetic confinement system (i.e. a tokamak or a stellarator) needs three magnetic field components:

Toroidal field (main function: reduce cross-field transport)
Poloidal field (main function: balance the plasma pressure)
Vertical field (main function: balance the hoop force)

The rotational transform The toroidal and poloidal fields together form a helical field. 'Rotational transform' is essential to eliminate charge separation due to drifts.
In a tokamak a plasma current is driven by induction, which requires a time-varying magnetic flux through the central hole. The can be achieved with a (iron yoke) classical transformer setup, or by dedicated coils that fit in that central hole: the central solenoid. The tokamak is a pulsed machine.
A stellarator creates the rotational transform through external coils only. The stellarator is in principle a steady state machine. The rotational transform is characterised by the field line winding ratio, or q-factor: the number of toroidal turns a field line needs to make 1 poloidal turn. In a tokamak q is typically around 1 on the magnetic axis and increases towards the plasma edge. In a stellarator the q-profile is externally determined, and typically quite flat.

Flux surfaces The field lines in a toroidal system lie on flux surfaces: toroidal surfaces that form a nested set. Temperature, density and pressure are uniform on a flux surface. Transport of heat and particles is therefore 1-dimensional, from surface to surface.
Flux surfaces express the topology of the magnetic field. They are traced out by field lines. The only way to find out where they are and what shape they have is by following a field line hundreds of times around the torus. We can do that in the computer to generate a Poincaré map. Particles in the plasma, due to their long mean free path, actually trace out the flux surfaces in the torus. That's why these flux surfaces have a very concrete meaning.

The complications: resonant surfaces. If a field line closes back on itself after a number of turns, the flux surface is called 'resonant'. Resonance occurs if q is a rational number: q=n/m. A resonant surface is 'topologically unstable': it can degenerate and form magnetic islands.

Magnetic islands Magnetic islands are flux tubes with their own (secondary) magnetic axis, that wind around the main magnetic axis with the same winding number m/n as the original resonant surface. In a poloidal cross-section of the plasma, the island appears as m bean shapes (not to be confused with banana-orbits). A small (O(0.001)) perturbation of B is sufficient to cause sizeable islands. Surfaces with simple rational q (1, 2, 3, 3/2, ...) can form large islands.
Magnetic islands can:

- reduce energy confinement: heat conduction from the inside to the outside of the island is along field lines, hence fast. Typical example: the neoclassical tearing mode. Meaning: limit to the achievable pressure in the tokamak.
- cause periodic instabilities: gradual growth, sudden collapse. Typical example: the sawtooth instability. Meaning: redistribution of pressure in central part of the plasma, keeps q above 1 (avoids continued peaking of temperature and current density).

- cause disruptions: if islands grow too large and start to overlap (usually initiated by the growth of a large m=2 island) the plasma can cool within milliseconds. Result: the resistivity increases, the current goes down, the plasma stops altogether. Disruptions can severely damage the reactor and must be avoided. They are an unsolved potential showstopper of the tokamak. Stellarators don't have disruptions.

Problems

7.1 The q-factor in cylindrical geometry

In cylindrical geometry it is easy to calculate the q-profile, i.e. q as function of the radius ($q(r)$), if the current density profile $j(r)$ is known. Consider a cylinder with radius a in which a total current I_0 runs in longitudinal direction, while there is also a uniform longitudinal B-field B_T, and use the definition $q = q_{cyl} = rB_T/RB_p$, where $B_p(r)$ is the field generated by the current:

(a) calculate $q(r)$ for the case that all current is concentrated on the axis of the cylinder (i.e. a delta-function)

(b) calculate $q(r)$ for the case that $j(r)$ is uniform

(c) calculate $q(r)$ for the case that $j(r) = 1 - (r/a)^2$

(d) give an expression for $q(0)$ and $q(a)$ in terms of $j(0)$ and I_0

(e) map a flux surface onto a rectangle with the toroidal and poloidal angels along the axes, and plot the trajectory of a field line on a flux surface for q=1, 2, and 3/2, respectively. (everything still in cylindrical geometry)

(f) The magnetic axis of a magnetic island still has the same q-value as the original flux surface. With this, and the picture you just drew, in mind, how many 'islands' do you see in a poloidal cross-section of a tokamak, for q=1, 2, 3/2 as above.

7.2 Flux surfaces and magnetic islands

The topology of the magnetic field in a tokamak or stellarator is that of nested toroidal surfaces. Field lines lie in those surfaces.

(a) The surfaces are called flux surfaces. Explain why. Which flux is constant on a flux surface?

(b) Explain why flux surfaces are isobaric as well as isothermal surfaces.
Now that is true in a steady state situation (why?). Of course, it is possible to perturb the density and temperature locally on a flux surface. That will set in motion transport processes parallel to the magnetic field that eventually will restore the isobaric and isothermal state.

(c) Which transport process will be faster: particle transport or heat transport. (we are running a little bit ahead of the chapter on transport here, but this is parallel transport, so the B-field does not play a role. Therefore, you can solve this without special knowledge, just common sense will do).

(d) We can fuel the plasma by shooting a frozen fuel pellet into it. The pellet ablates (due to the impact of the high energy electrons) which provides a strongly localised source of particles on the flux surfaces it intersects. Supposing that the flux surface will stay intact, describe the sequence of events. (hint: the pellet is cold, so it carries particles, but very little thermal energy).
Such a phenomenon, if it happens on a surface with a rational q value (q=m/n) will give rise to a perturbation of the temperature, hence the resistivity, which is localised in poloidal angle. Such a perturbation can easily give rise to the formation of a magnetic island.

(e) What is the value of the safety factor q at the flux surface where an m/n=2/1 island can form? What is the q factor of the field lines that live inside the island?

(f) In an island, there is transport along field lines (i.e. very fast) from one side to the other. Consider the separation between the magnetic island and the field around it. This separation is the imaginary surface called the separatrix. How many times does a field line on the separatrix have to go around the torus in toroidal direction to circle once around the island in poloidal direction?

(g) An island is formed by a perturbation of the poloidal magnetic field with respect to the ideal topology. This can be described as a surplus or deficiency of current density (along the field lines, so mainly toroidal) in the O-point of the island. Question: what is it – a surplus or deficiency? The answer depends on the radial dependency of q. Explain. (This is a tough question, but you can still solve it without any math, just imagine that you sit on a field line inside the island)

7.3 The sawtooth instability

The sawtooth instability is important for a number of reasons. Let's think it through.

(a) The sawtooth instability occurs when a $q = 1$ surface forms. Unless you take special measures, this automatically happens in a tokamak. Why?

(b) The sawtooth instability periodically mixes the plasma within a radius known as the 'mixing radius'. Sketch the evolution of the temperature at 5 different radii in the plasma: a) the center; b) the mixing radius; c) the radius in between the center and the mixing radius which is known as the 'inversion radius' (and explain its name); d) just outside the mixing radius and e) further away from the mixing radius. Explain why you drew these time traces the way you did.

(c) What is the typical duration of the 'crash' of the sawtooth instability?

(d) Make a reasonable assumption for the temperature profile just before and just after the sawtooth crash in order to estimate where the inversion radius is with respect to the mixing radius.

(e) The sawtooth instability has beneficial effects. Mention one.

7.4 The disruption

The disruption is one of the most important problems the tokamak has, and therefore you must familiarise yourself with its basics aspects.

(a) In a disruption, the plasma temperature drops very rapidly (the 'thermal quench'). How long does this typically take? And how does this give rise to the generation of strong electric fields in toroidal direction?

(b) One consequence can be the generation of a population of runaway electrons (remember those from the section on collisions?). When these lose confinement, that can be very damaging for the machine. Why – and why is that worse than dumping the same amount of energy on the wall in the form of thermal particles? What can be done to prevent the damage of runaway electrons?

(c) Another consequence, especially if during the disruption the vertical position control is lost and the plasma flies off, can be the generation of electrical currents in the vacuum vessel. Why is this a problem?

(d) How do the thermal energy content of the plasma and the magnetic energy of the poloidal field scale with the linear dimension of the reactor? Is the 'disruption issue' more or less severe for larger machines?

(e) Stellarators don't have disruptions, at least not the violent ones we know in the tokamak. Why not?

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8 Heating, fuelling and current drive

8.1 Ohmic dissipation and its intrinsic limitation

The plasma in a tokamak carries a toroidal current. Since the plasma has a finite conductivity (see Ch 5.5), this current will generate heat by Ohmic dissipation. The typical loop voltage (i.e. the electric field integrated around the plasma ring in toroidal directions, this is the field that drives the current) is 1 Volt in small tokamaks, and even lower, down to 0.1 Volt, in large ones such as JET or ITER. This leads to a typical dissipated power $P = I \cdot V$ of order 100 kW – 1 MW. Interestingly, this is not too dependent on the size of the tokamak, the reason lying in the strong decrease of the plasma resistivity with increasing temperature. A larger tokamak has better confinement, can reach higher temperature but therefore has lower resistivity. Thus, there is a self-limiting effect for the temperature that can be reached with Ohmic dissipation as a heating source. And this temperature limit, typically 1–3 keV, is too low for a fusion reactor. Note that one cannot cure this problem by increasing the plasma current, because this has to be in proportion with the toroidal field to ensure stability—see section 10.7 on operational limits.
Hence, Ohmic dissipation is useful to start up the plasma, but to reach reactor conditions external heating must be applied. Until eventually the fusion reactions provide enough power through the 3.5 MeV α-particles generated in the fusion reaction to self-heat the plasma. External heating methods come in shapes and sizes—here we discuss the most important ones.

8.2 Neutral Beam Injection (NBI)

One can heat a plasma by injecting high-energy neutral hydrogen. High energy is defined by two criteria. First, the energy of the injected particle must be much higher than the plasma temperature to achieve heating (obviously). In present day devices, the injection energy is typically 50–150 keV. Secondly, the energy must be chosen such that the particles do penetrate the plasma and preferably reach the core, but don't shine through, since in that case they could easily damage the vessel wall. The particles must be neutral because of the magnetic field. For ITER, the requirement that the beams penetrate deep enough calls for 1 MeV beams. That is a technology quite different from the present day NBI, and therefore the development of NBI for ITER is a major R&D programme.
To make high-energy neutral particles, we need a source of charged particles, which are accelerated and then neutralised by passing them through a gas cell where they pick up an electron. Figure 8.1 shows a schematic of a neutral beam injector. Note that there is massive pumping necessary to avoid excessive flow from the gas cel to the tokamak. And a dump for the ions that failed to be neutralised and are deflected by a magnet, to prevent them from entering the duct to the plasma where they would hit and damage the wall (when they enter the tokamak magnetic field).
Inside the plasma the fast neutral undergoes a charge exchange reaction with a plasma ion. This is not a collision – no momentum or energy is exchanged – but a resonant interaction, and therefore has a high probability. There is only the exchange of an electron, and the electron does not have a preference for the slow or fast ion, both are sluggish in the eyes of the electron.
As a result, the 'cold' plasma ion becomes neutral. It is not confined anymore and, depending on the size,

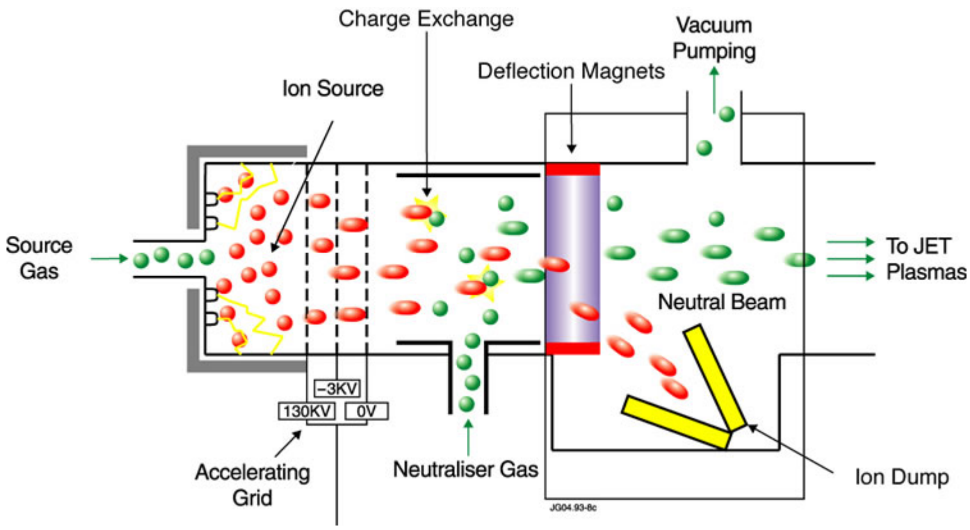

Figure 8.1: Schematic of the neutral beam injector. From left to right we see the particle source, the ioniser, the grids that produce the acceleration, the neutralisation cell and, very importantly, the magnetic filter that deflects the particles that failed to pick up an electron: these charged particles carry a lot of power and should be dumped before they can do damage, e.g. by hitting the duct between the injector and the plasma.(source: EUROfusion)

density and electron temperature of the plasma, it will either leave the plasma or be re-ionised on its way out. In the latter case, the plasma density has increased and the injection of the neutral beam is fuelling as well as heating the plasma.

The 'hot' injected particles become ionised and consequently are trapped in the magnetic field. Thus, the primary effect of the neutral beam injection is the creation of a population of highly energetic particles in the plasma. To achieve an increase of the plasma temperature, these particles still have to share their energy with the thermal plasma. This process happens through collisions with the thermal ions and electrons.

The question is where most of the energy goes: to the electrons or the ions? The answer is: it depends. Of course, energy is transferred much more efficiently in an ion-ion (like-mass) collision than in an ion-electron (large mass difference) collision. But we also saw that in a plasma, the collision frequency depends on the relative velocity v of the colliding particles as v^{-4}. Since the high energetic beam ions – depending on their energy compared to that of the electrons – can have a velocity that is much closer to that of the electrons (i.e. the ones that happen to move in the same direction) than of the ions, the probability of a beam ion - electron collision can be much higher than that of a beam ion - thermal ion collision.

The result of the two competing effects is that depending on the ratio of the injection energy and the electron temperature, the energy of the fast ion population is transferred preferentially to the electrons (for high injection energy) or to the ions (low injection energy), see figure 8.2. Of course, your have to account for the fact that the fast ions, when they collide with electrons, slow down and gradually start to heat the thermal ions, too.

Apart from heating and fuelling, if injected under an angle NBI also transfers momentum to the plasma. This can lead to very high rotation velocities, typically up to acoustic velocity. Such velocities, and the radial shear of the rotation velocity, are used to suppress turbulence. NBI is a reliable heating method. It is applied at almost every tokamak, the larger ones typically having installed up to tens of MW of NBI.

A final word on NBI: a neutral beam injector is necessarily a very large device, typically of the same dimension

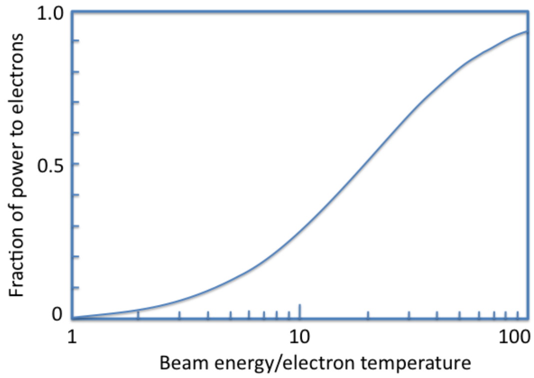

Figure 8.2: The fractions of beam power that go to the ions and to the electrons depend on the ratio of beam energy and electron temperature: high energy beams preferentially heat the electrons. Adapted from Wesson: Tokamaks

as the tokamak itself! And the beams that are injected have significant dimensions, too, so the port through which they enter is large. The size of a door, in ITER. Beams don't pass through windows, the access port has to be open. And that means that tritium from the main chamber will enter the neutron beam system. So this is part of the tritium containing system, with all the extra safety systems that come with that. Figure 8.3 gives an impression of the how large the NBI system at JET is. I selected a picture that highlights the 'rotary valve', the enormous vacuum valve that can separate the gas chamber of the NBI from the torus vacuum, to impress on you how large these structures are!

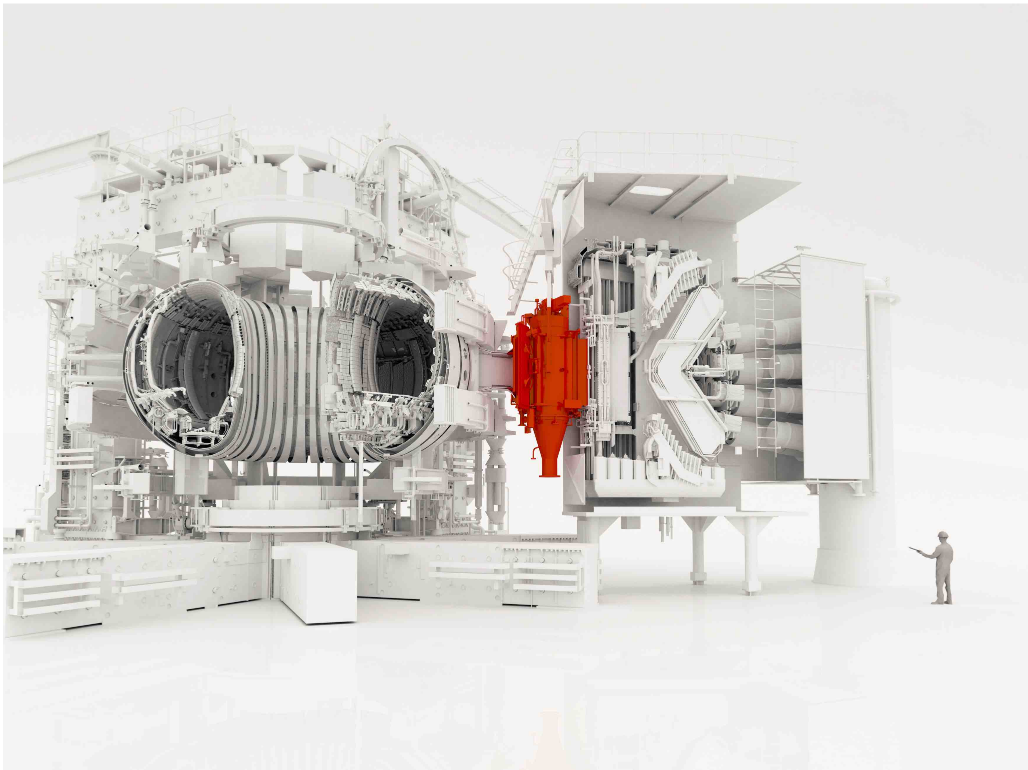

Figure 8.3: Computer rendering of JET (left) and the NBI system (right) separated by the 'rotary valve' (in red). Note how large the NBI is compared to the tokamak, that it needs to be placed directly next to it, and the size of the port/beam duct and vacuum valve – and compare this with the ECH system decribed below.(source: EUROfusion, JET)

8.3 Electron Cyclotron (Resonant) Heating (EC(R)H)

There is a whole range of heating methods that are based on the injection into the plasma of electro-magnetic waves that are absorbed on one of the many available resonances. An obvious choice is the electron cyclotron frequency, $f_{ce} = eB/2\pi m_e$: Electron Cyclotron Resonance Heating (ECRH), aka ECH (both acronyms are used, they mean the exact same thing). The electron cyclotron frequency is proportional to B, and therefore falls off from inside to outside of the torus as $1/R$ ($R =$ distance to principal axis of the torus). Hence, by choosing the frequency of the injected waves, one can choose where they are absorbed. This makes ECH the heating method of choice for precision heating, and unlike NBI, ECH exclusively heats the electrons. It is used to stabilise magnetic islands (which is achieved by heating them — not explained here) and to make local perturbations to the temperature for transport experiments (see Ch. 11). Moreover, by injecting the waves tangentially, it is also possible to locally drive current, giving further possibilities for control of the plasma.

So, how precisely can we deposit heat by this method? Let us first consider the geometrical aspect. For a magnetic field of 2 Tesla, the cyclotron frequency is at 56 GHz. In practice, it is better to use the second harmonic of the resonance (the injected frequency must be above the plasma frequency, so the second harmonic allows application with higher plasma density). In present experiments, the applied frequency lies in the range 110–140 GHz, while for ITER 170 GHz is the chosen frequency. The wavelength at these frequencies is 2–3 mm. The beam can be focussed to a few wavelengths, hence at the position of the resonance the beam size is a few cm or larger.

The resonance can be quite sharp; the injected waves normally have a very narrow spectrum. The resonance in the plasma is, however, broadened by two effects. First, the injected beam of EM waves, in any practical application, has curved (i.e. not planar) wave fronts. As a result, even if the beam is injected perpendicular to the magnetic field, the electrons see the beam under a small angle. The velocity of the electrons along the field then gives rise to a Doppler effect. (The perpendicular velocity of the electrons does not play a role: the absorption process by definition requires the electron to make many cyclotron motions in the oscillating EM field, hence the perpendicular velocity is averaged over the cyclotron motion and therefore is zero). Note that the Doppler effect can also be understood as resulting from the finite time an electron spends in the beam while it is passing through it: if it sees N oscillations of the EM field in that time, the natural line width is $1/N$. A sharper focus of the beam in toroidal direction therefore corresponds to an increase of the broadening of the absorption in radial direction (make sure you think this paradox through).

Secondly, the electron mass in the denominator of the cyclotron frequency is the relativistic mass, $m_e = \gamma m_0$. In a fusion plasma, the thermal energy of the electrons is easily a few percent of the rest mass ($m_0 c^2 \approx$ 0.5 MeV). Hence, the energy distribution of the electrons results in a mass distribution with a width of a few percent. In position, this translates in a broadening of the deposition radius of a few percent of R, the major radius of the tokamak. The combined effect of the Doppler broadening and the relativistic broadening gives an effective deposition width of typically a few cm in radial direction.

Sources that generate high-power EM waves in this frequency domain are technically very difficult to make. For ITER, 'gyrotrons' are under development that will deliver 2 MW continuous, at 170 GHz. These consist of an electron gun that sends an e-beam into a magnetic field, where the electrons gyrate and start to emit radiation at their gyro-frequency. The waves are contained in a resonator and coupled out through a diamond (!) side-window, the e-beam is dumped. on a collector (***why would the window be made of diamond - it is super expensive!***). Figure 8.4 shows a schematic and photograph. A total of 20 MW of ECH is planned for ITER.

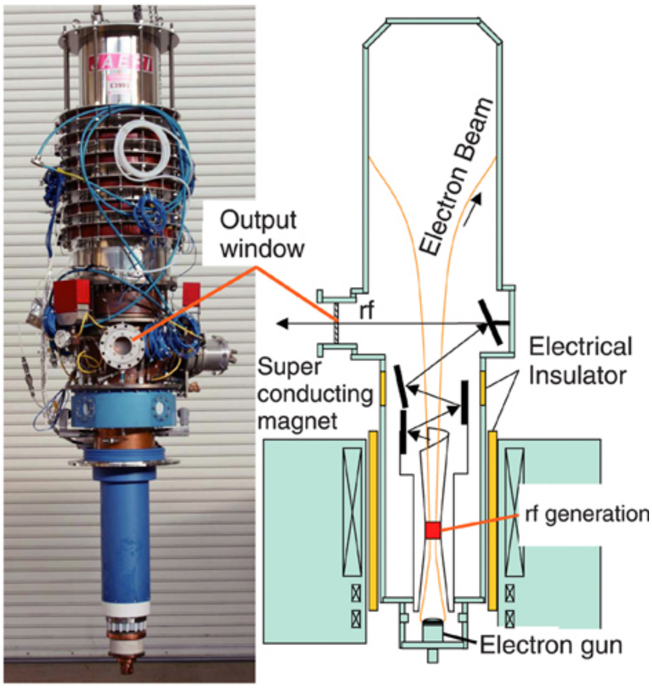

Figure 8.4: Schematic and photograph of a gyrotron. (source: JAEA)

The transportation of the power, on the other hand, is relatively easy. Waveguides, metal tubes with a width a couple of times the wavelength, can carry megawatts of EM power over long distances with relatively low losses. (Figure 8.5.

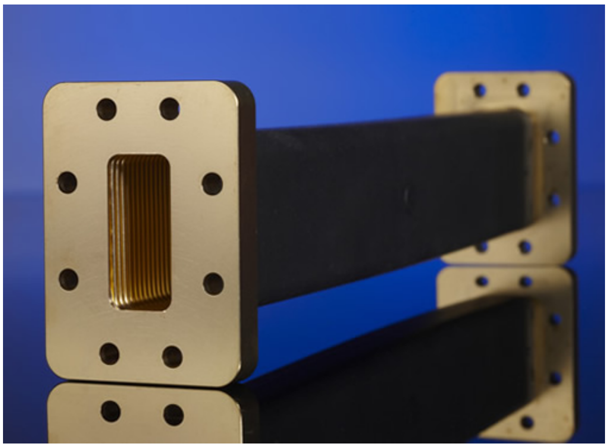

Figure 8.5: A waveguide for ECH waves. It is 'corrugated' on the inside to reduce losses through wall currents; it is rectangular to preserve polarization. It is typically a few cm across. (source: http-//www.flexiguide.com/products-rectangular-seamless.htm

That means that the sources and all control soft and hardware can be placed far away from the tokamak, in an easily accessible area. In ITER, the gyrotrons are placed some 100 m from the tokamak (see Figure 8.6. The waveguides, because they are thin, can pass the biological shield relatively simple with a 'dog-leg' construction to avoid neutron leakage. A third great advantage is that the access to the torus itself can be compact. A relatively small port is sufficient to bring the waveguides close to the plasma. Things become a little hairier, from a design perspective, if you want to have control over the direction and polarisation of the injected beam. This calls for steerable mirrors right next to the hot plasma. Not an easy thing to build, but for ITER these are indeed foreseen.

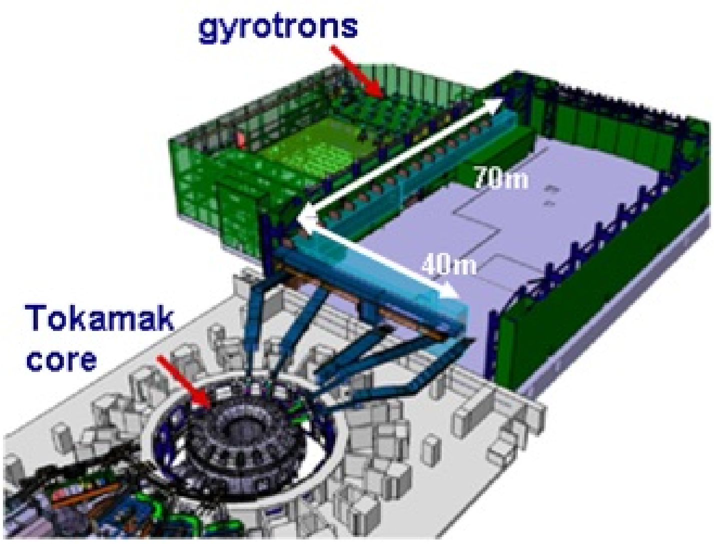

Figure 8.6: Overview of the ECH system in ITER: the gyrotrons – with a total output power of 24 MW – are placed in a separate building some 100 m away from the tokamak, their microwave power being transmitted to the plasma through low-loss waveguides (a total of about 4 km!) (source: www.usiter.org))

8.4 Heating: Ion Cyclotron Resonant Heating (ICRH)

Similar to ECH, we can use EM radiation at the much lower (by the mass ratio m_e/m_i) ion cyclotron resonance to heat ions. Here the wavelength is of order meters, so that sharp localisation of the heating is not possible. There are a number of aspects of the wave propagation of ICRH that fall outside the scope of this course (as it does not really treat wave propagation: we have a special course for this: 3MF130 Heating and diagnosing fusion plasmas).

Important to know is that this low frequency does not propagate in the layer of vacuum between the antenna and the plasma. Therefore, the antenna is carefully designed so that it can be placed very close to the plasma - where inevitably it will take a large heat load - and conversely, the operator must control the plasma position precisely to keep it at a predetermined distance from the antenna.

A second interesting aspect is that in most cases, it is better not to absorb the power on the hydrogen (isotopes) itself, but on a 'minority', e.g. a small percentage of helium mixed in the plasma. This absorbs the power very efficiently and then heats the rest of the plasma through collisions. ICRH does produce a strongly non-thermal tail on the ion velocity distribution– a fast ion population.

Sources for power at the ICRH frequencies are much less of a technological challenge than for ECH. ICRH is applied at many large experiments and is planned for ITER, too. Just to give you an impression of the state of the technology, I include a picture with some explanation of the ECRH antenna for ITER, taken from the ITER site: Figure 8.7.

8.5 Fuelling: Pellet Injection

A fusion reactor is operated with an extremely low inventory of fuel in the vessel, for seconds of energy production only. Moreover, the burn-up fraction of the gas that is pumped out is very low, at the percent level (it cannot be high, because the ash would choke the reaction). Therefore, there must be a continuous circulation of the fuel. The divertor serves to neutralise and pump off the gas. But how do we bring in fresh fuel?

One way of fuelling the plasma is by the injection of pellets of frozen hydrogen. This has to be done at very high velocity, because the pellets ablate (ablation = phase transition from solid state directly to gas) very fast under the energy transfer of the impinging electrons. For this purpose, a pellet injector is used that is

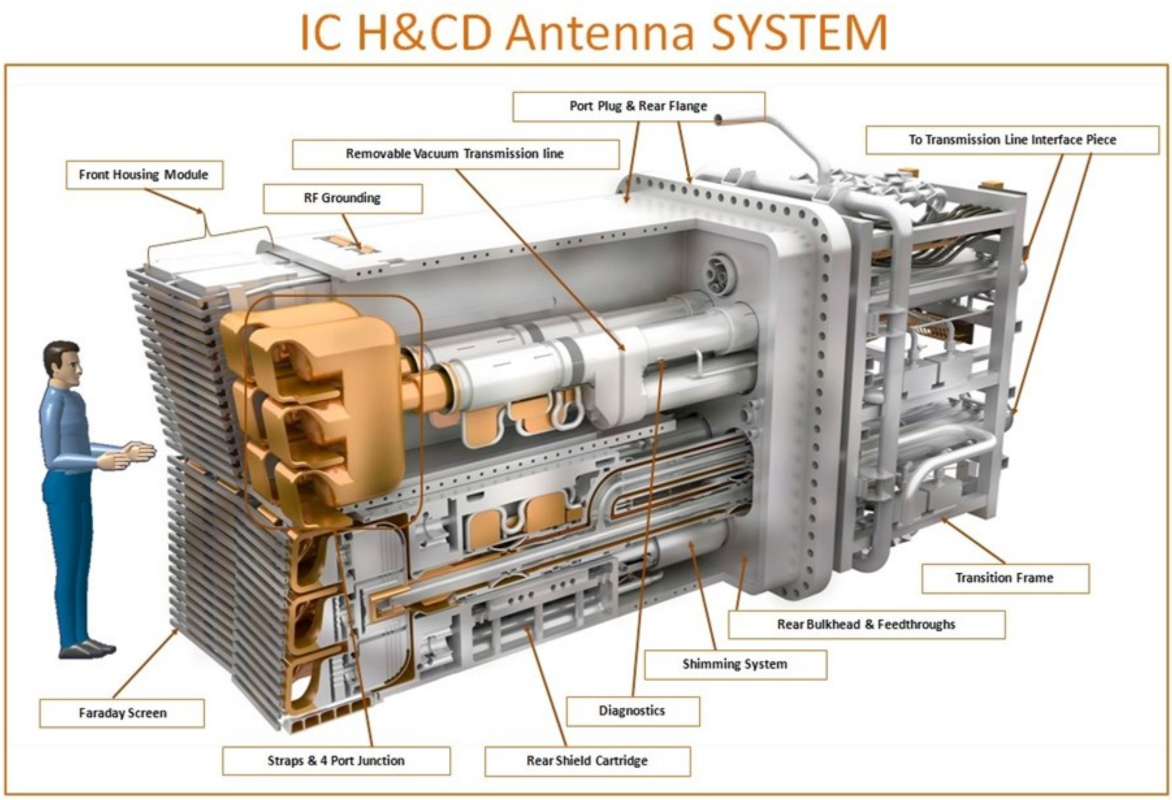

Figure 8.7: Image of the ICRH antenna for ITER. The actual antennas are the gold-coloured straps that face the plasma. They are shielded by a grill, the so-called Faraday screen. The power is delivered to the straps through a coaxial transmission line the size of a sewage pipe .(source: ITER)

basically a gun with pressurised hydrogen as the driver gas. (hydrogen, the lightest element, has the highest sound velocity, and this is the limitation for the speed that can be given to the pellet.) The ablation rate is a strong function of the electron temperature. Therefore, if a pellet is fired into the plasma and moves towards the centre, the ablation rate increases steeply until the pellet is fully ablated: in effect, the pellet ablates in a bright flash just before it dies. (Bright indeed, because the neutral gas that results from the ablation emits visible light). Thus, by choosing the speed at which the pellet is injected, the operator can select the position where the deposition of the particles peaks.

It should be noted that in a reactor, due to the plasma size and high electron temperature, the injection speed that would be required to make the pellet reach the centre is much higher than what can be achieved by the technique described above. There are other techniques to eject pellets, e.g. by fixing them on a fast rotating wheel and releasing them, but to reach the injection speeds required in a reactor remains a challenge. Pellets have other beneficial effects apart from deep fuelling. They can be used to control the density profile, and also to manipulate MHD instabilities.

Pellets are usually injected from the outboard side of the vessel. But it has been found out that injection from the inboard side is advantageous, as the dense plasma that forms where the pellet ablates has a tendency to drift outward, in the direction of increasing major radius. For a pellet that was injected from the inboard side, this means that the injected particles move towards the centre. Of course, the price paid is that it is technically much more challenging to realise inboard pellet injection. The pellet injector must fire the pellets into a long tube that, with gentle curves so as not to damage the pellet, follows the inside of the vacuum vessel and somehow launches the pellet under the correct angle into the plasma. This has been demonstrated on e.g. JET, but the experience is still limited. Yet, for full-blown fusion reactors, the size of ITER and larger, pellet injection from the inboard side is considered to be the only viable option.

8.6 Fuelling: Gas puff

A very simple technique to fuel the plasma is by simply injecting gas in the edge of the reactor. This will be partially ionised. By transport mechanisms that are not yet fully understood, this edge source of particles leads to a slight peaking of the density in the centre - an inward 'pinch'. But, for large reactors gas puff will not be a sufficient fuelling mechanism.

8.7 Fuelling: Neutral Beam Injection (NBI)

Finally, apart from heating NBI also brings particles into the plasma, when the plasma is large and hot enough to re-ionise the neutral particles that are created in the charge exchange reaction before they leave the plasma. For the experimentalist this coupling of heating and fuelling can be drawback. In particular, application of NBI can lead to unwanted and unavoidable increase of the density, which can send the plasma to a disruption if the operator is not careful.

8.8 Current drive

Since the tokamak – for the electrical engineers among us – is just a transformer, it will be clear that if the current is driven by induction only its pulse duration is limited. It may be tens of minutes, or even hours in a large reactor, but certainly not days, let alone a year. If we want to operate the tokamak in steady state we'll need to drive the toroidal current non-inductively. There are several ways to achieve that. NBI drives current if you inject tangentially: although the beam itself is neutral, the difference of momentum transfer to ions and electrons results in a current. And ECH can be used to drive current (then it is called Electron Cyclotron Current Drive – ECH), if the wave are launched tangentially to the plasma and through the Doppler shift are preferentially absorbed by electrons that move in one direction. These then become 'hotter' and as result experience less collisions, hence resistance, than the electrons that move in the other direction, with a net current as result. And, there are lower frequencies, too, which can be employed to drive current using phased antennas. Current drive technology is an active area of development, it is not at all a solved issue. It is outside the scope of this course to delve deeper into this now, suffice it to say that all of the current drive methods have in common that their efficiency (MA driven per MW) is too low, especially at the relevant (high) densities. Why is that a problem? Because we need power to run the current drive systems, which themselves have a finite efficiency, and we could easily end up using half of the electricity produced by a power plant for the current drive systems. Of course, that is not good for the overall plant efficiency, and eventually for the price of the electricity that is produced. So: progress is needed here!

Problems

8.1 Ohmic heating

Even if no other heating is applied, a tokamak plasma is heated by Ohmic dissipation. Straightforward, but it does have some interesting aspects.

(a) What is the cause of the so-called 'Ohmic heating' in a tokamak? Explain why it is not sufficient to reach the required burn temperature of some 15 keV. Would a stellarator exhibit Ohmic heating?

Consider two tokamaks, one small ($R = 0.3$ m) and one large ($R = 3$ m). Both have the same toroidal magnetic field B_T and aspect ratio R/a and have cylindrical cross sections. Both are operated with a current I_p such that the cylindrical safety factor $q_{cyl} = aB_T/RB_p$ is the same. Assume that the heat conduction coefficient κ is uniform, independent of temperature and the same in both devices. Show, considering only functional dependences, not the numerical values of constants, by which factor:

(b) the temperature in the large device is higher (or lower) than that in the small device

(c) the loop voltage (needed to induce the current) is higher (or lower) in the smaller device compared to the larger.

(This is back-of-the-envelope: you may think you do not have enough information to answer this, but you do).

8.2 Neutral Beam Injection

Neutral Beam Injection is applied to virtually every tokamak. But it gets harder when going to larger machines!

(a) Explain the processes of Neutral Beam Injection (NBI) heating that result in a population of fast ions in the plasma.

(b) How is the energy of this population transferred to the bulk plasma?

(c) Which species gets most of the power of NBI: electrons or bulk ions? Explain your answer.

(d) The NBI system at JET injects the particles with an energy of 140 keV. In ITER a much higher energy (500 — 1000 kV) will be used. Why?

(e) The JET NBI system is based on the acceleration of positive ions (D^+ for instance). Why can this principle no longer be applied in reactors the size of ITER or larger?

(f) Tangential NBI heats the plasma but also transfers momentum — which is useful if you want to make the plasma rotate in toroidal direction. For maximum momentum transfer at given heating power, would you maximise or minimise the injection energy (within the limits set by other considerations)?

(g) The neutral beam used for heating can also play a role in diagnosing the plasma. Which measurement is enabled by the neutral beams? (running ahead of the chapter on diagnostic techniques here — use your imagination).

8.3 Electron Cyclotron (Resonance) Heating

If heating is applied to simply increase the central plasma temperature, the deposition profile is not that important, as long as it is central. However, if heating is applied with the aim to manipulate — optimise — the temperature profile, it is essential that the heat deposition can be concentrated in a narrow region. The same holds *a fortiori* for current drive.

(a) Electron Cyclotron Heating (ECH) is a resonant process, which in principle yields strongly localised power deposition. Two mechanisms broaden the deposition (in radial direction). Which are those two mechanisms? Give a third reason — not a mechanism, really — why the power deposition profile is finite.

(b) Why is Electron Cyclotron Heating used for local heat deposition, whereas Ion Cyclotron Heating is much less suited for that?

(c) In a large, hot reactor plasma (in this context this means: the *optical depth* is very high), would ECH waves launched into the plasma in the mid-plane from low-field side be absorbed by electrons that have a relatively high energy (compared to the average in the distribution function), or just the opposite (relatively low energy)?

(d) Same question, but now in an optically thin plasma?

(e) Same question for the situation that the waves are injected from the high-field side?

(f) Calculate — approximately — the broadening (i.e. give a length) of the power deposition profile due to the finite toroidal width of the antenna pattern, taking this to be 0.1 m, with $T_e = 10$ keV, and major radius $R = 6$m.

(g) Why is ECH mostly used at the second harmonic frequency (as opposed to the first harmonic)?

(h) In a tokamak plasma with 'normal' magnetic shear (i.e. q increases with radius) the O-point of a magnetic island corresponds to a region with slightly reduced current density. How can ECH be used to suppress the island? Explain where and when the power must be applied, and why this suppresses the island.

(i) How, in your island suppression scheme, do you make sure that the ECH power is absorbed where you want it (i.e. you have described the actor, but we need a sensor to steer it).

(j) In general, the plasma and therefore the island will rotate in toroidal direction. How does this influence the island suppression scheme you described just now.

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9 How to take measurements in a hot plasma?

9.1 What do we want to measure, and why? (physics and control)

Roughly speaking, this is what we – researchers and operators – want to know:

1. Where is the plasma, what shape does it have, how much current runs in it, how large are the electric and magnetic fields applied to it, what is the plasma density? These measurements are all required to keep the plasma in stable confinement, under feedback at preset values.

2. What are the pressure, temperature, density, current density and flow velocity, as function of spatial coordinates (spatial resolution)? And how do these evolve as function of time? This information is required, in real time, to optimise the plasma conditions and steer close but not too close to operational boundaries. But also for fundamental studies of the plasma, such as transport processes, this spatially and temporally resolved information is necessary.

3. Is the plasma turbulent? If so, what fluctuates: density, temperature, electric and/or magnetic field? What do the fluctuating spectra look like? Broadband, or coherent modes? Do they evolve in time? Is there a relation with the sheared flow velocity? Or: what exactly happens in the edge of the plasma, where it is in contact with the wall? Which impurities enter the plasma, how deeply do they penetrate? Or: what are the properties of the fast particle populations in the plasma (resulting from fusion reactions, or neutral beam heating, or ICRH). And how do they interact with e.g. the formation of magnetic islands, or with turbulence? Or: what is the detailed spatial and temporal structure of the Edge Localised Modes?

The success- scientifically speaking – of any fusion experiment depends to a large extent on the variety, quality and completeness of the diagnostic equipment. If you visit a fusion experiment, you will probably be amazed by the fact that you have to look hard to actually see the tokamak or stellarator, because it is hidden under a layer of diagnostics! Figure9.1 gives an impression of the diagnostics that are mounted on JET, one of the best-diagnosed fusion experiments in the world. Plasma diagnostisation is almost a field of science in itself.

9.2 How to take measurements in a hot plasma?

A hot plasma is a nasty environment to take measurements in. You cannot stick anything in it! So what are the options? Outside the plasma we can measure electric and magnetic fields, using loops and coils. (We can even stick wires—so-called probes—a little distance into the cool edge of the plasma.) Or we can detect whatever comes flying out of the plasma: radiation (→ **spectroscopy**) and particles. Or we can shoot things into the plasma, photons or particles, and observe how they are absorbed or diffracted or scattered or deflected. In the special conditions of a fusion plasma, with strong magnetic and electric fields, all sorts of effects, such as Faraday rotation, the Cotton-Mouton effect, Zeeman-splitting etc, occur naturally and can be used to create a smart measuring technique.

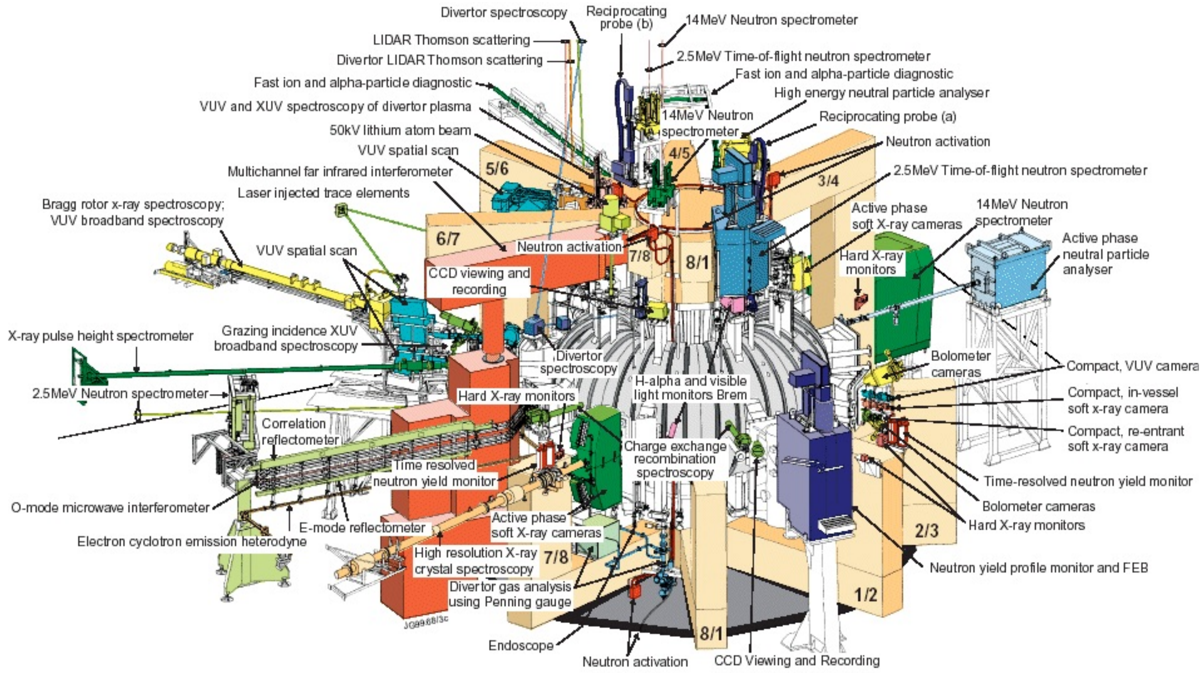

Figure 9.1: Overview of the diagnostic systems in use at the JET experiment. Take some time to read all the labels! (source: EUROfusion)

Below you find a short brainstorm list. It is far from complete, but gives an impression of the wide variety of techniques used to diagnose what goes on inside a hot plasma. The chapter on plasma diagnostics from the Carolus Magnus Summer school (see `www.carolusmagnus.net`) is recommended as background reading. And of course, if you want to know more about plasma diagnostics: join the course 3MF130!

Passive methods:

- Magnetic measurements: place pick-up coils around the plasma and measure the (rate of change of) magnetic field components. From this you derive the plasma current, the position of the plasma, its shape,$\cdots$
- Use flux loops to measure the rate of change of the flux through the central hole in the torus, to derive the loop voltage.
- Spectroscopy of the radiation emitted by the plasma. This is made up by a continuum stretching all the way from the far infrared to the X-ray region, and emission lines from not-fully-stripped ions in the plasma. The line radiation is emitted predominantly in the cold outer region of the plasma, inside the ionisation is almost complete and practically no spectral lines are emitted. The hot core does emit continuum radiation as a result of the collisions of the electrons with the ions (Bremsstrahlung), and this lies in the energy range of the thermal energy of the electrons: the keV range, or Soft X-rays. In addition, the plasma emits at the electron cyclotron frequency (mm-waves).
- Analysis of (neutral) particles that leave the plasma. As the hot plasma is not transparent for neutral particles (their mean free path for ionisation is a few centimetre) the neutral particles observed outside the plasma carry information about the edge of the plasma.

- Analysis of the neutrons emitted as a result of the fusion reactions. This will give you at least a time resolved measure of the fusion reaction rate, and if you set up the measurement properly, also of its spatial distribution.

Active methods:

- pass a laser beam through the plasma,
  - measure the phase lag by comparing to a reference beam and find the density$\rightarrow$ **interferometry**
  - measure the rotation of the polarisation plane and find the poloidal magnetic field $\rightarrow$ **polarimetry**
- launch mm-waves into the plasma,
  - reflect them off the plasma frequency and find the density profile $\rightarrow$ **reflectometry**
  - with waves at the electron cyclotron frequency, measure the absorption and measure the pressure $\rightarrow$ **Electron cyclotron absorption.**
  - measure collective (wavelength $>$ Debije length) scattering on electrons and find density fluctuations due to turbulence$\rightarrow$ **collective Thomson scattering**
- scatter (laser) light on plasma electrons (wavelength $\ll$ Debije length) and find the electron temperature and density $\rightarrow$ **Thomson scattering**
- inject neutral particles (not necessarily hydrogen) into the plasma, to generate light emission and measure the ion temperature, or the helium concentration, or the flow velocity $\rightarrow$ **diagnostic neutral beam, charge exchange recombination spectroscopy, or neutral particle analysis** (but now of neutral particles generated in the interior of the plasma by the beam).
- Inject a beam of heavy particles into the plasma to $\rightarrow$ **heavy ion beam probe**
  - measure their deflection due to the field and determine the internal plasma potential fluctuations
  - induce line radiation of these heavy particles by laser excitation and analyse the emission, e.g. to measure the poloidal magnetic field $\rightarrow$ **laser induced fluorescence**
- inject pellets and diagnose the light emitted by the ablation cloud.
- $\cdots$ *and many more*

Let's have a closer look at a few of those options.

9.3 Measurement of the density: interferometer

The refractive index of a plasma is a function of the density. Hence, by measuring the optical path length of a beam through the plasma one can determine the line integral of the density. Such a measurement is done with an interferometer: the light from a coherent light source is split into two beams. One, the reference beam, goes to the detector through air (effectively vacuum), the other—of equal physical length—through the plasma. The two beams are combined on a detector where they interfere, so that the detected intensity is a measure of the phase difference between the two legs. Figure 9.2 explains the set-up and measurement.

The measurement is calibrated before the plasma is started. During the build-up of plasma density, the phase of the plasma-leg will gradually change, and this is measured as a change in the phase difference between the two legs. For every phase shift of 2π the detection signal passes through a dark-light-dark cycle.
The basis for this measurement is the dispersion relation, which describes the relation between the frequency and the wavelength of a wave in the plasma. Perpendicular to the field this is

$$k = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2}, \tag{9.1}$$

where ω_p is the plasma frequency. By expressing the phase rotation the beam picks up during its passage through the plasma, and choosing a frequency much higher than the plasma frequency, $\omega^2 \gg \omega_p^2$, we find

$$\Delta\phi = \frac{1}{2c\omega}\int \omega_p^2 \, dl \tag{9.2}$$

The plasma frequency squared is directly proportional to the plasma density, no other plasma parameters come in. Hence, the phase shift does indeed provide a measure of the line integral of the density along the path of the beam. In this expression we only have the frequency of the probing waves and the velocity of light. Both are very well known, so that this measurement is absolutely calibrated. The only calibration needed is the 'zero', which is easily done by starting the measurement before the plasma is created.

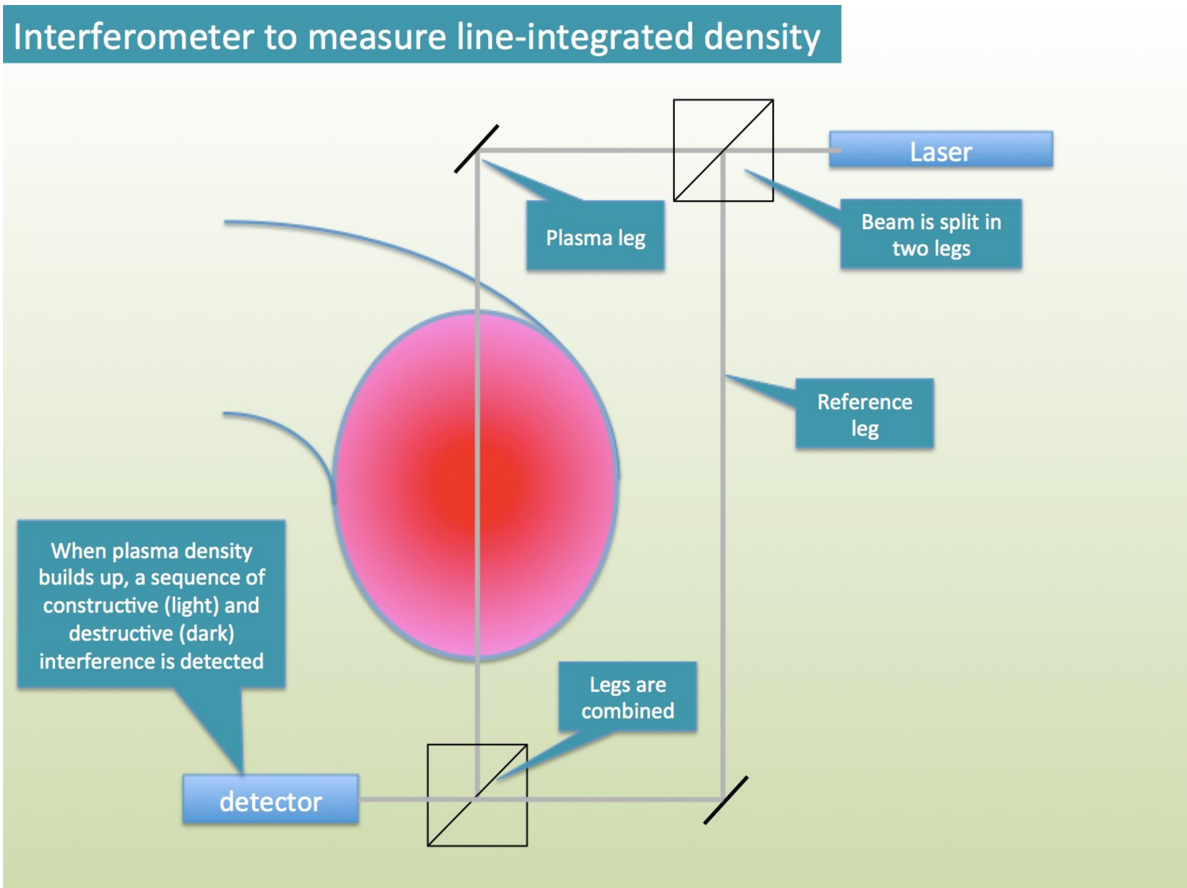

Figure 9.2: Schematic of a single-chord interferometer. This setup will measure the line-averaged density of the plasma along the probing chord. This is a robust measurement, indispensable for density control. To resolve the radial profile of the density a multi-chord interferometer must be used.

9.3.1 Abel inversion

Interferometry, or any other method that provides line-integrated quantities, does not give local information. If you want to measure the density profile, you'll have to measure the line integrals along many chords simultaneously. Here we are helped by the fact that the density is a flux surface quantity. In the simplification that the flux surfaces (projected on a poloidal plane) are concentric circles on which the density is constant, the map from the density profile to a line integral is a classical integral equation: the Abel equation. In our case, we have the inverse problem, i.e. we measure line integrals and want to recover the density profile. This reconstruction is called Abel inversion. The principle is very simple: the measurement of the outermost chord gives the density in the edge, which is then used to correct the next measurement inward for the contribution to this line integral in order to derive the density at a more inward radius. And so on. There are many ways in which this can be implemented numerically. But independent of the numerics, the method is only stable for a peaked density profile.

If the flux surfaces are not circular and not concentric, which is usually the case, the above procedure can still be carried out with only slight modification, as long as the shape of the flux surfaces is known. Essential is, however, that the density is constant on a surface. Otherwise the density distribution is truly 2-dimensional, or even 3-dimensional, and many more measurements are required.

9.3.2 Tomography

The more complex 2-dimensional density distributions (in the poloidal plane) often occur in perturbed, non-equilibrium plasma states. But these are the phenomena we are most interested in! Magnetic islands, disruptions, strong plasma movement, redistribution of density during pellet injection, etc. To reconstruct such 2-dimensional profiles we need to take many measurements, with chords crossing the plasma from different angles. Typically many tens of view lines are required, depending on the level of detail we want to recover. The generic term for this technique is 'tomography'. It is exactly the same technique used in medical scans of internal parts of the body. The computer reconstruction of the image is a well-developed field in which various mathematical and computational approaches are applied.

9.4 Density measurement: reflectometry

A beam of microwaves will penetrate a plasma as long as the frequency is above the plasma frequency. In a fusion plasma, the density - and therefore the plasma frequency - is normally low at the edge and high in the centre. Hence, a wave with a frequency that matches the plasma frequency at some point in the density profile will reflect from that point. The reflected wave can be detected. By measuring the optical path from sending to receiving antenna you can determine the distance from the reflecting layer to the antenna. This gives a measurement of the plasma density at one point (or rather, it gives a measurement of one location for a given plasma density). By doing this for many frequencies the density profile can be constructed. This can be done by sweeping the frequency.

The determination of the optical path length can be approached in two different ways:

1. interferometry - by comparing the phase of the reflected beam with that of a reference beam outside the plasma
2. time-of-flight measurement - by sending very short pulses into the plasma, and measuring the flight time of the reflected pulse (the further the reflecting layer, the longer the flight time).

The second method is more robust. Turbulence in the plasma may result in a scattering of the reflected beam, which messes up the interferometric measurement. The pulse radar technique (the method is completely analogous to radar) sends in many short pulses, of which the reflections are either detected or not. For those that are detected the analysis is straightforward. Sending in and detecting in ultra short pulses is challenging, but well possible with present day technology.

9.5 Temperature measurement: electron cyclotron emission (ECE)

The plasma emits radiation at the electron cyclotron frequency $\omega_{ce} = eB/m_e$ and its harmonics, as a result of the gyration of the electrons in the magnetic field. The frequency of this radiation, 28 GHz per Tesla, typically lies in the range 100–200 GHz. In this frequency range the plasma is even optically thick (i.e. there is so much emission and re-absorption of the emitted radiation that there is equilibrium between the radiation and the radiating medium: the radiation is described by Planck's law for black body radiation). Hence, the plasma is a black body radiator at the electron cyclotron frequency, and the intensity is follows Planck's law. We can make the approximation for long wavelength, the Raleigh-Jeans approximation, which results in a straight proportionality of radiation intensity and temperature:

Planck's black body law

$$I(\omega) = \frac{\hbar\omega^3}{8\pi^3 c^2} \frac{1}{e^{\hbar\omega/T_e} - 1} \tag{9.3}$$

Raleigh-Jeans approximation

$$I(\omega) = \frac{\omega^2 T_e}{8\pi^3 c^2} \tag{9.4}$$

Hence, a measurement of the intensity of the electron cyclotron emission is a measure of the electron temperature. Only an absolute calibration is needed.
In most cases the 2^{nd} harmonic of the cyclotron frequency is used for this measurement. The first harmonic is often shielded by the plasma frequency, which in a fusion reactor plasma happens to be in the same frequency band. Moreover, the signal of the 2^{nd} harmonic is stronger because of the ω^2-dependence. Higher harmonics become optically thin and are therefore not practical for temperature measurements. Figure **??** explains the set-up, how the temperature profile translates into spectra at the harmonics of the ECE-frequency, and how the treacherous harmonic overlap comes about.

Localisation, Harmonic overlap

This method gives a very accurate and localised measurement of T_e. The localisation results from the $1/R$ dependence of the toroidal magnetic field, just like with the ECR heating. In fact, for an accurate determination of the location of the resonance $B(R)$ must be calculated very precisely, taking into account the contributions of the plasma to the toroidal field: the para- and diamagnetic effects.
For an observation chord in the horizontal mid-plane, perpendicular to the toroidal field, the spectrum of the N^{th} harmonic ranges from $\frac{R_0}{R_0+a}N\omega_0$ to $\frac{R_0}{R_0-a}N\omega_0$, where ω_0 denotes the electron cyclotron frequency on the axis of the plasma ($R = R_0$). Depending on the aspect ratio $\epsilon = a/R_0$ there will be overlap of harmonics: e.g. the second harmonic at the outside of the plasma shields the first harmonic radiation from the inside, etc.

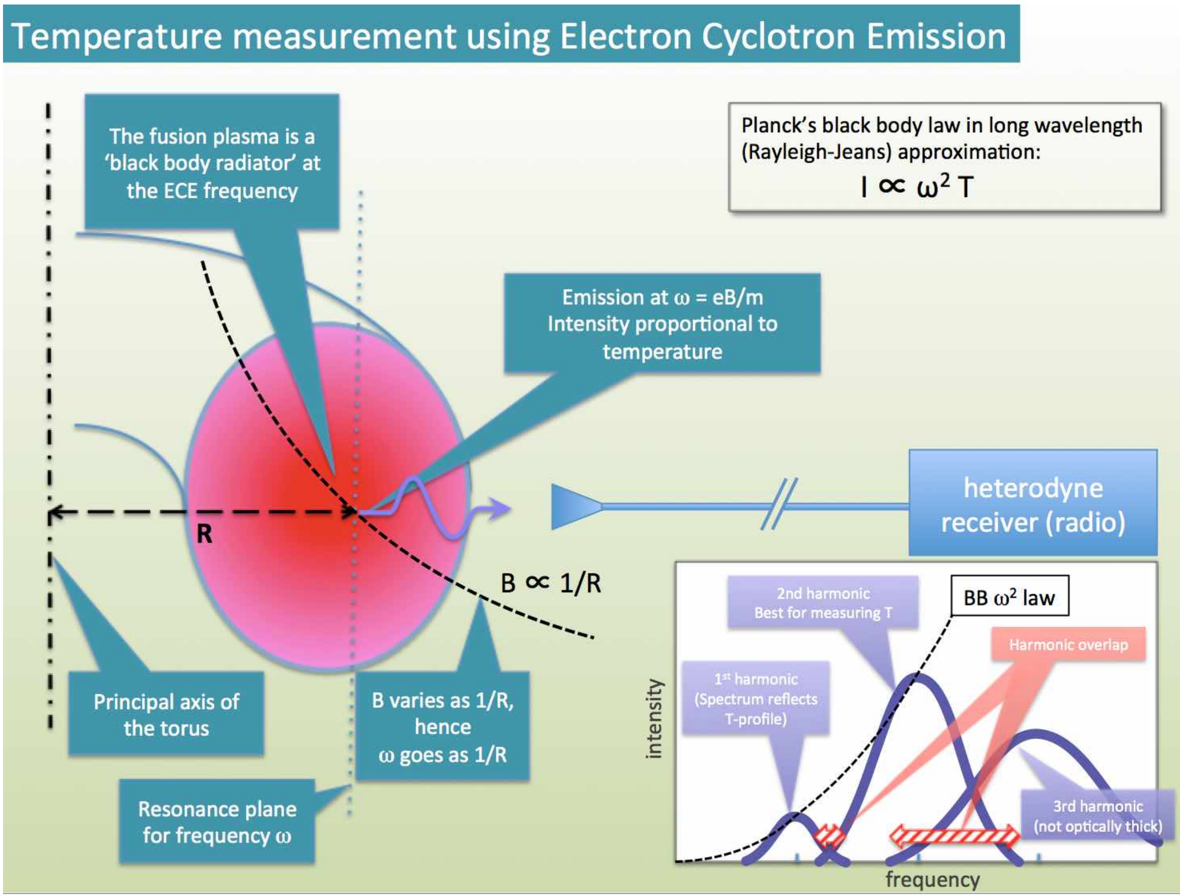

Figure 9.3: Schematic of ECE. Note that the spectrum – at each harmonic – reflects the temperature profile, but due to harmonic overlap only part of the 2^{nd} harmonic spectrum can be used.

Spatial resolution

The spatial resolution of the diagnostic is determined by the spectral resolution of the spectrometer (normally a heterodyne radiometer, with spectral resolution as good as you wish) and the line broadening mechanisms in the plasma. Just as with ECH, these are Doppler broadening and Relativistic line broadening.
Good ECE equipment realises T_e-measurements with a spatial resolution of order 1% of the major radius and a 'noise equivalent temperature' of only a few eV, at a bandwidth of hundreds of kHz.

9.6 Measurement of temperature and density: Thomson scattering

An elegant method to measure the temperature and the density of the electrons in the plasma uses the process called 'Thomson scattering', scattering of photons by free electrons. In this process, the free electrons oscillate in an applied EM field and as a result emit photons at the same frequency as the applied field. However, the emission is in all directions, whereas the applied field is normally a laser beam. The observer, looking at the laser beam under an angle, only sees the scattered light. The origin of those photons is precisely defined by the intersection of the laser beam and the view line, the spatial resolution can be as good as a few mm in all dimensions. Figure **??** explains the diagnostic.
The observer 'sees' the scattered photon at a frequency that may be somewhat different from the laser frequency, as the photon is emitted by a moving electron. The Doppler shift - due to the instantaneous

velocity - leads to a broadening of the observed spectral line, which carries information on the temperature of the emitting electron population. Thus, by measuring the spectrum of the scattered photons, one can derive a. the electron temperature (proportional to the square of the width of the spectrum) and b. the electron density (proportional to the total number of scattered photons). By imaging the laser beam on a 2-dimensional detector (one dimension for the wavelength, and one for the spatial coordinate, i.e. distance along the laser chord) n_e and T_e are measured along the laser chord in a single measurement.

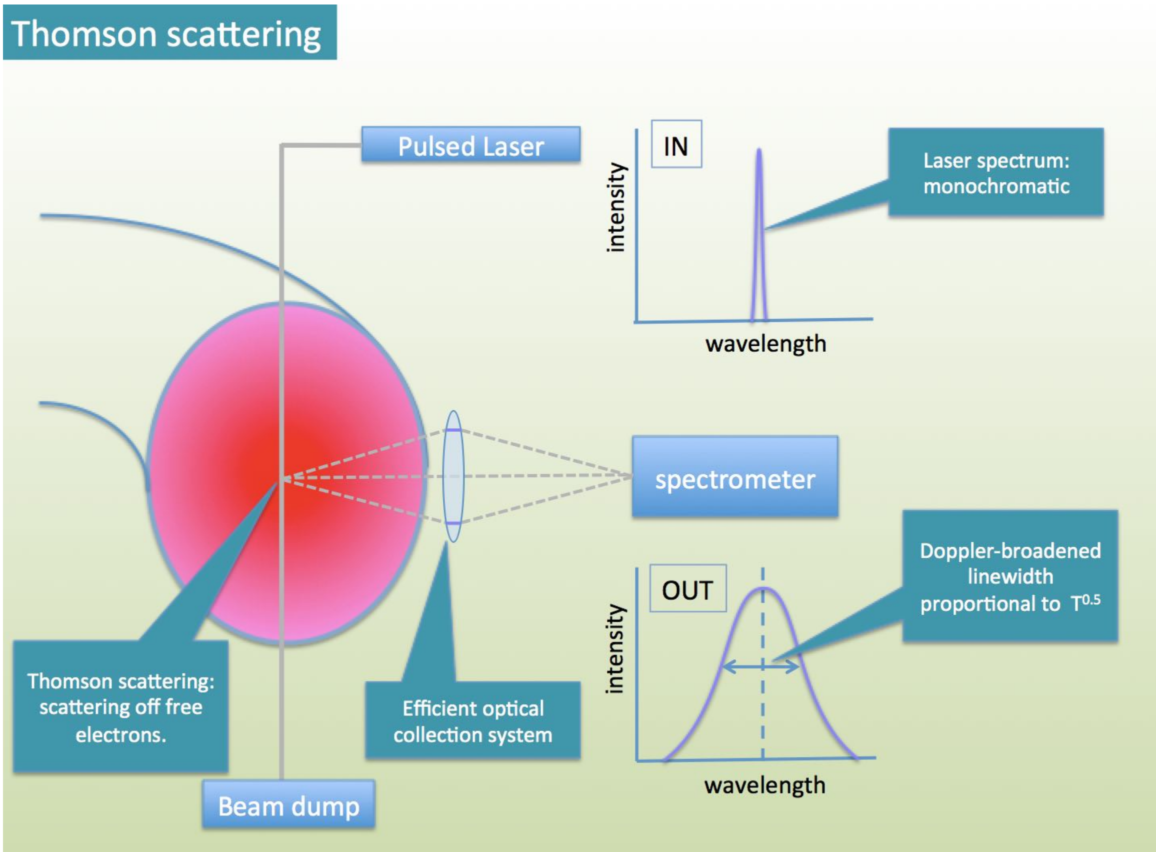

Figure 9.4: Schematic of Thomson scattering

So far it sounds easy. The basic difficulty of Thomson scattering is the tiny cross-section of the scattering process: in the tokamak plasma, from your typical observation length of a few mm of the laser chord, only a fraction of order 10^{-14} - 10^{-15} of the photons that you send in are scattered into your detector. (You may estimate this by taking the 'classical electron radius' – also called the Thomson scattering radius – to compute the cross-section for the scattering on a single electron, multiply this by the electron density and the length of your observation volume, and account for the collection angle of your viewing optics: only a small fraction of the scattered photons goes towards your collection lens). And on top, this very weak signal must compete with the background of light emitted by the plasma. To fight these problems, a very powerful pulsed laser is used. The high energy in the pulse maximises the number of scattered photons. The short pulse length allows you to gate the detector so that no signal is measured outside the time window of the laser: this minimises the detection of plasma light. Advantages of the method are absolute temperature calibration and excellent spatial resolution. The limitation is that each measurement provides a snap shot. With fast repeating lasers this can be made into a semi time-resolved measurement, with typically tens of Hz, but it is always far slower than e.g. ECE. Hence, both methods are complementary.

9.7 Magnetic measurements, coils and loops

Of course, even just to operate a tokamak we need accurate measurements of such basic operational parameters as the toroidal magnetic field, the plasma current, the loop voltage and the plasma position. These parameters are measured with magnetic coils and flux loops. You already know all about these: you figured them out in Problem 5.1 in Ch. 5.

Problems

9.1 Interferometry

Every tokamak or stellarator needs to have some form of density measurement, if only to know if the density limit is not approached. The most robust density measurement is performed with an interferometer.

(a) Give a schematic of the basic interferometer setup.

(b) The wavelength chosen for the laser in an interferometry system determines the maximum and minimum density that can be measured. For a tokamak plasmas the densities usually range from 10^{19} to $10^{21} m^{-3}$. What determines the maximum density you can measure?

(c) What determines the minimum density?

(d) What are these values for a laser in the visible wavelength (say λ = 500 nm)? What do you conclude from that?

(e) At several tokamak experiments an HCN interferometer ($\lambda = 337\mu m$) is used. What are the limiting densities for this diagnostic?

(f) How many 'fringes' are measured for a plasma with a (line-averaged) density of $1 \times 10^{20} m^{-3}$?

9.2 Thomson scattering

We consider the measurement of the electron temperature using Thomson scattering.

(a) Estimate the electron temperature obtained from Thomson Scattering in the example below.

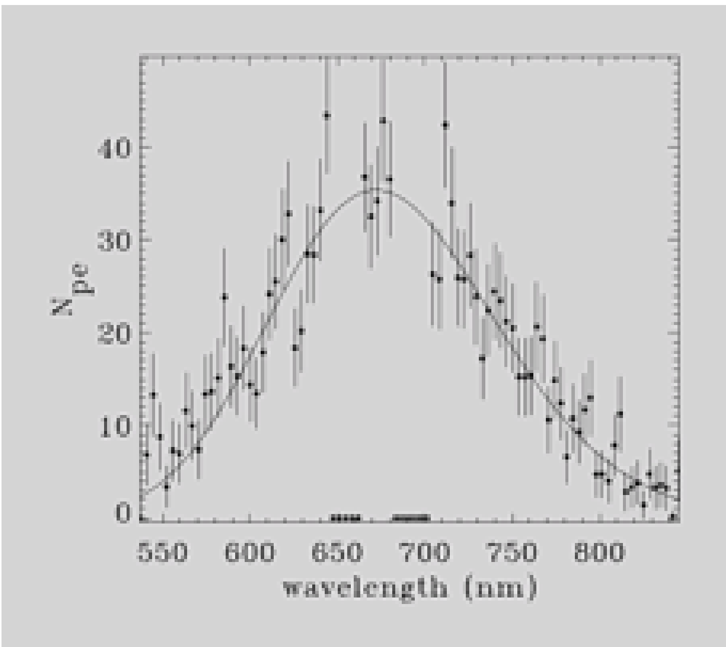

Figure 9.5: Scattered spectrum measured by a ruby laser TS system in which the viewing line is perpendicular to the laser beam. The vertical axis shows the signal strength (number of detected photons). The points are measurements, the line is a theoretical curve fitted to the measurements. The ruby laser has a wavelength of 694 nm

(b) The spectrum has two gaps (where the points have artificially been placed on the x-axis). Why would that be (Give separate explanations for each of the gaps)?

(c) The curve that is fitted to the measurements is asymmetric, where perhaps you had expected to see a — symmetric — Gaussian. Which phenomenon causes this asymmetry? Explain qualitatively how the asymmetry arises.

9.3 Spectroscopic measurement of the ion temperature

In principle, the ion temperature in a fusion plasma can be obtained from the broadening of a spectral line emitted by hydrogen.

(a) How?

(b) However, the hot core of the plasma in a fusion reactor does not emit much line radiation. Why not?

(c) What can be done to induce line radiation from the hot core of the plasma, in order to perform spectroscopy?

(d) Apart from the ion temperature also the toroidal plasma rotation speed can be measured by this method. How? And what does this mean for the geometry of the measurement?

9.4 Electron Cyclotron Emission spectroscopy

The hot core of the plasma does emit Electron Cyclotron Emission (ECE). This is an alternative possibility to measure the electron temperature T_e.

(a) How do you deduce the electron temperature from a measurement of the ECE intensity?

(b) Which condition must be fulfilled for this method to be applicable?

(c) How can ECE be used to obtain a temperature profile, i.e. a measurement of T_e as function of the location in the plasma?

(d) Draw a temperature profile (T_e as function of major radius) and indicate where the ECE measurement is difficult or impossible because of 'harmonic overlap' (and indicate which harmonics are overlapping).

(e) It can happen that during a tokamak experiment in which the density is increased, e.g. because a pellet is injected, the ECE signal suddenly disappears. Explain how this can happen.

9.5 Neutron measurements

One might say that in a power-producing fusion reactor the only diagnostic you really need is one that measures the neutron production — after all the neutron production is what we really care about.

(a) Since neutrons are not charged, they can only be detected in an indirect manner, i.e. by measuring the effect of their interaction with other particles. Give three examples of these interactions.

(b) Why is the neutron rate such a sensitive measurement for the ion temperature? Give the advantages and disadvantages of this diagnostic.

(c) Not only from the neutron rate, but also from the neutron spectrum the ion temperature can be obtained. Explain how.

(d) Give two examples of techniques to measure the neutron energy.

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10 Tokamak 2: Engineer's view

10.1 Parameters, dimensionless numbers, scaling laws

A large fusion experiment calls for a significant financial investment, and that certainly holds for a fusion power plant. The building cost of ITER is estimated at about 15 billion Euro's, to give a number. To make sure that the money is spent economically, we want to optimise—basically for cost-effectiveness—the parameters of the reactor before we order it. In the absence of a theoretical model that is so complete and reliable that the whole device can be computed, the optimisation can only be done by extrapolation of results of existing devices. Where it should be noted that this is not a simple extrapolation, but a multi-dimensional nonlinear scaling. This technique, the construction of scaling laws implies that the parameter to be optimised is written as a function of the engineering parameters of the reactor. By statistical analysis of a database that has been constructed with date from many different experiment, plus the application of additional constraints (e.g. arising from theory) the function can be determined.

As an example we shall consider the scaling law for an important quantity, the energy confinement time τ_E of a tokamak. But first we shall list the parameters that we can choose when designing a tokamak.

10.2 Parameters that define the tokamak

Geometry

R	major radius
a	minor radius
κ	elongation (the ratio height/width of the poloidal cross-section of the plasma)
δ	triangularity (of the poloidal cross-section of the plasma)

The elongation and the triangularity describe the shape of the cross-section. For a circular cross-section, $\kappa = 1$ and $\delta = 0$. High performance plasmas, such as ITER, have $\kappa = 1.7$ and $\delta = 0.3$–0.4. Note that the size of the vacuum vessel sets hard limits to the achievable plasma geometry, i.e. values of R, a, κ and δ. But the operator is free to make plasmas that have different values, as long as they fit in the vessel.

Engineering ('knobs in the control room')

B_T	toroidal magnetic field
I_p	toroidal plasma current
P	additional heating power And here we might add, within the constraints set by the vacuum vessel: R, a, κ and δ

Plasma (these parameters are not fully under the control of the operator)

n	density
Z_{eff}	effective ion charge, measure of the purity of the plasma

10.3 Dimensionless numbers

The parameters used above all have dimensions: Tesla, Ampere, Watt, etc. For scaling studies, and generally to get to the basics of a physics problem, it is important to remove as much as possible the dimensions by introducing dimensionless numbers. Different tokamak experiments may differ vastly in size and therefore in almost all dimensional parameters, while their dimensionless numbers may be very similar. To learn about the real scaling behaviour, we need experiments that differ in dimensionless numbers. Below a list of useful dimensionless numbers:

$\epsilon = a/R$	aspect ratio; Typical value $\epsilon = 0.2 - 0.35$.
q_a	'safety factor', i.e. the normalised pitch of the helical magnetic field, in cylindrical approximation given by $q_a = aB_T/RB_p$ As we shall see, q_a must be greater than 2, a typical value being a bit above 3.
β	kinetic pressure, normalised to the magnetic pressure. Typical value $\beta < 3\%$.
β_p	kinetic pressure, normalised to the pressure of only the poloidal magnetic field. Typical value $\beta_p < 1$.
l_i	normalised self-inductance, measure of the peakedness of the current density distribution in the plasma. Typical value $l_i \approx 1$.

Kinetic pressure, normalised to the magnetic pressure

$$\beta = \frac{p}{B^2/2\mu_0} \tag{10.1}$$

Kinetic pressure, normalised to the pressure of only the poloidal magnetic field.

$$\beta_p = \frac{p}{B_p^2/2\mu_0} \tag{10.2}$$

Normalised self-inductance

$$l_i = \frac{\int B_p^2 dV}{(B_p(a))^2 V} \tag{10.3}$$

10.4 Modes of confinement: Ohmic, L-mode and H-mode

Before we continue with a scaling law for the confinement of a tokamak, you must know that it has turned out that a tokamak can be operated in different modes of confinement. These are best appreciated when they are introduced in historical order. The first tokamaks were heated by the plasma current only. The only means to increase the input power was by increasing the plasma current – the two were intimately linked. This led to the so-called Ohmic Confinement. Ohmic confinement showed a linear increase of the stored energy in the plasma with heating power (something you might expect for a simple linear system, but not necessarily for a highly nonlinear, turbulent plasma). This extrapolated very promisingly to a larger plasma with more heating. But of course, the scaling suffered from the fact that the heating power and the plasma current were coupled: the confining field was increased together with the power!

To separate the two, the independent auxiliary heating sources discussed in 8 were developed, in the 1970-ties and later. A nasty surprise awaited: with additional heating the stored energy did not at all increase linearly with heating power, but more like the square root of the power. This was bad news indeed, because it meant that a lot more power, and larger tokamaks, were needed to meet the Lawson criterion. In 1981 it was discovered – in the ASDEX experiment in Garching – that it is possible to bring the tokamak in another regime of confinement. This so-called H-mode gives roughly 2 times better confinement than the other mode, which in retrospect was called L-mode (H for High, L for Low). This is a true bifurcation: the confinement is either H or L and can jump between the two, but there is no intermediate state. H-mode has since been the standard mode of operation and is the baseline for ITER. In the 1990ties and later yet other confinement modes were discovered, with even better confinement but also much more restricted operational windows. We will not discuss these here. But it is important to know the difference between the L and H-mode. Figure 10.1 shows the typical pressure profiles for the L and H mode. You see that the main difference is that the H-mode profile features a 'pedestal', which effectively shifts the entire pressure profile up. Now you may not be impressed by the magnitude of this shift, but you have to remember that in order to arrive at the stored energy we have to integrate this profile over the entire volume. And because of the cylindrical geometry, the edge puts a lot of weight in the balance. So the H-mode is good, except for the fact that the steep edge gradient gives rise to the 'Edge Localised Modes', which lead to high wall loads. More on these ELMs later.

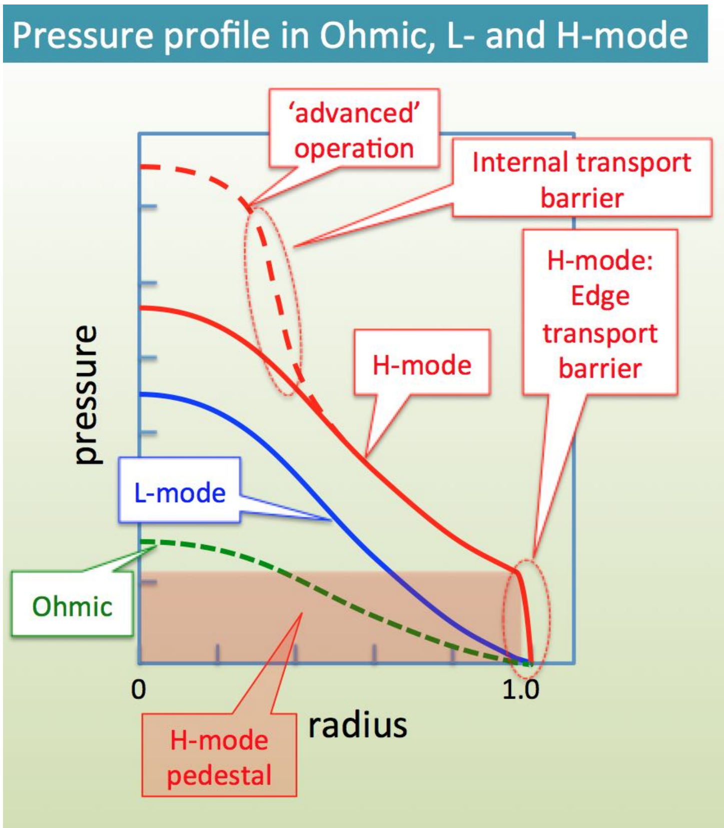

Figure 10.1: The pressure profile in H-mode is shifted up with respect to L-mode. This 'pedestal' is caused by a transport barrier – a thin region with very low thermal conductivity – close to the edge. Note that because most volume resides at the edge, the pedestal is much more effective at increasing the energy confinement time than might seem from the relatively modest increase of the peak pressure. It is possible to additionally induce an internal transport barrier, with higher peak pressure as a result.

10.5 The divertor

As said, the H-mode was discovered in a machine called ASDEX, and that was no coincidence. ASDEX is an acronym that stands for Axially Symmetric Divertor EXperiment, and it was the introduction of this 'divertor' that gave access to the H-mode. Figure 10.2 shows the two ways in which the edge of the plasma can be connected to the material world.

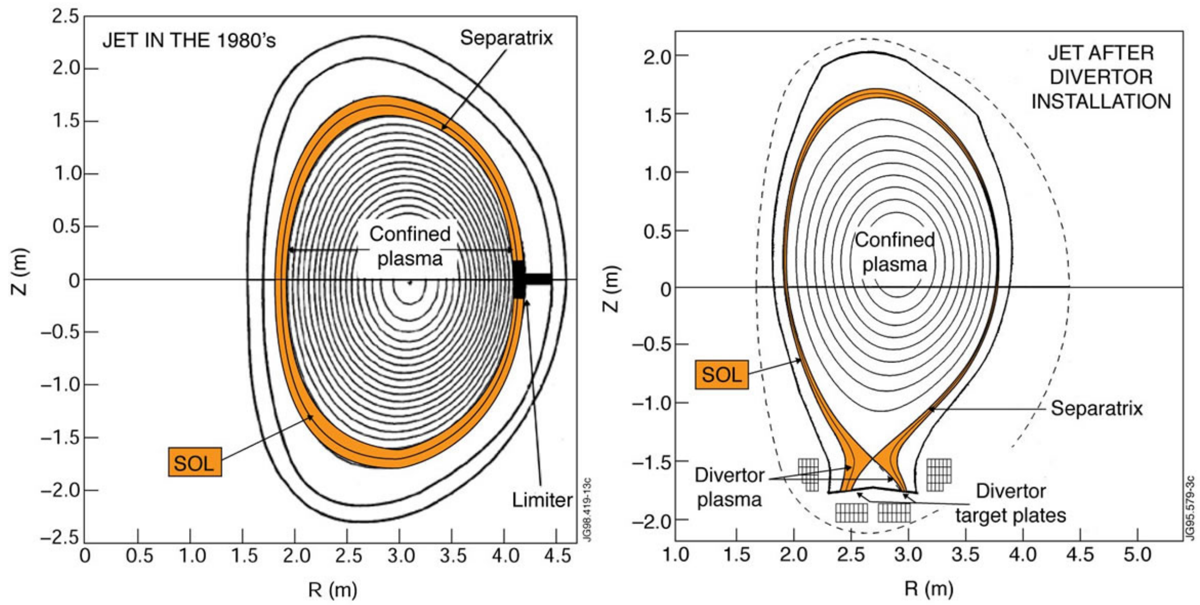

Figure 10.2: The two basic configurations of the magnetic field in a tokamak: the limiter is simple but unsuitable for a reactor; the divertor offers much better control and exhaust, which enables access to the superior H-mode (source: EFDA-JET)

The 'limiter' is a cooled piece of metal that defines the outermost flux surface. In principle it could sit anywhere around the torus, and in practice the limiters were often belts that went round in toroidal direction. The limiters have a great drawback, and that is that they are situated directly next to the plasma. The plasma wall interaction, the neutralisation of the plasma, the heat load on the limiter and the ensuing flux of particles – partly consisting of wall material – it all happens right next to the main plasma. There is no way to pump off the neutralised plasma, no way to remove the helium and impurities, no way to control the influx of wall material, and no density control. And these are the reasons why the divertor was introduced. In this geometry, the plasma wall interaction happens at a distance from the main plasma, in a geometry that can be optimised to handle the large heat and particle fluxes, the interaction region can be separated from the main plasma in such a way that there is no or little back flow of gas, and effective removal of the exhaust gas can be achieved with pumps which also gives the possibility of density control. These were the advantages that also made it possible to achieve H-mode in ASDEX, and shortly after that discovery most tokamak experiments made plans to rebuild their device to include a divertor. It may be clear that today, the divertor configuration is the standard. But do realise that a divertor is a complex device, too. It requires coils close to the plasma in order to create the 'X-point', complex cooled structures inside the vacuum vessel and ... it uses up volume filled with vacuum and magnetic field (i.e. volume with a very high price per liter) which could otherwise have been used to confine hot plasma in. You'll appreciated the difference in effective plasma volume between the two configurations depicted in Figure 10.2. A final remark about the divertor: this configuration requires the formation of a separatrix. On this separatrix, by definition, the safety factor q goes to infinity, as field lines on the separatrix do not go round in poloidal direction no matter how often the make a toroidal tour. Thus, the cylindrical approximation of q is not at all an estimator of the local 'field line q' on the separatrix, while it still is a good measure for the ratio of the magnetic field to the plasma current. To still give a meaningful value of q in a divertor tokamak, usually the q_{95} is quoted, i.e. the value of q at a flux surface close to but not at the separatrix (to be precise: at the flux surface which in terms of poloidal flux is at 95% of the distance between the magnetic axis and the separatrix).

10.6 A scaling law for the energy confinement time

The first effort to distill a scaling law from the experimental data concerned the energy confinement time, τ_E. This effort, initiated by Goldston in the early 1980ties, used engineering parameters and applied to L-mode. Now, there are a number of different forms for the L-mode scaling, but a well-known and much used example is the following:

Scaling law L-mode

$$\tau_E \propto I^{0.85} B^{0.2} P^{-0.5} n^{0.1} a^{0.3} R^{1.2} \tag{10.4}$$

(This scaling is called 'ITER98-P', see Freidberg p. 509, slightly simplified and restricted to cylindrical geometry by leaving out the elongation)

A paradox

Let us first note how absurd this scaling law really is. If we take it at face value, the – very expensive – magnetic field hardly contributes to confinement, and plasma heating is the last thing one should do as it only reduces confinement. Still, this scaling law gives a good description of the data in the empirical database. How is this possible? The way out of this paradox is that there are hidden relations between the dimensional parameters: the magnetic field and plasma current are coupled, for instance. Moreover, the heating power P occurs both in the definition of τ_E and on the right-hand-side of the equation, which is confusing. Therefore, let us transform this scaling law by replacing the engineering parameters as much as possible by dimensionless numbers. Because these have typical values, this exercise may bring out the physics.
To do this, we first eliminate P and then use the dimensionless numbers where possible. If I do the exercise for the above scaling law, I get approximately (in the end I leave out the small exponents)

$$\tau_E \propto a^2 q^{-1.7} \beta^{-1} \tag{10.5}$$

Why does this look so much prettier?
In the first place: confinement scales with the linear dimension squared, which is characteristic of diffusive transport. And that is good news in itself, because that means that at least in the size scaling we expect no surprises when investing in larger machines. Secondly, this scaling law seems to reflect physics. We must minimise q, i.e. run as much current as possible for a given B-field, and that makes sense as the current produces the confining poloidal B-field. Further, increasing the normalised pressure makes confinement worse, and this hints at the fact that the pressure (or the pressure gradient) will be the driving term for the turbulence, which in turn reduces the gradient.
So far we have considered L-mode scaling. These days H-mode is the standard operation mode and it certainly is the mode ITER will operate in. The scaling law that was used to design ITER is the so-called IPB98(y,2) scaling. The exact expression is as follows (don't memorise this!):

Scaling law H-mode

$$\tau_{th,IPB98(y,2)} = 0.00562\, I^{0.93} B^{0.15} P^{-0.69} n^{0.41} A^{0.19} R^{1.97} \epsilon^{0.58} \kappa^{0.78} \tag{10.6}$$

Figure 10.3 shows how well this formula describes the database that was used to construct it. But most of all, it shows how close ITER really is to the database. While the database spans a range of more than two orders of magnitude, the extrapolation to ITER is a mere factor 4 or so. This gives a high confidence that the performance of ITER will be close to the prediction.

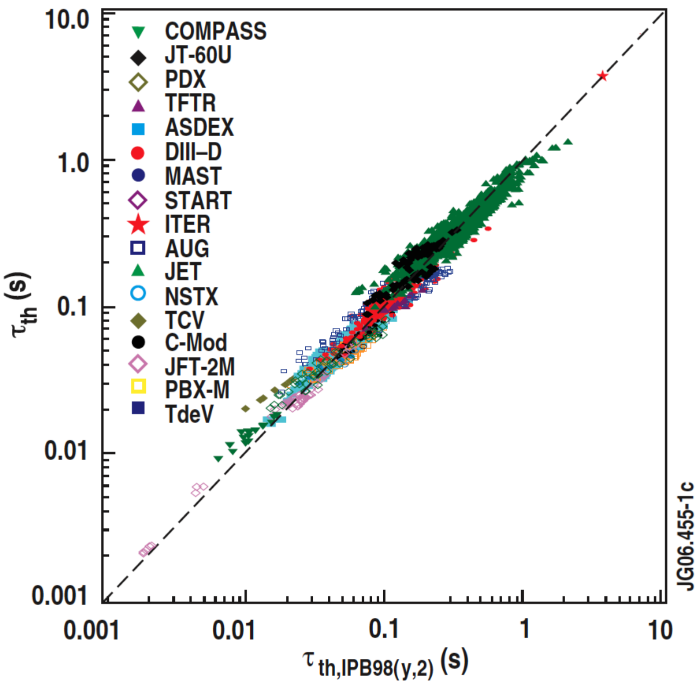

Figure 10.3: Confinement time versus the IPB98(y,2) scaling on a double-log scale, showing the database that was used to construct the scaling formula (10.6). The star denotes the ITER extrapolation point. (Source: EFDA)

Let me end by saying that of course the use of scaling laws is certainly not restricted to τ_E. Another very important parameter that is predicted using scaling laws is the height of the temperature pedestal, and yet another the critical power level that is needed to flip the discharge from L-mode into H-mode.

10.7 Operational limits

There are limits to a number of parameters, and these limits define the operational space of a tokamak. If you are to be the pilot of ITER, you better have a very good knowledge of these limits, otherwise you'll shake the machine – and probably lose your job. The most important limits concern the plasma current (or rather, the dimensionless number q_a), the plasma density n, the normalised pressure β, and the elongation κ. Within the boundaries set by these limits, the operator can tune the discharge.

10.7.1 Current limit, q-limit

The dimensionless number q, the 'safety factor' defined in Eq. (7.1), is of fundamental importance for the topological properties of the magnetic field in a torus, as we saw before. If q assumes a simple rational value at the edge of the plasma, magnetic islands will develop. For the higher rational numbers these are small and not dangerous for the plasma. So, when you start a plasma and ramp up the plasma current, the value of q at the edge starts out at some high value (remember that $q_a \sim 1/I_p$) and then decreases. The passage through $q_a = 7, 6, \cdots, 4$ is not a real problem, but to get past $q_a = 3$ already requires some skill from the operator. Ramping the current up further will bring you to $q_a = 2$, and this is such a strong resonance, and the $m = 2$ islands are so large and fast growing, that it is not possible to pass this barrier. Any operator who brings the plasma to close to $q_a = 2$ will be rewarded with a disruption and will have to start over.

Current limit:

$$q_a > 2 \tag{10.7}$$

In a realistic geometry (D-shaped plasma: elongation, triangularity etc.) q is not so easy to calculate. For our purposes we'll use the cylindrical approximation:

$$q_{\text{cyl}} = \frac{aB_T}{RB_p} \tag{10.8}$$

In cylindrical geometry we further have $B_p = \mu_0 I_p / 2\pi a$, hence $q_{\text{cyl}} = (2\pi/\mu_0)a^2 B_T / RI_p$. Hence, the q-limit describes how far you are allowed to ramp up the plasma current I_p for a given toroidal field B_T.

10.7.2 Density limit

The heat loss in a tokamak is dominated by the diffusive flux from centre to edge - the plasma is its own insulator. The central plasma does not radiate a lot of power, since it is fully ionised. Hence the efficient generation of radiative power through line radiation is not available. However, in the outer layer of the plasma the temperature approaches 'room temperature' (say 10^4 K), and here radiation becomes an important loss channel. In fact, we expressly arrange the edge in such a way that the radiation losses account for some 90% of the total power exhaust, because this reduces the conductive heat load on the divertor. (Radiation is spread out over a very large surface, whereas conductive losses are concentrated on a very narrow strike zone).

Now, since radiation is the result of a 2-body process the intensity is proportional to the product of the densities of the exciting and excited particles, hence roughly with n^2. So what happens if we increase the density of the plasma? If we keep the input power constant, the fraction of radiated power increases until at some point the losses exceed the input power. From then on, the plasma is losing energy and cools down. And the cooling occurs first where the energy is radiated away, i.e. in the edge. As a result, the resistivity in the edge increases and the plasma current is pushed inward. In other words, the plasma column shrinks, the effective radius of the plasma column, a, decreases.

The result of the shrinking is that the edge safety factor q_a decreases: $q_{\text{cyl}} \sim a^2 B_T / RI_p$, and a decreases while B_T and I_p remain constant. This process continues until the q-limit $q_a = 2$ is reached and a disruption ends the plasma.

This is the global picture of the density limit. There is no good theory of the density limit. But physicist Martin Greenwald published a very simple empirical expression of the density limit that so far seems rather universally right:

Density limit (aka Greenwald limit):

$$n < n_G = \frac{I_P}{\pi a^2} \tag{10.9}$$

with n in 10^{20} m^{-3}, I_P in MA and a in m. You will find n_G as a normalisation in practically every scientific paper on tokamaks.

10.7.3 Beta-limit

Although it is clear that from an efficiency point of view you want a high pressure in a reactor, you cannot increase the pressure of a magnetically confined plasma indefinitely. Clearly, the kinetic pressure (nk_BT) cannot exceed the total magnetic pressure ($B^2/2\mu_0$), so $\beta < 1$. In practice the pressure limit is much lower because in most magnetic configurations **B** is far from perpendicular to the current density **j**, so that only a fraction of the **B** field contributes to the confining $\mathbf{j} \times \mathbf{B}$ force: $\beta \ll 1$. When the plasma pressure becomes too high for the confining field, an MHD instability will set in. There are several different types of instabilities and which is the first to be triggered depends on details that need to be analysed by numerical computation. However, Troyon found that the trend is similar for all and he found, on the basis of extensive computational studies done in the mid 1980s, a universal pressure limit known as the Troyon-limit:

Beta-limit

$$\beta < 3.5\% \cdot \frac{I_p}{aB_T}. \tag{10.10}$$

for I_p in MA, a in m, and B_T in T. The limit has been confirmed in numerous experimental studies. Note that for a machine like JET, with $I_p = 5$ MA, $a = 1$ m and $B_T = 3$ T, the limit is around $\beta = 5\%$ only!

10.7.4 Elongation

Both for confinement and the other operational limits, it is very advantageous to give the plasma an elongated rather than circular cross-section. Most importantly, an elongated plasma can carry a higher plasma current for the same value of q. Elongated meaning: stretched in vertical direction. However, there is a limit to the allowable elongation, if you stretch the plasma too much it will break up in two rings. The Swiss experiment TCV (Tokamak a Configuration Variable) explores the more exotic plasma shapes and the effect on the quality of confinement. They realised values of κ up to 2 or slightly above. But at higher values, it becomes increasingly difficult to keep the plasma stable.

10.8 Vertical field

The two most important components of the magnetic field in a tokamak are the Toroidal field (B_T, generated with external coils) and the Poloidal field (B_p, generated by the plasma current). However, there is a third essential field component, which must prevent the plasma ring from expanding in radial direction. This is the Vertical field B_v. Like the toroidal field, it is generated by external coils. The vertical field must counteract the hoop force, a radial, outward force the plasma experiences due to two different effects. First, due to the kinetic pressure in the plasma, the plasma ring has a tendency to expand - just like the inner tube of your bicycle becomes larger upon inflation. Second, the current in the plasma ring exerts a Lorentz force on itself: like any current carrying ring it tends to expand. Both effects can be quantified by computing the change of the kinetic and magnetic stored energy for a small change of the radius of the plasma ring. This is an easy job in cylindrical geometry, but especially the magnetic component becomes a bit cumbersome in toroidal geometry. Anyway, the change of energy as a result of a small change of the radius gives an expression for the hoop force, and this must be balanced by the Lorentz force generated by the vertical field cross the plasma current. The vertical field is then found to be:

Vertical field

$$B_v = \frac{\mu_0}{4\pi}\frac{I_p}{R}\left(\beta_p + \frac{1}{2}l_i - \frac{3}{2} + \ln\frac{8R}{a}\right) \tag{10.11}$$

Note the following points:

- The vertical field scales with the plasma current. Thus, when starting up a tokamak discharge, the most important thing is to make sure the vertical field is proportional to the plasma current.
- Inside the brackets only dimensionless parameters are found, that are of order 1. The natural logarithm inside the brackets has to good approximation a fixed value of around 3.2.
- For constant I_p and R (i.e. by keeping the plasma in place under feedback) all the action is in the term $\beta_p + \frac{1}{2}l_i$. In other words, by measuring the current in the B_v coils needed to keep the plasma in its place, you can measure a combined change of the kinetic pressure and the peaking of the current density profile. To disentangle the two, you will need an independent measurement of e.g. the pressure.

Problems

10.1 The principal components of a tokamak

Let's review what you need to build a tokamak.

(a) What are the four principal (set of) items you need to make a tokamak? Just a tokamak to create a plasma in, nothing fancy, no fusion power or anything: the principle, not the technology. So think as a theoretical physicist would.

(b) Of each of the four items, describe their principal function

(c) Now, thinking as an engineer, your shopping list is much longer. Give a handful of indispensable bits of hardware the theoretical physicist may have forgotten, and explain your choice.

(d) In the picture below there are quite a few magnetic coils that go round in toroidal direction, the so-called PF coils. What is their function? Why so many?

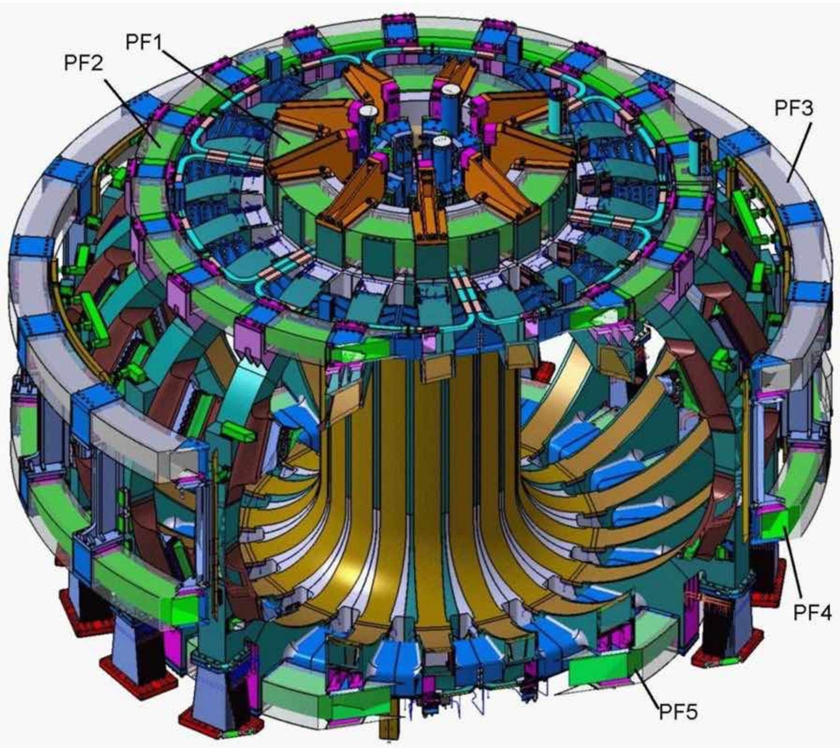

Figure 10.4: The poloidal field coils of ITER

(e) Apart from the PF coils that you see in the picture, there is another very important coil that encircles the central hole in the torus. What is that and what is its function?

10.2 Vertical field

The vertical field is not very strong compared to the toroidal field, but without it tokamak operation is not possible.

(a) Explain the function of the vertical field

(b) Derive an expression for the vertical field by approximating the torus by a cylinder if you want to compute the thermal and magnetic energy – while retaining the characteristic of the torus that gave rise to the need for the vertical field in the first place. Make sure to express the vertical field in terms of the dimensionless parameters β_{pol} and l_i, plus all the dimensional parameters you need. (This way you will get a simplified version of the equation given in the lecture notes)

(c) Rework your expression to find the B_V as function of the toroidal field (so, apart from that only dimensionless parameters).

(d) Give a typical value for the ratio of the three principal B-field components in a tokamak: B_T : B_{pol} : B_v

10.3 Scaling Laws

Scaling laws are commonly used in fusion research to find empirical expressions that relate e.g. the energy confinement time of a tokamak to its design parameters (size, shape, B-field) and operational parameters (plasma current, heating power, density). This is the engineering approach. By recasting the engineering scaling law into a form that uses physics parameters and dimensionless numbers, we may hope to get a hint of the physics behind the scaling law, and even the processes by which the confined plasma loses energy. The (simplified) L-mode scaling law given in the lecture notes reads:

$$\tau_E \propto I^{0.85} B^{0.2} P^{-0.5} n^{0.1} a^{0.3} R^{1.2} \tag{10.12}$$

(a) Bring this into a form that uses as much as possible the dimensionless numbers q, β, and ϵ, while eliminating P.

(b) Now multiply by nT: what do you conclude for the quantity we really have to optimise, the triple product that we know from the Lawson criterion?

(c) Repeat the exercise for the H-mode scaling, which you may simplify by forgetting about A (atomic mass) and κ (elongation), to:

$$\tau_E \propto I^{0.93} B^{0.15} P^{-0.69} n^{0.41} a^{0.58} R^{1.39} \tag{10.13}$$

(d) Comment on your result. Compare to L-mode scaling.

(e) Do you note anything peculiar about the density dependence in your result for the Lawson criterion, i.e. for the condition required for ignition?

10.4 The safety factor in an elongated plasma

You may wonder why tokamak plasmas are always designed to have a large elongation. Let's find out by doing a thought experiment.

(a) Consider a tokamak in which rather than one, two identical plasmas with circular cross-section sit on top of each other, i.e. they have the same major axis but one is at a different vertical position than the other. They see the same toroidal B-field, carry the same toroidal plasma current, hence they have the same – cylindrical – q_a, for which we'll arbitrarily assume $q_a = 2.5$. Now, we move the plasma rings together, so that they merge into a single plasma ring with elongated cross-section ($\kappa = 2$). What is the q-value of this newly formed plasma?

(b) What are the values of the plasma current and the Toroidal B-field in this new plasma ring?

(c) What is the maximum achievable value of β in the new situation, compared to that of the separate plasma rings?

(d) What do you conclude for the κ-dependence of the β-limit?

10.5 The Density limit

The density limit implies that we need to have control over the density when running a tokamak. For control you need actuators and sensors.

(a) Mention and briefly describe a sensor for the electron density that could be used in a control loop.

(b) Give three different actuators that can be used to increase the density

(c) But given that the density limit is an upper limit, perhaps we should be more concerned with actuators that decrease the density. Discuss the possibilities for such actuators.

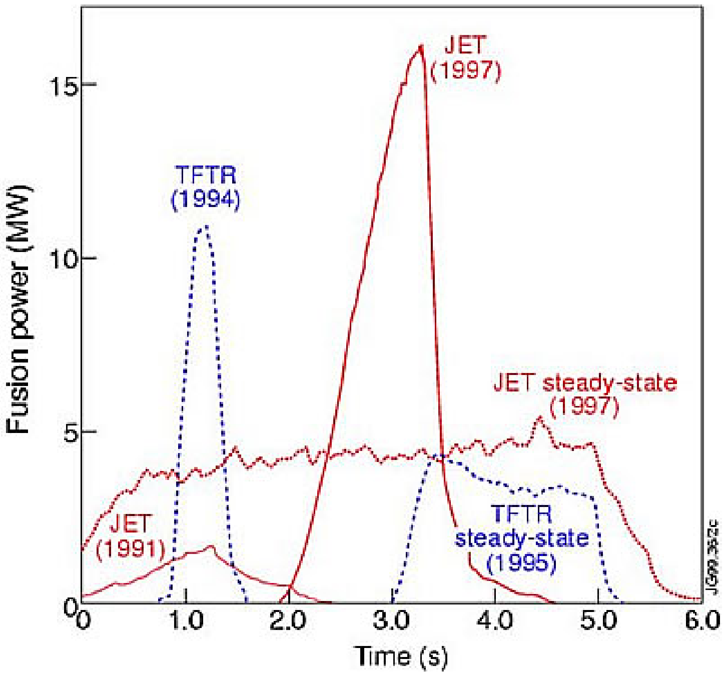

Figure 10.5: In 1997 the JET experiment achieved the highest fusion power in a magnetic confinement device ever: 16 MW. To achieve this, about 25 MW of input power was needed, so Q=0.64 – not quite unity but exceeding the design value of JET

(d) In the famous world-record d-t discharge in JET, which produced 16 MW of fusion power, the high performance lasted only for about a second. Why – do you think – could this not be sustained longer? (In the same graph you see that a sustained fusion power could be achieved at a level of about 4 MW.)

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11 Transport

11.1 Laws of Fick, Fourier en Ohm, the Transport matrix

The classical transport phenomena are: conduction, convection and radiation. For the tokamak, we'll concentrate on conduction, because this is what brings energy from the hot plasma in the center to the edge, where it is exhausted. Diffusion also governs the transport of particles, such as the helium 'ash' that needs to be removed from the plasma somehow. The basic relations are the proportionality of flux (of particles, energy or charge) and the thermodynamic forces: gradients of density and temperature, and the electric field. Those are the well-known laws of Fick, Fourier en Ohm. In the tokamak plasma we consider the particle and energy fluxes across the flux surfaces, and the current density (charge flux) parallel to the field.

Particle flux (Fick's law)

$$\mathbf{\Gamma} = -D\nabla n \tag{11.1}$$

Heat flux (Fourier's law)

$$\mathbf{q} = -n\chi\nabla T \tag{11.2}$$

Current density (Ohm's law)

$$\mathbf{j} = \sigma\mathbf{E} \tag{11.3}$$

Here, D is the particle diffusion coefficient, χ the thermal diffusion coefficient or diffusivity, and σ the electrical conductivity. (Take care not to confuse the heat flux vector $\mathbf{q}$ with the safety factor).

For the heat flux, we must in general distinguish two channels: via the electrons and via the ions. The two species can have different temperatures, are often heated by different mechanisms and form two separate conduction channels. They do interact through collisions, but due to the mass difference, the energy transfer is not very efficient. However, if the time constant associated with this energy transfer (τ_{ie}) is short compared to the global energy confinement time (τ_E) of the plasma, the two species are thermally coupled and their temperatures will become almost the same. In that case it is no longer possible to distinguish the two channels. This condition is met in large devices (long τ_E), such as JET and ITER.

In contrast, particle transport is described by a single expression. Because electrons and ions are charged, any difference in flux would lead to a strong electric field that would force the fluxes to stay equal. As a consequence, the net transport is determined by the species with the slowest diffusion.

In a simple physical system, the relation between the fluxes and gradients can indeed be described as above. However, as we shall see, in a plasma cross terms exist. For instance, a particle flux towards the centre of the plasma, driven by the toroidal electric field (that was meant to drive the toroidal current). And likewise, a radial density gradient drives a toroidal current!

In general, the flux-gradient relation is therefore written in matrix form:

$$\begin{pmatrix} \mathbf{\Gamma} \\ \mathbf{q}_i \\ \mathbf{q}_e \\ \mathbf{j} \end{pmatrix} = - \begin{pmatrix} D & ? & ? & ? \\ ? & n\chi_i & ? & ? \\ ? & ? & n\chi_e & ? \\ ? & ? & ? & \sigma \end{pmatrix} \begin{pmatrix} \nabla n \\ \nabla T_i \\ \nabla T_e \\ -E \end{pmatrix} \tag{11.4}$$

Now the challenge is, on the one hand, to find theoretical expressions for the coefficients in the matrix, and on the other hand, to measure these coefficients. We'll start with the theory: classical (collision driven transport in a homogeneous **B**-field) and its extension to a toroidal geometry, 'neoclassical' transport. But eventually, transport will turn out to be dominated by turbulence, and there is no *ab initio* theory that accounts for the full richness of turbulent transport yet.

11.2 Classical transport

Charged particles follow field lines (that is, in a homogeneous, straight field), so the cross field transport would be zero if it were not for the collisions, which can make the particles hop from one field to another. The average displacement during a collision is the gyroradius (ρ). The time between the collisions is the collision time (τ). The classical heat diffusivity therefore is estimated by $\chi_e \approx \rho^2/\tau$. This is a very small conductivity! For electrons in a fusion plasma, typically $\rho = 0.1$ mm, $\tau = 10$ μs, so $\chi_e = 0.001$ m^2/s. The ions have a larger heat conductivity. Both their gyroradius and their collision time is larger than that of the electrons by the square root of the mass ratio $\sqrt{m_i/m_e}$. Because the gyroradius appears squared in the diffusion coefficient we have

$$\chi_i \approx \chi_e \sqrt{\frac{m_i}{m_e}} \tag{11.5}$$

These expressions are for so-called test particle transport: the diffusion of single particles in a background that is not influenced by their movement. And this is the appropriate expression for heat transport, as particles can transfer energy without actually moving themselves (in a solid state that is the usual situation). However, for mass flow this is not true. In this case, the transport is limited by the particle with the lowest (test particle) diffusion coefficient, which is the electron. Hence, the electrons glue the ions to the magnetic field! The particle transport coefficient D, which is the same for electrons and ions because they have to move together in order to preserve quasi-neutrality, is therefore estimated by $D \approx \chi_e$

11.3 Neoclassical transport

The curvature of the field in toroidal geometry, and associated with that the unavoidable inhomogeneity of the field, result in corrections to the classical transport theory. This so-called 'neoclassical transport theory' predicts enhanced cross-field diffusivities, while the parallel conductivity is reduced. And although we may think of neoclassical theory as a correction of classical theory taking into account the effects of geometry, this correction is of the order of a factor 10 for the cross-field diffusivity, and a factor 2 for the parallel conductivity. Hardly what you would call a correction! It is all the result of the combination of two effects. Firstly, particles that travel around the torus experience an increasing B-field if they move towards smaller radius, and hence can be reflected by the magnetic mirror effect. Hence, rather than travelling around the torus, they bounce between two reflection points. Secondly, they are always drifting in vertical direction due to the curvature and gradient of the field. Hence, as we shall see below, their orbit takes the form of a banana (in projection on the poloidal plane). These particles are 'trapped', they do not carry toroidal current, and the cross-field diffusion step size is now the width of the banana orbit instead of the gyroradius. Apart from the obvious effects on conductivity and cross-field diffusivity, the banana orbits give rise to other, more exotic effects: a radial density flow directed inward (i.e. up the gradient!) driven by the parallel electric field, and a toroidal current driven by the radial density gradient. Two fluxes that are actually desirable in a reactor, driven by gradients that are perpendicular to the resulting flux! Welcome to the world of the cross products.

11.3.1 Banana orbits

In a toroidal geometry, the magnetic field strength is higher on the inside of the torus than on the outside. Simply put, the coils are closer together there. Hence, particles that follow the helical field line (by approximation) will see an increasing magnetic field if they move inward, and because of the mirror effect, they will reflect at some point. Where that point is depends on the ratio between parallel and perpendicular velocity, as given by the mirror ratio (see chapter 5). Those particles will bounce between the two reflection points (see figure). Note that the connection length between the reflection points can be quite long: e.g. at the $q = 2$ surface, a field line has to go twice around the torus for one full poloidal turn, hence the distance along the field line between bounce points that are roughly on top and bottom of the flux surface, is about one toroidal circumference. For ITER, with a major radius of 6 m, that is about 40 m!
In a fusion plasma the mean free path of the charged particles, proportional to T^2, is so long that particles in banana orbits can bounce many times before making a collision. They are said to be trapped.
The fraction of trapped particles is given by the mirror ratio, see Eq. (5.37), between the smallest radius ($R = R_0 - r$) and largest radius ($R = R_0 + r$) ends of the magnetic surface. Due to the $B \propto 1/R$ behaviour of the magnetic field, we find that the trapped particle fraction is given by

$$f_t = \sqrt{1 - \frac{B_{\text{low}}}{B_{\text{high}}}} = \sqrt{1 - \frac{R_0 - r}{R_0 + r}} = \sqrt{\frac{2r}{R_0 + r}} \approx \sqrt{2\epsilon}, \tag{11.6}$$

with ϵ the inverse aspect ratio. That is a very appreciable number! In a magnetic surface with $\epsilon > 0.2$, the fraction of trapped electrons can easily exceed 50%. So this dramatically changes our picture of the plasma in a tokamak: most of the particles are not circling the torus, but moving in banana orbits. So, every reason to have a serious look at the consequence of this phenomenon.

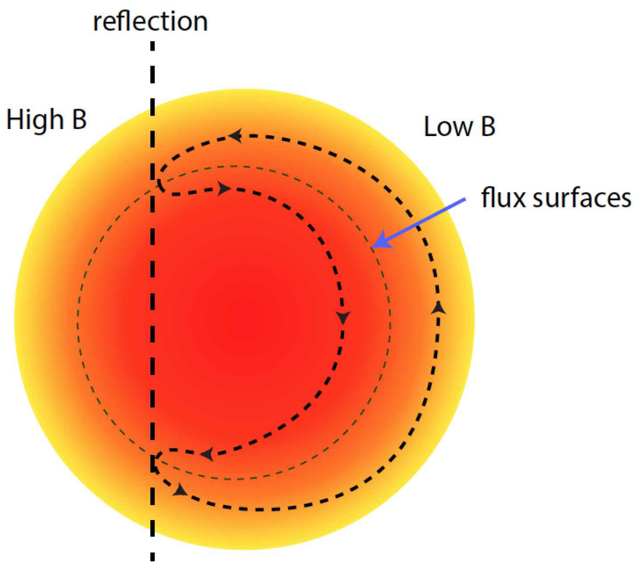

Figure 11.1: Banana orbit projected on the poloidal plane. The actual orbit is stretched toroidally. The reflection location depends on the particle's pitch angle.

Three warnings should be issued with this picture:

- although there is some likeness in shape, the banana orbit has nothing to do with a magnetic island. Nothing at all.
- always keep in mind that the picture is a projection on the poloidal plane. In three dimensions the banana orbit is wrapped around the torus, often a full toroidal turn or more. The main component of the orbit is always toroidal.
- the particles do not deviate very much from the magnetic field lines. Hence the particle's turning point (the tip of the banana in the poloidal plane) is also it's toroidal turning point: trapped particles do not continue in the toroidal direction and therefore do not contribute to the total plasma current.

11.3.2 The width of the banana; Neoclassical cross-field diffusivity

You may ask why the electron does not travel up and down between the bounce points along the flux surface. In other words, why is the projection of the orbit shaped like a banana, with finite width, and not simple like a curved line? The reason is that the electron is always experiencing the curvature and gradient-B drift, in vertical direction. The net effect is exactly compensated during the completion of a banana orbit, so the orbit is closed. But the drift does give the banana its width. There are various ways to calculate the actual width of the banana orbit. The result is very simple: the width of the banana orbit w_b is approximately equal to the gyroradius, but using the poloidal magnetic field instead of the total field. Which in turn can be expressed using the dimensionless numbers q and ϵ:

$$\text{Banana width:} \quad w_b = \frac{mv}{eB_p} = \rho_c \frac{q}{\epsilon}. \tag{11.7}$$

So, we see that in the relevant part of the plasma, where $1 < q < 2$ and $0.1 < \epsilon < 0.3$, the banana width is typically some 5 times larger than the gyroradius, and this is the cause of the increase of the cross-field diffusivity. Trapped electrons clearly travel between the inside and outside of a banana orbit, which is a radial excursion with the size of the banana width. This now takes the role of the step size in the diffusion process. The neoclassical diffusivity due to the trapped electrons is therefore some 25 times larger than the classical diffusivity. Some correction! Remember that this is purely a geometrical effect of the curvature of the field. It is the price we pay for getting rid of the end losses.

11.3.3 Parallel conductivity

As said, the trapped electrons cause the electrical resistivity along the field lines to go up. Why is that? You might intuitively attribute the increase to the fact that trapped electrons do not go around the torus and therefore cannot carry toroidal current. That is perhaps too simple an explanation, or at least deserves some consideration. You may recall that the parallel resistivity does not depend on the electron density ($\eta \propto Z_{\text{eff}} T_e^{-3/2}$). And this was logical: the resistivity is the result of collisions between electrons and ions only, and if you increase the electron density - and so increase the number of current carrying particles - you also increase the number of ions, so the mean free path of each electrons is reduced. So this is the trick: in the case of electron trapping, the number of current carrying electrons is reduced without the accompanying reduction of the ion density.

11.3.4 Ware pinch, Bootstrap current

The banana orbits produce some rather interesting off-diagonal elements in the transport matrix: fluxes driven by 'other gradients'. Two of these are very important.

First, to drive the plasma current in the tokamak we have induced a toroidal electric field. This E-field will accelerate trapped electrons on one leg of the banana orbit, decreasing their pitch angle. And it will slow them down on the return leg, thereby increasing their pitch angle. As the mirror ratio depends on the pitch angle, the top and bottom reflection points are no longer on the same major radius. In effect, the banana orbit is tilted a little bit.

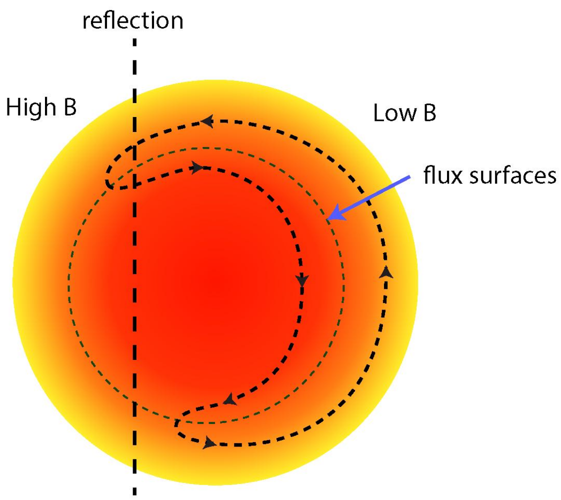

Figure 11.2: Ware pinch. Reflection depends on pitch angle $v_{\|}/v_{\perp}$: deeper for electrons that are accelerated by $E_{\|}$ on the outer leg, less deep for electrons slowed down on the inner leg.

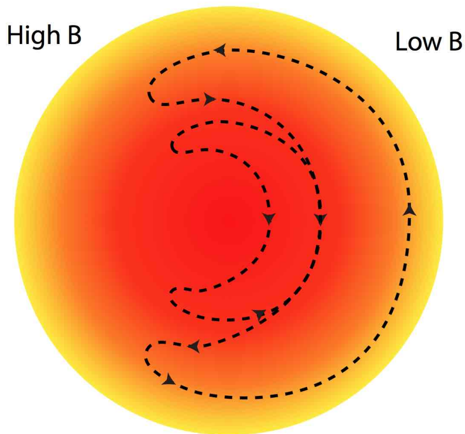

Figure 11.3: Bootstrap current. Two touching banana orbits: net toroidal current on the intersecting legs.

It is only a small effect, because the loop voltage is 0.1 V or so, where as the electron energy is in the keV range. But, it is enough to break the symmetry, and the drift of the electrons no longer cancels out over an orbit. Due to the E-field, there is a small net inward drift. The banana orbit is not closed exactly anymore, the electrons wiggle their way to the centre of the torus. This inward flux is called the 'Ware pinch', after is discoverer Ware (and pinch refers to the resulting contraction of the plasma).

The Ware-pinch leads to a peaking of the density profile, also in absence of a central particle source. Not only is this effect indeed observed, the experimental value is even larger than the neoclassical prediction. It is an important effect.

The Ware-pinch is a radial particle flux driven by a toroidal electric field. Its twin brother (if you want: the Onsager symmetric element in the transport matrix) is a toroidal current driven by a radial density gradient. The torus is filled with a continuum of banana orbits. Now consider two that touch as in the figure below. Electrons in the inner leg of the outer banana travel at the same radius as those on the outer leg of the inner banana, but in opposite direction. Remember that the motion is mostly toroidal. So, if both bananas contain an equal density of electrons that move along the field line equally fast, the net toroidal current in the overlapping region is zero. However, if there is a density gradient (a temperature gradient also contributes), the net toroidal current will be non-zero. This is the Bootstrap current. (Named after the strap on the back of a boot, used by Baron von Münchhausen to pull himself up).

So this is interesting: by making sure that there is a density gradient, we can let the plasma generate its own toroidal current! The Bootstrap current, as measured in the experiment, is well described by the neoclassical expression. Again, this is not a small effect. Not at all! By organising the discharge in such a way that the largest density and pressure gradients are located where the fraction of trapped particles is largest (i.e. in the outer part of the plasma column, but sufficiently far away from the edge, where the collisionality is too high due to the low temperature) we can get the Bootstrap current to contribute some 80% of the total plasma current. This allows for a major extension of the pulse duration of the tokamak.

11.4 Anomalous Transport; Turbulence

So, we understand classical diffusion. It is driven by collisions, the fundamental step-size is the gyroradius and we have to distinguish heat transport from particle transport. Heat transport uses two channels - where the ions carry heat more efficiently than the electrons by the square root of the mass ratio - whereas particle transport is ambipolar and therefore determined by species with the lowest diffusion coefficient, i.e. the electrons.

We also understand neoclassical transport. It is very similar to classical transport but takes into account that in hot, toroidal plasmas a fraction (roughly the square root of the aspect ratio) of the particles is trapped in banana orbits. For those particles the step size in the diffusion process is the banana width instead of the gyroradius, which makes cross field transport much faster. But the trapped electrons do not carry current, so the parallel resistivity increases. Plus we get the extras: the Bootstrap current and the Ware pinch.

So far so good. The neoclassical theory is a good, complete, theory and it forms the absolute rock-bottom of transport - lower transport is not possible. And do realise: if the neo-classical prediction for thermal transport in a tokamak could be realised in the experiment, things would look really bright for the fusion reactor! Now let us have a look at the experiment.

In the experiment cross-field transport turns out to be significantly higher than the neoclassical prediction. Approximately, the experimental values are

Anomalous diffusion coefficients

$$D \approx \chi_e \approx \chi_i \approx 1\mathrm{m}^2/\mathrm{s} \tag{11.8}$$

Also the Ware pinch appears to be larger than the prediction. In contrast, the parallel transport (electrical conductivity and the Bootstrap current) is well described by neoclassical theory. The additional cross-field transport (the bit in excess of the purely collision-driven diffusion) is attributed to turbulence. Turbulence is a rather generic term. In order to be more specific you need to specify the spatial and frequency spectrum, correlation times and lengths, perhaps structural information: spatial structure such as stratification, or temporal structure such as intermittency. Depending on these characteristics the resulting transport may be diffusive in nature, but it certainly does not need to. So, when you enter the realm of turbulence, be prepared for some interesting, intuition-defying experiences. But these are outside the scope of this course.

Here we limit the discussion to the distinction of two main types of turbulence: 1. electrostatic and 2. magnetic.

11.4.1 Electrostatic turbulence

In electrostatic turbulence, the flux surfaces are intact. The enhanced cross-field transport is due to fluctuating electric fields, which are generated when electrons exhibit collective motion with respect to the ions. The fluctuating E-fields lead to fluctuating $E \times B$ drifts, which move the plasma perpendicular to the flux surfaces. Now, you may argue that that by itself is not enough for net transport, as the plasma fluctuates around an average position. But if there is also a density fluctuation – and this is likely, because electron density fluctuations were the cause of the fluctuating E-fields in the first place – there can be a phase relation such that more plasma moves in one direction than back: net transport. The most common class of electrostatic turbulence is 'drift wave turbulence' (a term not explained here) and more particularly the turbulence driven by the Ion Temperature Gradient: ITG turbulence. This is presently the standard model of turbulent transport in tokamaks. In the past decade great advances have been made in the numerical calculation of this turbulence driven transport. Figure **??** shows a early, but classic, results of such a computer simulation, showing how sheared flows in the plasma tear the turbulent eddies apart with a much reduced radial transport as result.

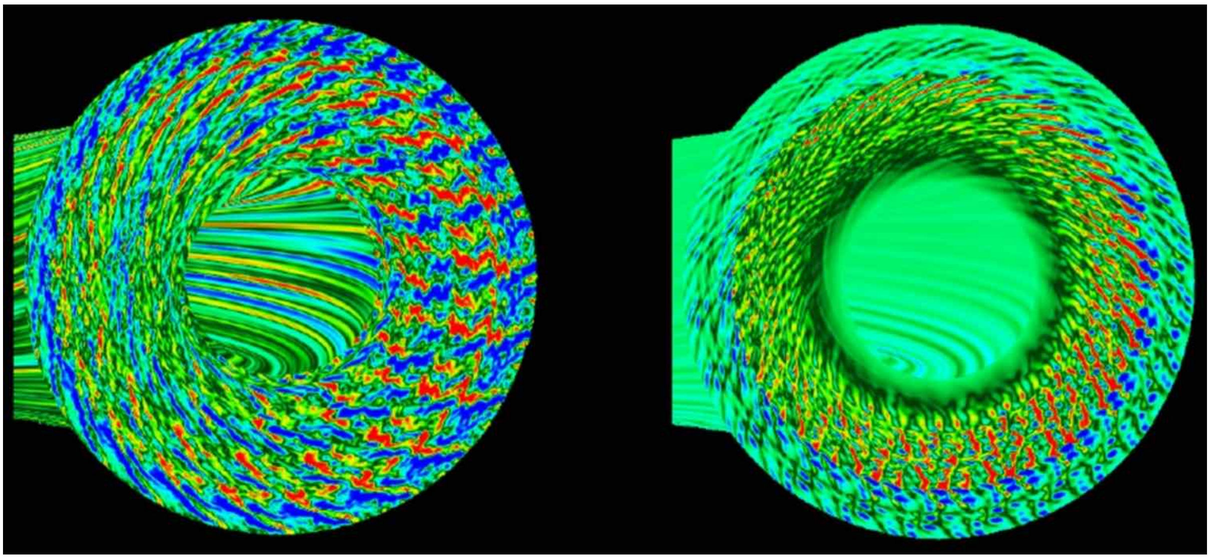

Figure 11.4: Numerical simulation of electrostatic turbulence in a tokamak. This classic picture, published in 1994, shows how the fully developed turbulence on the left has lang radial correlations, which lead to strongly enhanced transport. In the picture on the right the effect of sheared flows in the plasma were taken into account. These destroy the radial correlations and thereby suppress the turbulent transport. (source: Waltz et al, Phys Plasmas 1994)

11.4.2 Magnetic turbulence

Quite another situation is created when there are magnetic fields that perturb the perfect topology of nested toroidal flux surfaces. Practically any perturbation field can lead to the degeneration of a flux surface if the mode structure (the toroidal and poloidal mode numbers n and m) is resonant with the field, i.e. when $q = m/n$ at that flux surface. In that case magnetic islands are formed, as described in Chapter 7. And electrons, being practically collisionless with a mean free path of a kilometre or so, can travel from the inside of an island to the outside along a field line, thereby making a radial step of the width of the island. Which can easily be many centimeters. Islands are therefore shortcircuits for heat, you effectively lose the width of the island for the insulation of the hot core. In other words: An island with a width of 10% of the plasma radius reduces the central temperature by some 10% (and the fusion yield, which scales approximately with T^2 by 20%). But things get really hairy if there are islands at several flux surfaces at the same time. If these islands grow so fat that they overlap, we get regions with chaotic field and effectively we create a thermal path from the hot center to the edge. This leads to a loss of temperature in milliseconds: the plasma disruption discussed earlier.

11.5 How to measure transport coefficients in a hot fusion plasma?

It may be clear that the experimental determination of the value of the cross-field transport coefficients – as a first step in the unravelling of turbulent transport – is a very important branch of fusion research. If you want to keep a plasma at 250 million K, you want to know where the leak is! There are two essentially different methods to determine a transport coefficient. We restrict the discussion here to heat transport - but everything applies to particle transport, too, with minor modifications.

11.5.1 Power balance

In a thermal equilibrium situation (time derivatives zero, the power input exactly balances the losses) and assuming that the off-diagonal terms in the transport matrix are small, the relation between heat flux ($\mathbf{q}$) and temperature gradient is – locally and per species – simply given by

$$\mathbf{q} = -n\chi\nabla T. \tag{11.9}$$

To evaluate the heat diffusivity χ we need to determine the following:

- n, the density, can be measured by e.g. interferometry or Thomson scattering
- ∇T, the temperature gradient. Of course we can measure the temperature in various ways, but do realise that determining the gradient requires a differentiation and leads to a large increase of the error bar: you take the small difference between to measurements of T that are close together, and also the determination of the spatial separation between those points has an error bar. A local determination of ∇T with reasonable spatial resolution is therefore in principle difficult. To circumvent that problem one can fit a smooth function (e.g. a Gauss function) to the measured T-profile. In that case the determination of the gradient is very stable, but of course not local anymore.
- $\mathbf{q}$, the heat flux, cannot be measured directly. In the geometry of the nested flux surfaces, one can evaluate the volume integral of all heat sources (e.g. heat deposition by external heating) inside a surface with radius r, and that gives an estimate of the total heat flux through that surface. The main heat sources are Ohmic dissipation (due to the plasma current) and the additional heating by Neutral Beams, ECRH or other means. It is, however, often quite complex to determine the power deposition profile of these sources. Apart from the sources, there are also heat sinks. The most important heat sinks are radiation, and in the outer edge of the plasma, dissociation and ionisation.
- If the plasma is not completely in equilibrium and there are small time derivatives of e.g. the temperature, the $\partial(nT)/\partial t$ term must be included in the evalation as a heat sink or source.

If the power balance is evaluated separately for the ions and the electrons, the energy exchange between those species is an important factor in the equation: the energy exchange is a heat source for the one and a heat sink for the other. In this case a further complication is that we need to evaluate accurately how the additional heating power is distributed over the species.

In practice it is often very difficult to separate the ions and electrons in a power balance analysis, in which case the two are lumped together and some average heat conductivity is calculated. This does not mean that the intrinsic transport coefficients are the same for the ions and the electrons. Only that it is impossible to measure them separately. Despite its limitations, the 'power balance analysis' is quite generally applied to estimate the transport coefficients in fusion plasmas.

11.5.2 Perturbative methods

An alternative, much more elegant, way to determine the heat diffusivity makes use of a perturbation analysis. By superimposing a – preferably periodic – perturbation of e.g. the temperature on an equilibrium situation, time dependent signals are generated that carry information on the transport coefficients. In practice, one can e.g. modulate the power of the ECRH with a frequency much higher than the inverse confinement time. This wil result in a local temperature variation in the plasma, which will subsequently spread over the plasma by diffusion. If you take a time-resolved temperature measurement at some distance from the modulated power source, you'll see an oscillation. This temperature oscillation can be characterised by an amplitude and a phase. By taking such measurements at two or more locations, the decay length of the perturbation and its phase velocity can be determined. From each of these we can determine the heat diffusivity (heat conduction coefficient).

The equations we need to describe this process are simply the continuity equation,

$$\frac{3}{2}\frac{\partial}{\partial t}(nT) = -\nabla \cdot \mathbf{q} + S, \tag{11.10}$$

(where the power density S vanishes in a region free of heating sources), and Fourier's law, which the heat flux to the temperature gradient:

$$\mathbf{q} = -n\chi\nabla T. \tag{11.11}$$

In cylindrical geometry the vector differential operators complicate the math a little (not difficult, just a little tedious), so we'll solve in slab geometry, and assume that n and χ are homogeneous in the coordinate x (i.e. perpendicular to the flux surfaces). Thus, inserting Fourier's law into the continuity equation, we get

$$\frac{3}{2}\frac{\partial T}{\partial t} = \chi\frac{\partial^2 T}{\partial x^2} \tag{11.12}$$

Note the brand mark of diffusion: single differentiation with respect to time, double differentiation with respect to the spatial coordinate. Solving with the boundary condition $T(x=0,t) = T_0 e^{i\omega t}$ (harmonic perturbation at $x=0$) results in a harmonic oscillation everywhere, with the same frequency ω. But the amplitude decays exponentially as function of x, while the phase lag with respect to $x=0$ increases linearly with x:

$$T(x,t) = T_0 e^{-x/\lambda} e^{i(\omega t - kx)}. \tag{11.13}$$

By substituting this solution into the differential equation above we find:

$$\begin{aligned} &\text{Decay length:} \quad \lambda = \sqrt{\frac{4\chi}{3\omega}} \\ &\text{Phase velocity:} \quad v_\phi = \frac{\omega}{k} = \sqrt{\frac{4\chi\omega}{3}} \end{aligned} \tag{11.14}$$

Each of these equations can be used to extract χ from experimental data. But note that to determine the phase velocity it is not even necessary to have an absolute calibration of the temperature measurement at the various locations: just finding the phase is enough. If you want, you can multiply both equations to find an ω-free expression for the diffusivity: $\chi = \frac{3}{4}\lambda v_\phi$.

These relations express some fundamental properties of diffusive transport. For increasing frequency, the phase velocity increases but the decay length shortens. In other words: higher frequencies spread faster but penetrate less deep. That means that if your initial perturbation has a complex spectrum of frequencies rather than being a single harmonic oscillation, at sufficient distance from the source you always end up with

a sinusoidal signal at the fundamental frequency of the spectrum. So, it does not really matter whether your perturbation is a pure sine, or a sawtooth, or a block function.
To analyse the data and extract phase and amplitude information as a function of frequency, the experimental signals – normally these are T_e-signals measured with ECE – are Fourier analysed. In fact this was the application Fourier invented his Analysis for in the first place! (not for the application in tokamaks of course, but to analyse heat transport problems).

11.5.3 Comparison of the Power Balance and Perturbation Analyses

So we have two different way to evaluate the heat diffusivity. But on closer inspection, the two methods evaluate different quantities:

$$\begin{aligned} &\text{The power balance yields:} && \chi_{pb} = -\frac{q}{n\nabla T}. \\ &\text{The perturbative technique yields:} && \chi_{\text{pert}} = -\frac{\partial q}{\partial(n\nabla T)}. \end{aligned} \tag{11.15}$$

Figure 11.5 illustrates the typical cases we need to distinguish. In the simple case of a heat flux which is linearly proportional to ∇T, the two diffusivities are the same. That would be the case for a piece of copper. However, if the flux does have a linear dependence on ∇T, but the transport matrix contains non-zero off-diagonal elements, then the two χ values are different by definition. In that case the heat flux $\mathbf{q}$ is offset; the off-set is that part of the flux that is driven by other gradients than gradT. In that case a perturbative method measures the diagonal element of the transport matrix (or, to be exact, an eigenvalue), whereas the power balance yields an ill-defined linear combination of the different elements in the transport matrix. However, if χ itself depends on ∇T (and if the transport is turbulent, and the turbulence is driven by the temperature gradient, that is very likely the case) then, too, χ_{pb} is by definition different from χ_{pert}. Finally, the nonlinearity may take the form of a 'critical gradient', with a very steep rise of q when a critical gradient is exceeded. In such a case it is not very useful to measure either of the two χ-values - they have no predictive value. The system is determined by the critical gradient itself. In this case the temperature profile is called 'stiff'.
In conclusion, a perturbative measurement gives access to the transport coefficients in the transport matrix. The power balance gives a number that is representative of the net flux, but cannot easily be related to the underlying transport.

Problems

11.1 Classical transport and confinement

The purpose of this exercise is to let you discover how heating, ohmic dissipation, and transport through classical diffusion hang together in a fusion plasma. You'll also practice the art of working with proportionalities — very useful if all you have is the back of an envelope to write on. My answers are one-liners. Now grab an envelope and turn it over.

Let us assume that the cross-field thermal transport is solely due to classical diffusion. (i.e. no neo-classical effects, no turbulence). The collision times for ions and electrons are given by the following formulas:

Electron collision time for singly charged ions: $\tau_e \propto T_e^{3/2} n^{-1}$
Ion collision time for singly charged ions: $\tau_i \approx (2m_i/m_e)^{0.5}\tau_e$.

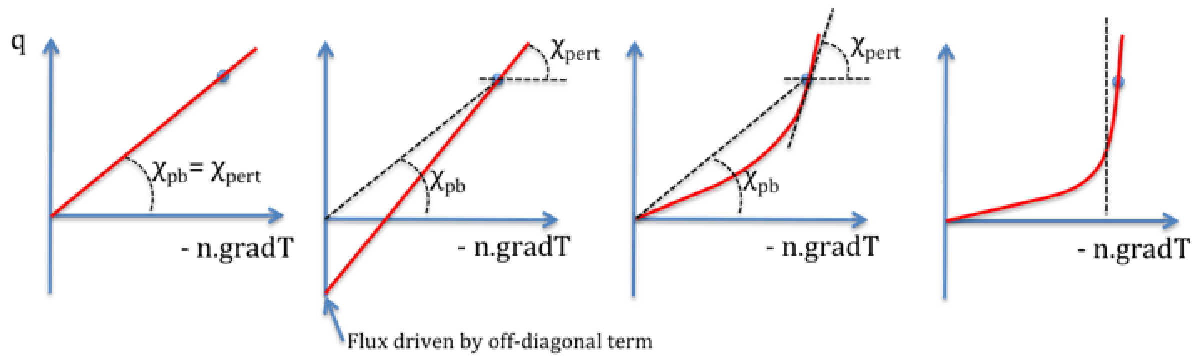

Figure 11.5: Whereas in a classical medium, such as a piece of conductor, the relation between the heat flux and temperature gradient is a simple proportionality (i.e. the heat conduction coefficient), in a turbulent plasma the relationship may be off-set linear, or nonlinear, or even dominated by the onset of turbulence at a critical gradient.

(a) give the generic expression for a thermal conductivity χ, expressed in a mean free path λ and a collision time τ.

(b) Which species (electrons or ions), still in the classical case, give the dominant contribution to heat conduction? (Prove your answer)

(c) Derive a scaling law for the global confinement time τ_E based on classical diffusion only. Consider only the dependence on the minor and major radius a and R, the magnetic field B, and the plasma parameters n and T. And as always: only the proportionalities, not the constants.

(d) Now consider a tokamak that is purely heated by the Ohmic dissipation of the toroidal plasma current. Express the heating power P in terms of the plasma current I, the temperature T, the density n and the geometrical parameters a and R (again: only the proportionalities.)

(e) This power has to be conducted by the plasma. Express (only the proportionalities) the thermal conductivity χ in terms of T, n, B and the dimensionless numbers $\epsilon = a/R$ and the safety factor q.

11.2 Determining the heat conductivity: Power Balance analysis

For a power balance analysis we need to evaluate the heat sources and sinks in the plasma. Heat sources are: Ohmic heating and auxiliary heating. Heat sinks are: Radiative losses. Further, the exchange of energy (through collisions) between ions and electrons constitutes a heat sink for the hotter species, and an equally large heat source for the colder species. Consider a tokamak plasma which is heated – in addition to the Ohmic heating – by Neutral Beam Injection, which you may assume to have the center of its deposition profile at half radius, and ECH which is resonant at $x = r/a = 0.7$ (with r the radial coordinate, running from 0 at the magnetic axis to a at the edge of the plasma). Assume that the tokamak is large, e.g. the size of JET or ITER.

(a) Sketch, as function of the normalised minor radius of the torus (i.e. as function of x) the power density of the sources and sinks, for the electrons and ions. Plot the sinks as negative sources, i.e. negative values of the power density.

(b) Sketch, again as function of x, the total radial heat flux, i.e. the integral of the sources and sinks. (make sure that you get the right values at x=1).

(c) How would these graphs change if you consider a much smaller tokamak?

(d) To complete the power balance analysis, you also need to find the local temperature gradient. You know how to measure the local temperature (confirm this for yourself). Explain why the measurement of the temperature gradient is so much more difficult, that is: its statistical error is much larger than that of the temperature measurement itself and it is sensitive to systematic errors, too. How does the accuracy of the temperature gradient evaluation depend on the spatial resolution with which you want to determine it?

11.3 Determining the heat conductivity of a deserted island

You are marooned on a deserted island. The classical tiny white beach island with a palmtree in the middle. Or a modern variation (Figure 11.8).

Figure 11.6: marooned but not bored!

Hot days, cold nights, the works. You are bored stiff, but fortunately you find a thermometer in your pocked and you realise that you can use that to measure the heat conductivity of the sand of the beach. Write down the relevant equations (in the sand), describe your measurement procedure and how you derive the heat conductivity of the island from these measurements.

11.4 Determining the heat conductivity: Perturbative methods

You have two measurements of the electron temperature in a tokamak as function of time, taken at different values of the radius r: r_1 and r_2. Think for instance of ECE as the diagnostic to take these measurements. The distance Δr between the two points is small compared to the minor radius of the plasma (a) and the region in between is source-free, i.e. there is no volume heat source there. Because someone is applying modulated ECH in the centre of the plasma (i.e. at $r = 0$), the temperature measured at r_1 shows a harmonic oscillation with amplitude ΔT and frequency f_{mod} around its average value (T_0). Assume that the propagation of the temperature perturbation in the region $[r_1, r_2]$ can be considered in slab geometry. So now you look at a slab of plasma with a measurement of $T(t)$ at either end.

(a) Sketch the $T(t)$ signals at r_1 and r_2, indicating relative amplitude, frequency and phase.

(b) Derive (i.e. do not look up) the formula that allows you to calculate χ from the measured phase of the temperature oscillation.

(c) What is the advantage of using the phase information to arrive at χ, compared to using the amplitude information?

(d) How would you choose the modulation frequency f_{mod}? (To which quantity would you have to compare it?). What happens to the phase and amplitude measurement if one modulates too fast, or too slow, respectively?

(e) And here is a really tricky one: because of the modulated heating, the total kinetic energy – hence the poloidal β – of the plasma will be modulated too, and this will lead to a small in-out oscillation of the plasma. How would this affect the measurements?

(f) Sometimes the ECH power is modulated in order to determine where the power is deposited. ECE in that region is used as $T(r, t)$ diagnostic. How would you choose the modulation frequency in this case: higher or lower than for the perturbative transport measurement? Give considerations.

(g) Can you predict — in this case — the phase of the temperature modulation in the deposition region, with respect to the phase of the power modulation?

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12 Aspects of theory: a brief introduction

This course is strictly 'back-of-the-envelope' and contains no derivations that require more than just that, and no calculations that require a calculator. A thorough treatment of theoretical aspects of fusion science is clearly outside the scope of this module. But I would still like to give you an idea of the sort of theory you might expect to encounter or become involved in should you chose to do a specialisation course in this direction, or a special project. So in this chapter you'll find an introduction to some important concepts. It will take you from a cyclist who cycles into a rain shower to the equations of magnetohydrodynamics, and from the simple plane wave in vacuum to the regular zoo of waves that can propagate in a magnetised plasma. This chapter is a guided tour only – for those want to delve deeper we have specialisation courses, theory-oriented internships and graduation projects on offer.

12.1 The distribution function

A plasma consists of very many particles, and to make things worse, each particle in a fusion plasma is in interaction with some 10^8 other particles. Clearly, you don't want to try to describe a plasma by solving the equation of motion for all those particles. So what can you do? Well, to start you resort to the use of the distribution function, which describes the density of particles in phase space $(\mathbf{x}, \mathbf{v})$ in some statistical sense: $f(\mathbf{x}, \mathbf{v}, t)\, d^3x\, d^3v$ is the number of particles in phase space volume element $d^3x\, d^3v$ at time t.
($\mathbf{x}$ and $\mathbf{v}$ are both vectors in a 3-dimensional space: real space and velocity space, respectively. The $(\mathbf{x}, \mathbf{v})$ phase space is therefore 6-dimensional)
Integrating over $\mathbf{x}$ and $\mathbf{v}$ gives the total number of particles N in the plasma:

$$N(t) = \iint f(\mathbf{x}, \mathbf{v}, t)\, d^3x\, d^3v. \tag{12.1}$$

Integrating over $\mathbf{v}$ alone gives the particle density (i.e. particles per m^3) at location $\mathbf{x}$.

$$n(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{v}, t)\, d^3v. \tag{12.2}$$

The entire function can have a time dependence.
So, now we have to describe the evolution of the distribution function under the influence of the dynamics in the plasma. Much more doable than following individual particles, but still a formidable job!

12.2 The material derivative

The second practical thing to do is to borrow a mathematical trick from our colleagues in the fluid dynamics department. We introduce the 'material derivative'. In real space, this is an obvious thing:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{d\mathbf{x}}{dt} \cdot \nabla \tag{12.3}$$

or in words: if a function G has a spatial and temporal dependence, then the time variation of G as experienced by an observer travelling with velocity $\mathbf{v}$ is given by:

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G \tag{12.4}$$

Have a look at Figure 12.1. If you are cycling and you experience that the weather is dry at first, then it starts to rain and some time later it is dry again, it does not mean that the whole of the Netherlands saw the same sequence. You may have cycled through a stationary, local rainstorm. You can't tell the difference.

Figure 12.1: The rain intensity this cyclist experiences as function of time is due to both his moving through the rain gradient, and the possible time dependence of the rain in the frame of the observer

In the 6-dimensional phase space it is no different, but now we have to take the gradient in all 6 dimensions; 3 spatial, 3 velocity, and the time derivative of the velocity is the acceleration **a**:

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + \mathbf{v} \cdot \frac{\partial G}{\partial \mathbf{x}} + \mathbf{a} \cdot \frac{\partial G}{\partial \mathbf{v}} \tag{12.5}$$

where we wrote $\partial / \partial \mathbf{x}$ for the gradient.

12.3 The Boltzmann equation

Applying the material derivative to the distribution function, we obtain the so-called Boltzmann equation. This is a very generic equation, applicable in many fields and situations - not just plasma physics! To obtain it, we have to take only two steps:

- use $\mathbf{a} = \mathbf{F}/m$ (trivial: Newton)
- realise (this is a real step) that external forces make particles move and accelerate together and therefore do not change the distribution function. The only thing that does fundamentally alter the distribution function are collisions.

Thus, the Boltzmann equation reads

Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\partial f}{\partial t} \right)_c \tag{12.6}$$

where the last term denotes the rate of change due to collisions. Specialising now to the case of a plasma, the most important step is to take the Lorentz force to be the force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. The greatest simplification is achieved by assuming that collisions do not play a role at all. This yields the Vlasov equation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0. \tag{12.7}$$

Indeed, there are many phenomena in plasmas that occur at time scales shorter than the inverse collision frequency in which the Vlasov equation is of great use.

But in the more generic case that the collisions cannot be neglected, we can still be specific to the plasma case by introducing a collision term that takes into account the Coulomb collisions. In this case the equation is called the Fokker-Planck equation. We will not work with it here, but it is fundamental to much plasma theory.

12.4 From the Boltzmann equation to fluid equations

12.4.1 Moments of the distribution function

One of the uses of the distribution function is that you can define its 'moments', or more precisely 'velocity moments', which turn out to be useful quantities with a clear physical meaning. Here are the first 3 moments, respectively:

$$0^{th} \text{ moment:} \qquad \int f \, d^3v = n \tag{12.8}$$

$$1^{st} \text{ moment:} \qquad \int \mathbf{v} f \, d^3v = n\mathbf{u} = \mathbf{\Gamma} \tag{12.9}$$

$$2^{nd} \text{ moment:} \qquad \int v^2 f \, d^3v = n\overline{v^2} \tag{12.10}$$

where n is the particle density, $\mathbf{u}$ the average velocity of the ensemble of particles, also called the 'fluid velocity', $\mathbf{\Gamma}$ the particle flux density, and p the pressure.

12.4.2 Continuity equation

For a Maxwellian velocity distribution (i.e. the normal distribution, which you get if the plasma is sufficiently collisional), the second moment is a measure of the temperature: $p = nk_BT$ (which is only defined for a Maxwellian distribution). For other distribution functions is still a measure of the average kinetic energy of the particles, which for distribution functions that are not too far off a Maxwellian is normally equated to 'temperature'.

Now, we can integrate the Boltzmann equation over the velocity. In other words, we take the 0^{th} moment of the Boltzmann equation. This is the outcome (not derived here) for the four terms in the equation:

1. $\displaystyle\int \frac{\partial f}{\partial t} d^3v = \frac{\partial n}{\partial t}$
2. $\displaystyle\int \mathbf{v} \cdot \nabla f \, d^3v = \nabla \cdot \int \mathbf{v} f \, d^3v = \nabla \cdot (n\mathbf{u})$
3. $\displaystyle\int \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{v}} d^3v = \int \frac{\partial}{\partial \mathbf{v}} \cdot (\mathbf{F} f) \, d^3v = 0$ (Electromagnetic forces are non-dissipative: $\displaystyle\frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{F} = 0$)
4. $\displaystyle\int \left(\frac{\partial f}{\partial t}\right)_c d^3v = 0$ (Collisions do not change the number of particles)

So, by taking the integral of the Boltzmann equation we find an old friend, the *c*ontinuity equation:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0. \tag{12.11}$$

12.4.3 Equation of motion

Now, things become a little more interesting if we take the next moment of the Boltzmann equation. First multiply the Boltzmann equation by $m\mathbf{v}$ and then carry out the integration over $\mathbf{v}$. We don't derive this here, but using the moments of f defined above you'll be able to see (you'll easily find the terms below, but there are a few others of which you have to show that they are zero) that the result of this operation is the equation of motion:

$$mn\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla p + \mathbf{F}_{ij} \tag{12.12}$$

Note that this is indeed the equation of motion, i.e. an equation that follows $\mathbf{F} = m\mathbf{a}$, as the left hand side represents the mass times the acceleration, while the right hand side collects all forces (actually force densities, Newtons per cubic metre):

- $qn(\mathbf{E} + \mathbf{u} \times \mathbf{B})$ is the Lorentz force with $\mathbf{u}$ the average velocity of particles under consideration, i.e. the fluid velocity,

- ∇p is the gradient of the pressure (note that this is a special case, only true for an isotropic Maxwellian distribution function. The more general term would be $\nabla \cdot \mathbf{P}$ where $\mathbf{P}$ is the stress tensor (a 3×3 matrix) that includes contributions of pressure and e.g. viscosity. The simplification also implies that we can represent the plasma kinetic energy with a temperature T, which together with n describes the entire velocity distribution function)

- $\mathbf{F}_{ij}$ represents the force on species i due to collisions with particle of another type j, i.e. in our case the momentum transfer to the ions if we consider the equation of motion of the electrons.

Things simplify further if you introduce the mass density $\rho = mn$, and d/dt for the material derivative:

$$\rho \frac{d\mathbf{u}}{dt} = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla p + \mathbf{F}_{ij} \tag{12.13}$$

Note that this form is very similar to the Navier-Stokes equation for fluids – which is also just the equation of motion – and for that reason plasmas behave in many ways similar to fluids.
Now, it is important to realise that so far we have written down the equation for one type of particle, e.g. the ions, at the time. We can stick with that description and solve the equations for the different particles in the plasma, where the interactions between them are represented by $\mathbf{F}_{ij}$. However, since we have both types of particles in equal amounts, we can perform the following trick. We add the equations for the electrons and the ions, and in doing so we realise that

- The electrostatic forces qE cancel (because now the total ensemble is neutral).

- The mutual momentum transfers $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ cancel out (action equals minus reaction).

- The fluid velocities of electrons and ions will in general be different and this difference represents the electric current density $\mathbf{j} = ne(\mathbf{u}_i - \mathbf{u}_e)$.

- As before, we assume an isotropic Maxwellian distribution function, so that the scalar parameters n (density) and T(temperature).

By doing so, we describe the plasma as a 'single fluid', i.e. the ions and electrons are lumped together in a single, neutral ensemble and the fact that they are really species with different charges is represented by the capacity of the ensemble to carry a current density $\mathbf{j}$. The Lorentz force now acts on $\mathbf{j}$, instead of the individual moving particles. The equation of motion now takes the form:

$$nm_i \frac{d}{dt}\left(\mathbf{u}_i + \frac{m_e}{m_i}\mathbf{u}_e\right) = \mathbf{j} \times \mathbf{B} - \nabla p \tag{12.14}$$

where on the left hand side we have the rate of change of the total momentum of the fluid. Because the ions are so much heavier than the electrons, the total momentum is completely dominated by the heavy particles, i.e. the ions. Lumping everything together and assigning the symbol $\mathbf{u}_f$ to the fluid velocity, we finally get:

$$\rho\left(\frac{\partial \mathbf{u}_f}{\partial t} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f\right) = \mathbf{j} \times \mathbf{B} - \nabla p. \tag{12.15}$$

In any stationary situation, in particular the magnetic equilibrium in which the magnetically confined plasma is meant to sit still, the time derivatives should be zero (that is the definition of 'stationary'). So the equilibrium force balance is simply:

The Pressure Balance

$$\mathbf{j} \times \mathbf{B} = \nabla p \tag{12.16}$$

This formula is the basis of magnetic confinement: if you want to maintain a pressure gradient (which you do, because you deal with a plasma that is hot in the centre and cool on the outside) you'll need a force to keep the plasma from expanding, and that force is the Lorentz force $\mathbf{j} \times \mathbf{B}$.

12.5 The equations of Magneto-Hydrodynamics.

Taking the equation of motion together with the continuity equation, and combining those with the four Maxwell's equations and finally the equation of state

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0, \tag{12.17}$$

with $\gamma = \frac{5}{3}$ the ratio of the specific heats, we get a set of equations that form a theoretical framework known as Magneto-Hydrodynamics, MHD for short. It describes the plasma motion as that of a fluid, but a fluid that can carry electric current and is sensitive to electromagnetic fields. This is a very successful description for basically all plasma dynamics in fusion. Interestingly, the MHD equations are scale-invariant: waves on the surface of the sun are the same problem as those in a tokamak plasma.

12.6 MHD: a very successful description of the plasma equilibrium, stability, and the occurrence of 'MHD-modes' in tokamaks

As said, the MHD theory is a very successful theory, the basis for the understanding of magnetic confinement. This includes:

- the equilibrium state: shape and position of the plasma and the internal flux surfaces,
- the occurrence of instabilities: e.g. the kink- and ballooning instabilities that are fundamental limits to stable confinement

- the occurrence of perturbations of the equilibrium system known as MHD modes
- the occurrence of MHD waves: e.g. the so-called Toroidal Alfven Eigenmodes, which are easily detectable with a magnetic pickup coil and can be analysed to reveal precise quantitative information about the interior of the plasma.

12.7 Waves in plasmas

In these back-of-the-envelope lectures, we have so far stayed away from the propagation of waves in plasmas. Yet, this is a very important topic, essential for a variety of diagnostics as well as heating methods. The reason that we don't treat it here is that waves in plasmas are complex. I don't know of any reasonable way of discussing them that does not go a lot deeper than we intend to do in this course – this we'll do in the module on 'Heating and Diagnosing Fusion Plasmas'. So consider this section as a peek preview. I remind you of a few definitions: ω is the wave frequency, k the wave vector, i.e. a vector that points in the direction of the propagation of the wave, and in absolute value is equal to the 'wavenumber', i.e. $1\pi/\lambda$ with λ the wavelength. The propagation velocity of the wave, the 'phase velocity', is given by $v_{ph} = \omega/k$, whereas the group velocity is $v_g = \partial\omega/\partial k$. The group velocity cannot exceed the velocity of light c, but there is no such limit to the phase velocity. The refractive index (N) is defined as the ratio of c to the phase velocity, N=c/v_{ph}. In a material such as glass, N is always larger than unity, but in a plasma N can actually be smaller than 1. The dispersion relation, finally, relates the wave frequency to the wave number. Now let's start the tour.

If we have just a plasma, no magnetic field, it is not so bad. There is basically the plasma frequency which acts as a cut-off. There is no propagation for waves with frequency below it, waves that encounter the plasma frequency when moving in the direction of increasing density (hence plasma frequency) reflect. The dispersion relation is simple – a graphical representation is given in the first panel in Figure 12.2

But if we add a magnetic field the complexities appear. We must distinguish between waves that

- travel parallel and perpendicular to the field
- are polarised perpendicular or parallel to the field (for perpendicular injection) or
- have left- or right-handed circular polarisation (for waves that travel parallel to the field),

and to boot there are essential differences in wave propagation between 'cold' plasmas and hot plasmas. Furthermore, with a magnetic field we have additional resonances due to the cyclotron motion of the electrons and ions.

The result of all of these propagation modes is an interesting zoo of different waves that can propagate in the plasma, and a number of different cut-off frequencies (where waves reflect: $N \to 0$, $v_{phase} \to \infty$, $k \to 0$) and resonances (where waves are absorbed; $N \to \infty$, $v_{phase} \to 0$, $k \to \infty$). Finally, a wave, upon hitting the right condition, can break up into several new waves, that travel in other directions. As you can imagine, in a plasma that does not only have a magnetic field, but one that is inhomogeneous and curved, this zoo of waves is interesting, indeed. And because of the density and temperature gradients, the waves will be refracted, too. Having said this, I should add that the propagation of waves in plasma is very well understood, despite its complexity. It is normally not possible to calculate the propagation of a wave in the plasma by analytical means, but there are so-called ray-tracing codes that do this for you.

To give you a peek preview of all the beauty, I include two graphs which depict the dispersion relations for a cold plasma, see Figure **??**. You'll see that two important new frequencies emerge that we haven't introduced, the so-called upper and lower hybrid frequencies. The lower hybrid oscillation is, as the name says, a hybrid oscillation of which the frequency is a mix of the ion plasma frequency and the cyclotron frequencies of both the electrons and the ions – hence it depends among other things on the ion mass. (The ion plasma frequency is the analog of the normally used electron plasma frequency, and is just a factor of $(Z^{-1}m_i/m_e)$ smaller). Similarly, there is an upper hybrid frequency, which is the quadratic sum of electron cyclotron and plasma frequencies. So, larger than but mostly relatively close to the largest of the two.

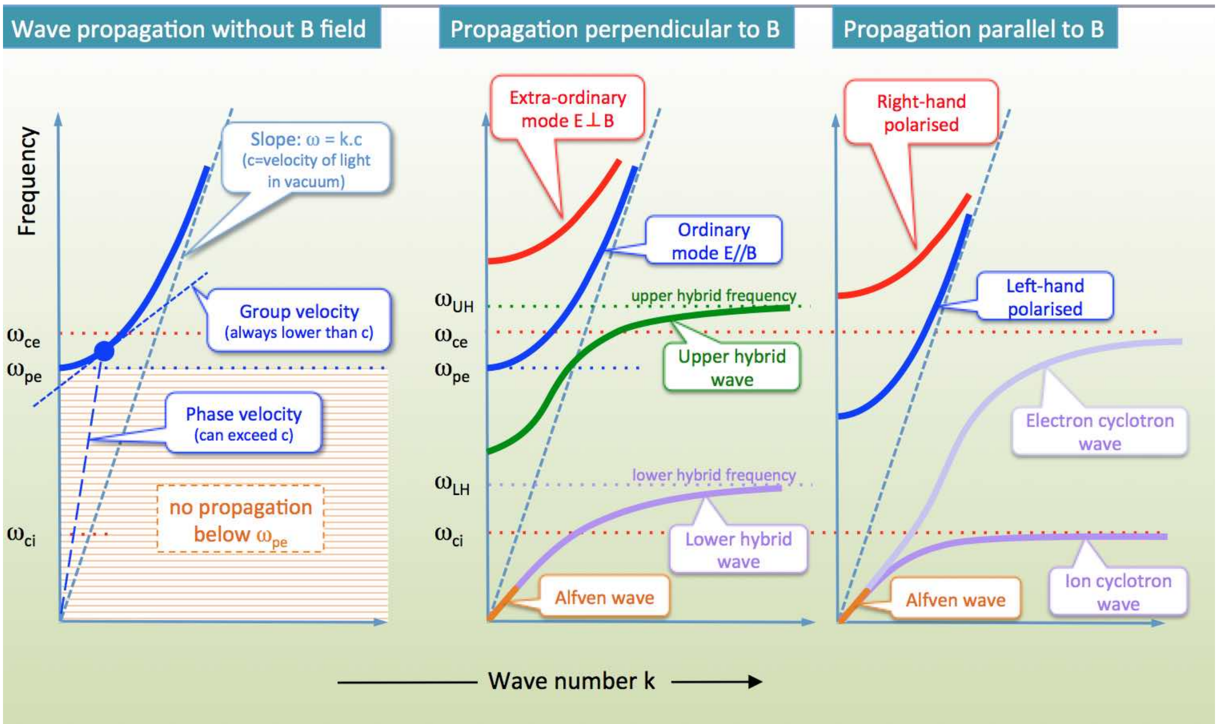

Figure 12.2: Wave propagation in a plasma without (left) and with a B-field

Don't memorise these graphs – but do give them your full attention for about ten minutes.
To connect this world of dispersion relations to the practical life of a fusion scientist, I have also indicated the frequencies that are used for plasma heating. Electron cyclotron waves and, in particular, lower hybrid waves are also used to drive current.

Problems

12.1 Making friends with the Maxwellian velocity distribution

This may seem to be an elementary exercise in integration rather than a fusion problem. Yet, by doing this you will discover somewhat unexpected and very important properties of the well-known and omnipresent Maxwellian velocity distribution.

(a) Consider a 1-dimensional Maxwellian velocity distribution $f(v) = Av^2e^{-\frac{mv^2}{2kT}}$, centered around $v = 0$: Calculate the following five quantities:

- the constant, considering that integrating f over all v must yield the density n.
- the average speed and the average absolute speed

- the most common absolute speed
- the absolute speed for which the kinetic energy equals kT
- the average energy of the particles

(b) repeat in 2 dimensions and plot $f(|v|)$

(c) repeat in 3 dimensions and plot $f(|v|)$

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13 Plasma Wall Interaction in Fusion Reactors

13.1 The issues

13.1.1 Fusion is a compact heat source, which is nice, but the wall loads are high

However big ITER may seem, its reaction chamber is still much smaller than the combustion chamber of a coal fired power plant, as is demonstrated in the Fig. 13.1 (the two have similar power output, and about one thousand times more than the windmill that happened to be in the same picture). This goes back to the fact

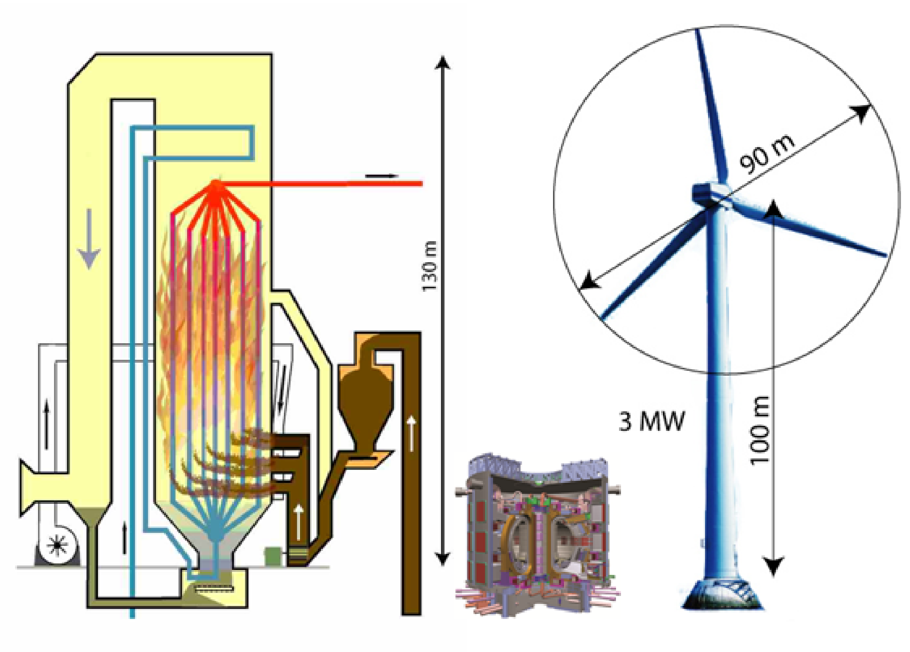

Figure 13.1: Fusion is a compact power source. ITER can comfortably make a pirouette inside the combustion chamber of a coal plant with comparable power output. Note —for comparison— that wind has a very low power density: about 1000 windmills as depicted are needed to produce that same power output.

that in a fusion reaction the binding energy is due to the strong nuclear force, which releases about 10^7 times more energy per reaction than a chemical (oxidation) reaction, which in turn is a much more compact storage of energy than moving air. So the tokamak is a compact heat source. This has nice aspects —fusion power does not require much surface area— but there is a flip-side to that medal: the compact generation of power results, inevitably, in a high heat load on the wall of the vessel. And that will lead to issues of wear, reliability and maintenance. Moreover, if the wall erodes under the influence of the plasma, stuff will be released —wall material— that, if it enters the hot plasma in the reactor, will strongly decrease the fusion power output. So the exhaust of power and the ensuing power load to the wall is much more than a maintenance issue, it is right in the heart of the feasibility question of fusion power!

13.1.2 Magnetic confinement – which is good – makes the wall load much worse!

But things are even worse than they may seem at first sight. Inside the confined plasma the heat generated by the fusion reaction diffuses out radially. But when it crosses the outermost closed flux surface it starts to flow along field lines that end up in the divertor. Now, because the parallel heat conduction is so much better than the cross-field diffusion, all that power will flow in a thin layer just outside the last closed surface. The heat flux density in that 'scrape-off layer' (SOL) is tremendous: about 1 GW/m^2 (see Exercise 13.1). No material can withstand such a heat flux, not by a long way. So we have to do something about it.

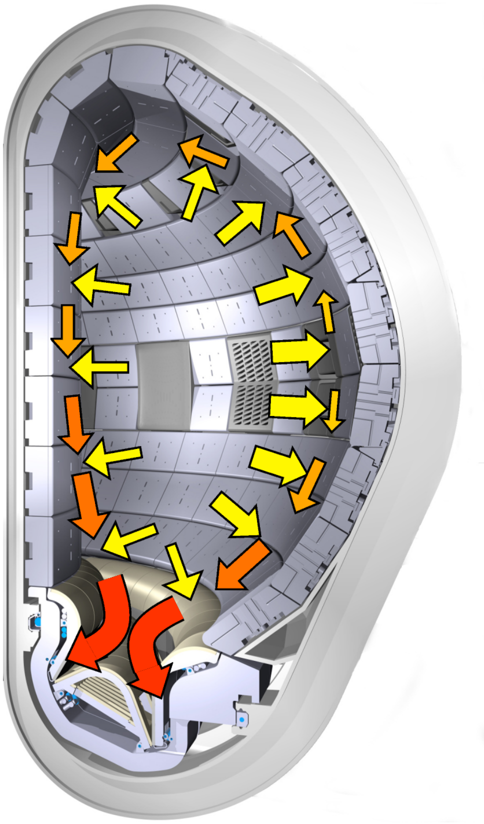

Figure 13.2: The heat produced inside the fusion reactor first diffuses radially outward, but once it crosses the last closed flux surface it flows in a very thin layer —the Scrape-Off Layer— parallel to the field lines to the divertor in the bottom of the reactor. (from ITER, adapted)

13.1.3 Two tricks to reduce the power load: target tilt and radiation

Two tricks are applied in a tokamak to make the heat flux density to the wall in the divertor bearable:

1. target tilt (Fig. 13.3, left): by placing the wall under a small angle with the field lines, we spread out the power over a larger area. Effective, but there is a limit to this trick: the smaller the angle of incidence, the more sensitive the system becomes to tiny misalignment of the wall elements. Moreover, any imperfection of the surface will result in a local amplification of the power density, with melting and redeposition of material as a result. Therefore, the angle of incidence of the field lines on the surface is a few degrees at minimum. Here you have to factor in the fact the while the figure shows the impact angle in a projection on the poloidal plane, the field lines are predominantly going in toroidal direction, so the real impact angle is much smaller than the one in the picture. As a result, the reduction of the power density due to the target tilt is limited to about a factor of 10.

2. line radiation (Fig. 13.3, right): close to the divertor the plasma has cooled down to a temperature that is so low that it emits line radiation: the atoms are not all stripped any more. This is why you

see so much light emerging from the divertor region in images of tokamak plasmas. This process can be enhanced by creating a high density in this region: this cools the plasma and enhances the radiated power. By controlling this process, we can radiate about 90% of the power flux: another factor 10 reduction of the heat load on the divertor wall. The light is emitted in all directions, so that the power is spread over a large surface and the power density is no issue. The operator must take care, however, not to come too close to 100% lest the plasma could disrupt. Another point of consideration: it may not be that easy to radiate enough power. The radiating volume is but small, so a high density of radiating atoms will be required. And as a consequence, now the issue is if the atoms will not contaminate the core of the plasma too much.

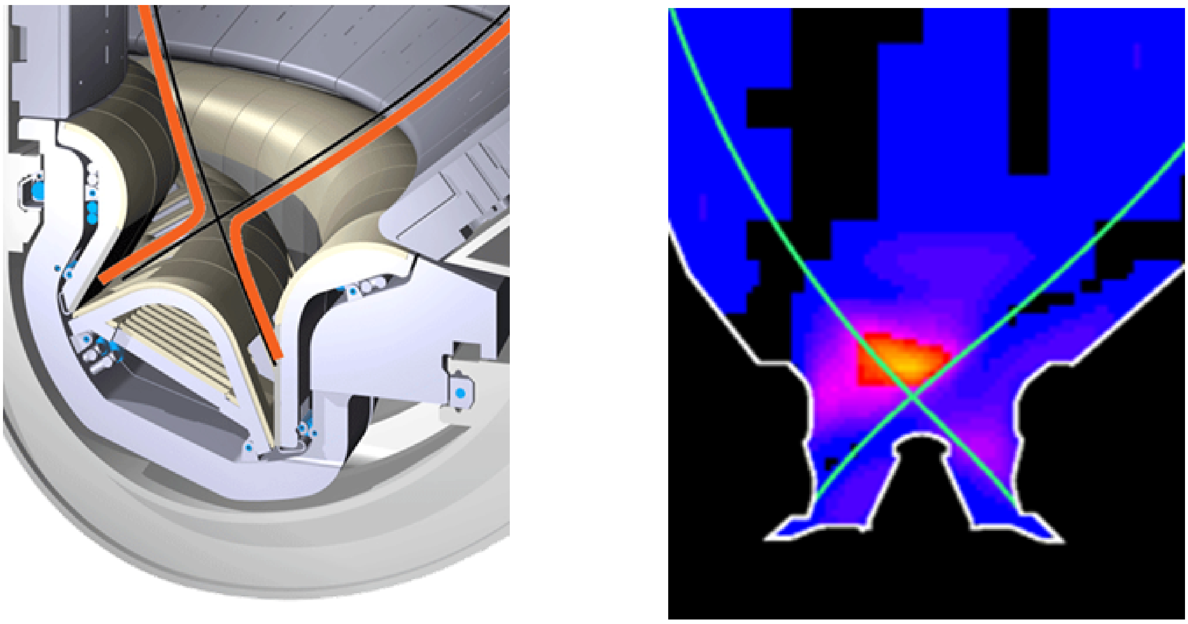

Figure 13.3: Illustration of the two methods to reduce the power density on the target plates of the divertor. Left: shown is how the field lines hit the ITER divertor plate under a small angle, thus reducing the power density. But realise that the figure shows the projection in a poloidal plane. In fact the incidence angle is much smaller, because of the toroidal component of the field. Right: A bolometric measurement of the radiative power density in JET, showing that the emission is highly localised in a small radiating volume. By carefully controlling the density in this part of the plasma some 90% of the power can be radiated away (courtesy Christian Ingesson).

Thus, the power flux density to the divertor is brought back to about 10 MW/m^2, and this is indeed the design value for ITER. This is still a huge power flux density, but with very intense cooling this can be handled.
There is a third method, or rather a family of methods, to reduce the heat flux density on the target plates. These manipulate the magnetic configuration with purpose designed coils. In the 'super-X' divertor (under investigation in the MAST-Upgrade experiment in the UKAEA fusion lab in Culham) the field lines are guided outward, to lower field, so that they spread out and distribute the power over a large surface area (flux expansion). In the 'snow-flake divertor' (in the TCV experiment in Lausanne and the NSTX experiment at Princeton) a complex structure of multiple X-points is created which also spreads the power over a larger surface area, while other schemes –such as the Dynamic Ergodic Divertor (in the now defunct TEXTOR tokamak in Julich) — aim at ergodising the magnetic field in the edge plasma. All these schemes are under investigation, which goes to show that the heat load in the divertor is seen as one of the most serious issues in fusion research.

13.1.4 Edge Localised Modes: pulsed heat loads

But we are not out of the woods yet! Another complication arises because of the Edge Localised Modes (ELMs), the solar flare-like instabilities in the edge of the plasma. Due to these ELMs there are energy bursts on top of the steady 10 MW/m^2 flux. In an ELM, the flux density is enhanced by two orders of magnitude —so we are back at the 1 GW/m^2 level— during about 1 ms. This poses a different kind of challenge: cooling does not help against a pulse, so it is unavoidable that the surface temperature of the wall material will make a large excursion. This causes strong temperature gradients in the material, resulting in thermal stresses, cracking and other unwanted phenomena. The short of it is that big ELMs are intolerable; we need to find ways of suppressing the ELMs. In ITER, the energy deposition per ELM will have to be brought down by a factor of 30 compared to present expectation. This is an unresolved issue. But ITER is an experiment, right?

13.1.5 Issue 1 — heat load: melting and evaporation

So what are the issues with a high power load on the wall? First: recrystallisation, melting and evaporation. One obviously has to stay away from the melting temperature of a material, and already long before the material melts it starts to evaporate rather quickly. This is a very steep function of the surface temperature. Even for materials such as tungsten and carbon the surface temperature must not exceed 1500 °C or so. Figure 13.4 shows the vapour pressure as function of temperature for some materials. Moreover, at typically 30% of the melting temperature, the material starts to recrystallise, thereby losing the good (mechanical) properties it had when it was prepared in the factory. The evaporation rate is proportional to that:

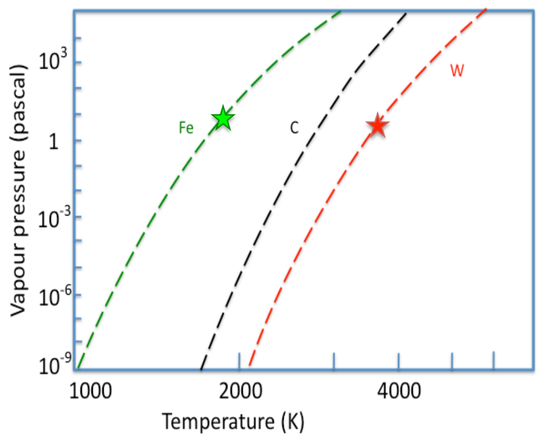

Figure 13.4: The vapour pressure as function of temperature for the two materials of choice for the ITER divertor: carbon and tungsten. Iron is shown for comparison. The stars indicate the melting temperature. Note that carbon does not melt, it ablates when overheated. Note, too, the extremely strong temperature dependence of the vapour pressure: a 10% temperature variation results in several orders of magnitude variation of the vapour pressure. Material properties deteriorate due to recrystallisation at a temperature that is typically only 1/3 of the melting temperature.

$$R_e = 1.7 \times 10^{24} (A\,T)^{-0.5} p \quad \text{in atoms m}^{-2}\text{s}^{-1} \tag{13.1}$$

where R_e is the evaporation rate, T is the surface temperature in K, A the atomic mass and p the vapour pressure in Pascal. If you do the calculation (Exercise 13.1), it is clear that one has to stay clear of the melting point (the star in the figure) by a large margin. The very steep dependency also makes clear that a temperature excursion —e.g. due to an ELM— can make a big difference: a few hundred degrees temperature increase can boost the evaporation rate by several orders of magnitude.

If incidentally you do hit the melting point, bad things happen. Droplets of metal may fly off the wall and, as a small droplet already contains more mass than the plasma (see Exercise 13.4), these will kill the discharge immediately.

The bottom line is that you want to stay below the melting temperature by a wide margin. Figure 13.5, taken from the Alcator C-Mod tokamak (MIT) which has molybdenum walls, shows that it is indeed possible to locally melt the wall material, even if it is made of heat resistant material such as molybdenum. You may

Figure 13.5: Photograph of a Molybdenum tile from Alcator C-Mod (MIT) of which the surface has been melted by the plasma. (see `www.psfc.mit.edu/research/alcator/`)

say: big deal, so cool harder. But there is a simple limitation to the effect of cooling: heat conduction. If the maximum allowable surface temperature is given (say 1500 °C) and that of the heat sink too (e.g. 300 °C), then the total heat flux through the layer between surface and heat sink is simply given by the heat conductivity of the material and the thickness of the layer. Even for materials with very high heat conductivity this means that 10 MW/m^2 translates into a maximum layer thickness of 1–2 cm (Exercise 13.4). And this, combined with the evaporation rate, tells you how long the wall element will survive. In practice, this requirement alone limits the choice of wall materials in the high heat flux zone to refractory metals and carbon.

13.1.6 Issue 2 — erosion: physical sputtering and chemical erosion

There is an entirely different aspect of the heat load, or rather the associated particle load, and this is the energy with which the impacting ions hit the surface. This, as we shall see, is determined by the plasma temperature close to the wall. The temperature of the electrons, to be precise. The point here is that if the impact energy of the particles exceeds a threshold, they can directly knock atoms out of the surface: physical sputtering. The threshold depends on the material (binding energy in the lattice and mass of the atoms) and on the mass of the incoming particles (see Fig. 13.6). So, if the target is made of light atoms such as carbon or beryllium, the threshold for physical sputtering by incoming deuterium atoms is some 10–20 eV, whereas molybdenum or tungsten targets can handle deuterium atoms with energy in excess of 100 eV. Note that for very high energies the sputtering yield decreases again - then the incoming particles penetrate so deep into the target that the probability that their impact results in the ejection of a surface atom reduces. But we certainly don't want to go there!

We need to avoid physical sputtering —as this causes far too rapid erosion— and therefore we must keep the temperature close to the wall low, effectively in the order of a few eV. This can be done by injecting neutral gas. This increases the density and brings down the temperature. Moreover, it enhances radiation, which reduces the conductive heat flux as we have seen already.

As is clear from Fig 13.6, keeping the impact energy of the ions below the sputtering threshold will effectively

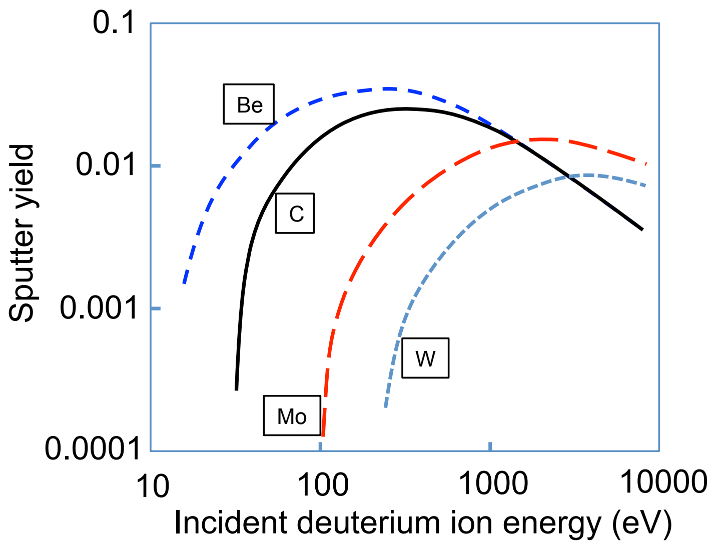

Figure 13.6: The physical sputtering yield (number of sputtered atoms per incident deuterium ion) as a function of projectile energy for different substrates. Note that below a threshold energy the sputtering yield essentially vanishes. So to avoid the heavy wear due to sputtering, the edge plasma in a reactor must be conditioned in such a way that the ions reach the wall with an impact energy below that threshold.

shut down physical sputtering as a cause of erosion. But this does not mean that erosion does not occur at all. The particles that land on the surface, hydrogen radicals mostly in the case of the fusion reactor, can form chemical bonds with atoms from the wall, thus forming molecules that leave the surface. This is a particularly important process in the case of a carbon wall: hydrogen and carbon like each other! This process forms hydrocarbons that enter the plasma, or may be redeposited in remote places. And that process, apart from causing erosion, will also bind fuel —in particular the precious and radioactive tritium— to carbon and take it out of the fuel cycle. Which brings us to the third issue: tritium retention. (In fact – running ahead of the story a bit – this is such an important issue that it has become clear that carbon cannot be used at all).

13.1.7 Issue 3 — tritium retention

The third issue is: tritium retention. In the case of a carbon wall, hydrogen will react with the wall and form hydrocarbon molecules that come off the surface and will be redeposited somewhere else, let's say in the bottom of the machine. In JET, after two years of operation, about a kilogram of hydrocarbon dust was recovered. In ITER, a similar amount could be produced in a single pulse. The retention of tritium by carbon turned out to be so severe that carbon has been taken out of the ITER design altogether: the nuclear authorities would not allow it at all. But tritium retention is not restricted to deposits of hydrocarbons. If the wall material is tungsten, tritium could diffuse into the material, in particular if the surface is cracked due to temperature excursions. And where liquid lithium is considered as a wall coating, it will bind tritium, too. Why is tritium retention a problem? First, because tritium is difficult to make, and second because it is radioactive. Tritium is produced inside the reactor, in the blanket, in a reaction of a neutron from the fusion reaction with a lithium nucleus. So, every triton that fuses with a deuteron can produce exactly one new triton. Of course, that would not work, as you cannot avoid losing the odd neutron and that would be the end of the process. It is, however, possible to multiply the neutrons: one energetic neutron can be used to make a few slower ones. But nonetheless, it is a very big challenge to reach a 'breeding ratio' of 1.1 or so, so we have to be extremely stingy with the tritium. Closing the tritium cycle is one of the critical issues of the development of fusion. (see Exercise 13.5).

Moreover, tritium is radioactive: it is a weak β-emitter, i.e. it decays spontaneously (its half-life is about 12.3 years) under emission of an electron with an energy in the keV-range. At this energy, a sheet of paper, or skin, is more than is needed to stop electrons. Outside the body tritium is therefore not dangerous, but when it enters the body through inhalation or digestion —as part of a water molecule— it effectively acts as a highly poisonous chemical. Which means that it must be treated with great care. Therefore, tritium bound in hydrocarbon dust that is distributed over the entire vacuum system is not acceptable. Here, we must also introduce the notion of nuclear safety and standards. As tritium is a radioactive material, its use is subject to strict regulations. Issues such as the possibility of uncontrolled production of tritiated dust would be a reason for the regulatory body to withhold license to operate.

Note also that the production of hydrocarbon dust per se would pose a serious safety issue. Under normal operating conditions there is no problem at all, but in case of a loss of vacuum, the dust could explosively react with the oxygen (air) that enters the vessel.

For all these reasons, carbon – although a perfect wall material from the plasma operational point of view – has been banned from the ITER design and is not considered for future fusion power plants.

13.1.8 The constraints: neutron flux and plasma compatibility.

On top of these very challenging requirements, the material will have to function in an environment with a very high neutron flux. This can affect the material properties, such as hardness, brittleness and in particular, dimension: metals swell under neutron irradiation. Further, neutrons can lead to transmutation, through inelastic collisions with nuclei of the metal. Tungsten, for instance, in the course of years gradually changes into other metals, and the resulting alloy has very different properties than pure tungsten, being much softer and having a low melting point. Moreover, there are many elements that become radioactive under neutron bombardment, and these cannot be used either. For the same reasons the materials that can be used must be exceedingly pure, to avoid the activation of impurities.

Finally and decisively, the plasma-facing material must also be compatible with the plasma. That is: materials that erode must be low-Z, otherwise the atoms that enter the plasma will radiate too much power. That limits the choice of materials to carbon, beryllium, and refractory metals, of which tungsten - because of its very high melting temperature and neutron compatibility - is the only one considered for ITER. Beryllium and tungsten are normally applied in the form of a coating on the heat sink material.

As you can imagine, these material issues are the subject of some kind of ping-pong game between the plasma physicists and the material scientists. The plasma physicists say: 'this is our plasma, these are the boundary conditions, give us a material that can deal with all of these issues'. But the material scientists tend to say: 'these are the best materials we can possibly make, so deal with it, organise your discharge in such a way that both your plasma and our material survive'.

13.1.9 Which processes control plasma surface interaction, which knobs can we turn?

To solve all of these puzzles, we need to address the following physics issues. First, the all-determining factor is the so-called sheath, a very thin (a few Debye lengths) interface layer between plasma and wall. Here the electron temperature is the control parameter, which determines the flux that is transmitted through the sheath as well as the impact energy of the particles on the wall. Second, the transport in the scrape-off layer translates the sheath conditions to boundary conditions of the core plasma. And finally, materials science can help us design wall materials suited for the purpose.

The box below summarizes the issues associated with the exhaust of power and particles in a fusion reactor.

Causes

- fusion is a compact power source
- the power flows out through the SOL which is only <2 cm thick
- ELMs cause bursts of energy deposition ⇒ surface temperature excursions
- and of course: the fusion reaction produces high energy neutrons

Complicating circumstances and constraints

- Neutron fluence ⇒ strong limitation on applicable materials
- Intense flux of hydrogen radicals (very reactive, corrosive)
- Intense heat flux density (due to combination of high power and thin SOL) which is pulsed due to ELMs
- Wall must be cooled ⇒ need plasma-facing material with high heat conductivity
- Plasma-facing material must have high melting point
- Plasma-facing material must have low Z, or extreme care must be taken to avoid contamination of the plasma with high-Z material

Issues

- Surface melting ⇒ droplets that are ejected into the plasma ($\mathbf{j} \times \mathbf{B}$ forces!)
- Recrystallisation ⇒ well below the melting temperature the materials degrades due to recrystallisation.
- Physical sputtering ⇒ bring plasma temperature close to wall down to 1–10 eV
- Chemical erosion, especially in the case of carbon (formation of hydrocarbons) ⇒ don't allow carbon
- Tritium retention ⇒ trapping inside a metal (tungsten, lithium)

Physics

- Sheath physics: transmission of power and particles through the sheath
- Scrape-off layer physics: conduction and convection towards the sheath
- Material science: tritium retention, erosion

13.2 Where plasma and wall meet: the sheath

Whichever way we turn it, the plasma has to get rid of the power it produces. So in the end we need a process that transfers energy from the plasma to the wall. There is very well established theory about this process. Very generically, when a plasma sits close to a wall, the following happens. First, the more mobile species, i.e. the electrons, are lost preferentially to the wall, which is a 'sink' for plasma particles (Pay attention: the wall is a perfect sink for charged particles, which recombine on the wall. But they may come back into the plasma as neutral particles, and can then be ionized. This phenomenon, recycling, is discussed later). So the plasma charges up positively, until the thus formed electric field holds back the electrons and so establishes a situation in which the ion and electron fluxes to the surface are equal.

So, a potential difference develops between the plasma and the wall. This happens in a layer that is —for obvious reasons— a few Debye lengths thick (i.e. really thin, a fraction of a millimeter). This is called the 'sheath'. A good understanding of the sheath is of fundamental importance when dealing with plasma wall interaction. Chapter **??** is devoted to it. Without going into detail here, these are the main things to take home about the sheath:

- the sheath potential is a few times the electron temperature (the voltage drop increases until it stops almost all electrons —also the electrons in the tail of the distribution function— the total allowed particle flux being determined by the sluggish ions).
- The ions enter the sheath with the acoustic speed, i.e. the thermal velocity of the ions computed by taking the average of the electron and ion temperature.
- The impact energy of the ions on the surface is determined by the sheath potential, hence by the electron temperature!
- The energy transferred through the sheath by the electrons and ions together is typically (this is not so obvious, but you will calculate this in Chapter **??**) 8 times the electron temperature times the particle flux (Γ), which in turn is the collective ion velocity (and as we said above, this is the sound speed) times the density. In formula:

$$q \approx 8\Gamma kT_e = 8nc_s kT_e, \tag{13.2}$$

  where c_s denotes the ion sound velocity.

In summary, the wall acts as a perfect sink for the charged particles that land on it. The sheath transfers the power to the wall. This process is controlled by the electron temperature. The electron temperature, by setting up the sheath potential, also determines the impact energy of the ions that hit the surface. Therefore, it is the electron temperature close to the wall that must be controlled and brought down to such a low value that the ion impact energy is below the sputtering threshold. In practice, this means that the temperature close to the wall must be in the eV range.

13.3 Between plasma and sheath: the scrape-off layer (SOL)

13.3.1 The footprint of the scrape-off layer is only ∼1 cm thick!

We said earlier on that the SOL is found to be thin, its footprint on the strike zone in the divertor is $\approx 1\,\text{cm}$ thick with no strong dependence on machine size (we shall be more precise on this score in Chapter **??**). This is the basis of the problem with the high energy flux density and resulting wall load. Can we do a back-of-the-envelope estimation of the SOL width? The answer is that this is indeed possible, and quite straightforward. The basic line of reasoning is that if we treat the SOL as a homogeneous slab of plasma, with length L and thickness d, where heat is entering in one corner, the ratio L/d must be equal to $\sqrt{\chi_\parallel/\chi_\perp}$, where $\chi_\parallel$ and $\chi_\perp$ are the heat diffusivities along and perpendicular to the field lines, respectively (see e.g. Exercise **??**). Here we have silently assumed the density to be uniform (which it is not, but the approximation is ok). For the heat diffusivities we shall take the classical expressions, which might well be a good estimate. In any case, in this cold and dense and therefore very collisional part of the plasma neoclassical effects will not be important, and parallel to the field lines classical theory basically is always good. We are, however, neglecting the effect of drifts - which we shall make good on in Chapter **??**. Further, a big question is of course whether there will be turbulent transport across the field, something we cannot easily predict. For now we'll just start by assuming classical transport and see where it brings us.
So we find the SOL thickness from the expression

$$d = L\sqrt{\chi_\perp/\chi_\parallel} \tag{13.3}$$

To estimate L, we must realise that the particle still follows the field line, hence moves predominantly in toroidal direction. The 'connection length', i.e. the distance along the field line from point of SOL entrance to the divertor, is the toroidal circumference (2πR) times the safety factor q, i.e. a few times around the torus. For ITER, with $R = 6\,\mathrm{m}$, this would typically result in $L \approx 100\,\mathrm{m}$ (it also depends on where the particle enters the SOL). For the classical cross-field transport we must take the ion heat diffusivity, it being larger than the electron diffusivity by the square root of the mass ratio:

$$\chi_\perp = \rho_i^2/\tau_i \tag{13.4}$$

where ρ_i denotes the ion Larmor radius and τ_i the ion collision time, which is approximately $1.3\sqrt{m_i/m_e}$ times longer than the electron collision time τ_e.
In the classical parallel diffusivity the electron channel is dominant:

$$\chi_\parallel = v_e^2\tau_e. \tag{13.5}$$

We can reduce these expressions, by simple algebra (do try this at home!), to arrive at one that only contains the mass ratio, the electron gyro frequency $\omega_{ce} = eB/m_e$, and the electron collision time τ_e:

$$d = L(\omega_{ce}\tau_e)^{-1}(m_i/m_e)^{1/4} \approx 6L(\omega_{ce}\tau_e)^{-1}. \tag{13.6}$$

Now, the parameter $H = \omega_{ce}\tau_e$ is the so-called Hall parameter. It measures if the plasma is magnetised, i.e. if $H \gg 1$ an electron makes many gyrations between collisions, and the magnetic field therefore dominates the transport. To evaluate it, we first calculate ω_{ce}, which – apart from the natural constants m_e and the elementary charge, depends on the magnetic field only. At $B = 5\,\mathrm{T}$, $\omega_{ce} \approx 10^{12}\,\mathrm{rad/s}$. The electron collision time, on the other hand strongly varies with temperature and density. As we shall see in Chapter **??**, the typical value in the SOL goes from 10^{-6} s at the SOL entrance to 10^{-8} s close to the divertor, i.e. $H = 10^4$–10^6. (Note in the passing that this means that the SOL is magnetised, and so satisfies the condition for using the classical cross-field diffusivity in the first place.) I deliberately say 'close to the divertor', as in the short distance just before the SOL reaches the sheath the temperature takes a drop (see next subsection) while the density rises, with a further strong increase of the collision frequency as a result. But the SOL width is determined upstream. Below we shall derive that the temperature along the SOL goes as $(x/L)^{2/7}$, where x is the distance from the target. Averaging the Hall parameter over this profile, we find that the typical SOL width, in this simple estimation - would be millimetres, typically. This estimate also appears to imply a size scaling, with $d \propto L$, i.e. proportional to the linear size of the machine. However, in larger reactors, certainly if they are burning fusion reactors, the heat flux that must be conducted through the SOL also scales with size, resulting in a higher temperature at the SOL entrance, which in turn would reduce the SOL width. As we'll see below this dependency the temperature on the heat flux density is quite weak ($T \propto q^{2/7}$), but it is enough to compensate the size scaling (work this out for $P_fusion \propto R^3$). As a result, no or a very weak size dependency remains.

A word of caution is in place here. The above estimation is truly back-of-the-envelope. It only considers classical heat conducion, where in fact the transport processes in the SOL are more complex, involve drifts, convection and possibly turbulence, and it is not immediately clear which of the different processes will be dominant. Moreover, the geometry of the SOL is complex, the power flows to the inner and outer divertor leg are different, etc. So what we did is an estimation, it is not a theory. Having made that caviat, the results of the estimation line up well with experimental results. In the experiment the width of the strike zone is usually measured, and in different machines this comes out at 1 cm. However, this includes the spreading of the field lines due to the tilt of the divertor, and/or flux expansion. Factoring in that effect, the SOL itself is only a few mm wide. For present experiments we can rely on measurements, but we are really interested in predictions of the SOL width for ITER and other large reactors. The take-home message for now is that the SOL is thin because the parallel heat transport is much faster than the cross-field transport, and that an estimate using classical diffusion coefficients gives a prediction that is in the right ball park (millimetres) has a weak size dependence.

13.3.2 Heat conduction in the SOL: 1 GW/m^2

Taking the footprint of the SOL to be 1 cm wide, it is easy to estimate the power density in the SOL. For a full-blown fusion reactor, producing 5 GW of fusion power in a torus with 8 m radius the numbers are as follows. Thermal power: 20% of the total power, hence 1 GW (the remaining 80% is in the neutrons) needs to be conducted out of the plasma through the SOL. The footprint of the SOL on divertor is simply $2\pi R d_{\mathrm{SOL}}$. With $d_{\mathrm{SOL}} = 1$ cm, doubled because there are two sides, and $R = 8$ m, the power flux density comes out as 1 GW/m^2. Hence the problem! And this is including the spreading of the field lines over a wider area. In the SOL itself, the parallel heat flux is several times larger. The same estimation for ITER, which at full performance will produce 500 MW of fusion power, and including the heating power, a total thermal power flux of 150 MW that needs to be conducted out, shows that the heat flux density in the SOL would still be hundreds of GW/m^2. So it is manifest that special measures are needed to bring down that heat flux density before it hits the divertor plates. And it is also clear why the prediction of the SOL width is of such crucial importance – if we get that wrong by a factor of 2, it could be the difference between a functional divertor, or one that doesn't survive operation at full power.

13.3.3 The temperature at SOL-entrance is always 100-300 eV.

The power is transported through the SOL by electron heat conduction until the sheath is reached. The parallel heat conductivity described by classical diffusion as before:

$$\chi_{\parallel,e} = v_{th,e}^2 \tau_e \propto T^{5/2}. \tag{13.7}$$

Under the assumption that only conduction carries the heat flux, we find the temperature as function of the coordinate (x) along the field line by integrating the temperature gradient (the heat flux $q = \text{constant} \cdot T^{5/2} \cdot dT/dx$ is constant), starting from the foot point in the sheath, where T is close to zero:

$$T(x) = T_0 (x/L)^{2/7}. \tag{13.8}$$

($x = 0$ at the sheath entrance, $x = L$ at the SOL entrance, and L is the connection length as before).
The gradient dT/dx at the SOL entrance is then

$$\left.\frac{dT}{dx}\right|_{\text{SOL entrance}} = \frac{2}{7}\frac{T_0}{L}. \tag{13.9}$$

This determines the temperature at the entrance of the SOL T_0: the total heat flux q must be transported according to

$$q = -n\chi_{\parallel,e} k \frac{dT}{dx} \propto T_0^{7/2}, \tag{13.10}$$

in other words: a small variation of T_0 is sufficient to accommodate great variations in q. Conversely, T_0 is very insensitive to the power flux: $T_0 \propto q^{2/7}$ and this explains the observation that the temperature at the entrance of the SOL is always typically 100 eV in present day machines. For a full-sized fusion reactor which needs to exhaust more than a GW/m^2 through the SOL, this would be 200–300 eV. Only!

13.4 Particle fluxes, recycling

What we have left out of the picture so far is the particle transport. There is a mass flow in the SOL due to the acceleration of the ions, and this particle flux reaches the surface. In fact, this 'exhaust' is a very important function of the divertor, because in order to get the burned-up fuel, i.e. the helium formed in the fusion reaction, out of the system, we need to set up a steady flow of gas out of the reactor. Therefore, the diverter region is pumped heavily, while the plasma is refueled by neutral beams or pellet injection. But the total particle flux that comes out of the plasma and reaches the SOL is determined by the particle diffusion in the core plasma and is not directly related to the power loss that needs to be exhausted. In fact, the particle confinement time is typically 10 times longer than the energy confinement time. If it were not for the accumulation of helium and other impurities, there would be no reason to pump the diverter and we did not need to have a particle flux to the wall at all. But then, who would transfer the energy to the wall?

So, we have two fluxes that need to be exhausted from the core: the heat flux (which is fully determined by the power that needs to be exhausted, i.e. governed by the burn process) and the particle flux (which is determined by the particle confinement time, and perhaps the pumping efficiency of the wall or divertor). So two fluxes that are determined by independent physical processes, and yet have to match each other in the SOL and sheath. The system appears to be overdetermined. But there is a final degree of freedom for the plasma: recycling. Particles that have landed on the surface have become neutralised, and may have formed molecules, can be knocked off again. In this case there is a probability that they become ionised again in the plasma. This raises the local plasma density while it lowers the temperature (the ionisation takes energy from the electrons). Recycling is the perfect closure of the set of equations, and the plasma uses it all the time to adapt to changing conditions.

In an experiment one can measure the rate of recycling because it is one of the processes that lead to light emission. So the burst of energy deposition by ELMs causes a rapid increase of the recycling — just to keep the energy and particle fluxes consistent. And this is indeed the most common way of characterising ELMs: by plotting the signal of the light emitted by neutral hydrogen atoms that have come off the wall and are in an excited state (the H-signal). Fig. 13.7 shows such signals, taken from the JET experiment, for three different types of ELMs (unfortunately, the ELM phenomenon comes in a variety of shapes and sizes. They have been classified as 'type I', 'type II' etc. — basically the fast, small ELMs are tolerable, the low-frequency large ELMs are not).

13.5 Summary: the overall picture

The burning plasma in a fusion reactor has to get rid of the energy it produces, of which 20% comes in the form of heat. Due to the magnetic geometry, the power flows in the thin layer just outside the last closed flux surface, the SOL, where the power flux can be of order $1\,\mathrm{GW/m^2}$. This is one boundary condition of the problem: the SOL has to conduct this power and will adapt its parameters to make this happen. Apart from the energy exhaust we also want to set up a particle out-flux, but this is a much less stringent condition. The other boundary condition is the sheath: here the power is finally transferred to the wall, through this ingenious mechanism of the sheath potential and the acceleration of the ions towards the surface. The sheath physics and associated with that, the erosion of the surface, is determined primarily by the electron temperature. And that, in turn, adjusts itself to accommodate the incoming heat flux. The temperature in the SOL can be understood by integrating the gradient —necessary to conduct the given heat flux— starting from the sheath, where it is very low, to give a temperature of 100-300 eV at the last closed flux surface. To make consistency

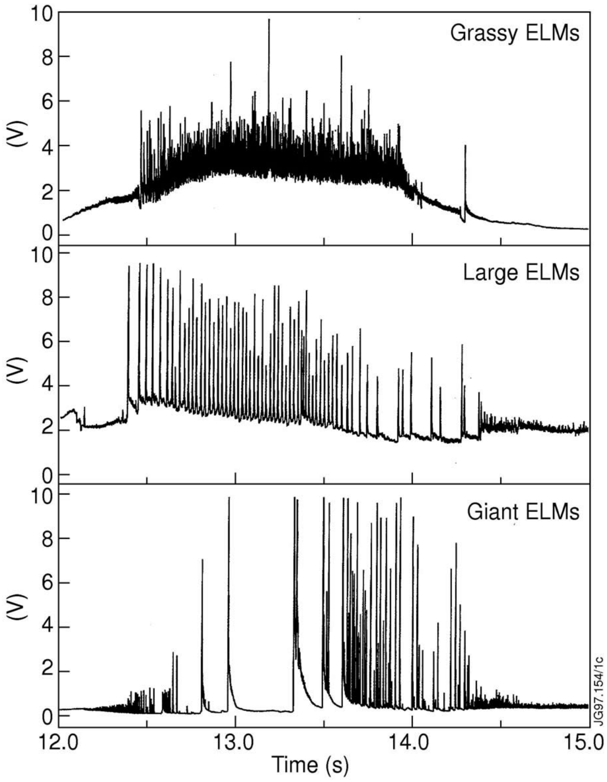

Figure 13.7: Time traces of the emission of Balmer-α light that characterize different ELM types measured at JET (courtesy JET).

between energy and particle fluxes, finally, particles can recycle. In this way the sheath can pump energy to the wall while sustaining a net particle flux that appears to be (much) smaller than is needed for the energy flux. To complicate things further, the heat flux from the core plasma to the SOL comes in bursts, during the ELMs. To make the energy transfer to the wall follow the energy burst, during the ELM the temperature in the sheath must go up – leading to higher ion impact energy, which could result in enhanced sputtering – and the recycling must increase. The increased recycling - seen as strongly enhance emission of H_α-light, is indeed the fingerprint signal of the ELM. And an unavoidable result of the pulsed heat flux is that the surface temperature of the plasma-facing components will make excursions, no matter how well we cool. And that is a serious concern for the materials. In the following chapters each of these ingredients will be dealt with in more detail.

Problems

13.1 Heat flux in the SOL (scrape-off layer).

The power generated in a fusion reactor comes in two forms: (i) the neutrons from the fusion reaction (80% of the power) are not confined by the magnetic field and are absorbed in the wall (in specially designed wall elements called the 'blanket modules'), and (ii) the α-particles from the fusion reaction (20% of the power) are confined and transfer their energy to the plasma. This power sustains the burn temperature. But it is transported to the plasma boundary by conduction. There, it enters the SOL, where it is transported along the field lines to the divertor.

(a) For a reactor with major radius of 8 m, compute (roughly) the connection length, i.e. the length a particle has to travel along a field line from the moment it crosses the separatrix until it hits the wall in the divertor. Consider the ratio of the poloidal and toroidal magnetic field.

Measurements show that the thickness d_{SOL} of the SOL is about 1–2 cm, rather independent of the machine size. In the lecture notes an estimation of the thickness of the SOL is given based on the parallel and perpendicular conductivities, which results in a number that is compatible with the experimental finding.

(b) Demonstrate that if d_{SOL} does not depend on machine size the power density on the part of the wall that takes the heat flux that leaves the plasma by conduction scales as R^2 (major radius), whereas the power itself scales as R^3. Conclusion?

(c) Assuming that the total power produced in this reactor is 5 GW, calculate the heat flux density that would hit a surface if it were inserted in the SOL perpendicular to the field lines

(d) What is done in a tokamak to reduce this heat flux before it hits the target plates in the divertor? Give two different methods. Can you think of a third method?

(e) What is the disadvantage of placing the divertor tiles under a very small angle with the incoming field lines? Explain. Also explain the role of the toroidal field in this matter.

13.2 What is hot? Erosion and evaporation. Get a feel for the numbers. Passive cooling (by radiation).

Whereas in the plasma we are often content with estimates or calculations that are correct within an order of magnitude, when it comes to materials absolute numbers become crucial. Material properties are highly nonlinear in the temperature, even a few hundred degrees can move a material out of its operational temperature window. This exercise will provide a coarse calibration of your intuition.

(a) Give (find on the web) the expression of the power radiated by a 'black' surface as function of temperature. (i.e. assume that the surface is a black body radiator).

(b) The time-averaged power density to the divertor is estimated to be 10 MW/m^2 in ITER. Assuming that all of this must be radiated away, calculate the surface temperature.

(c) Compare this to the melting temperature of the following materials:

i. Tungsten
ii. Iron
iii. Carbon
iv. Beryllium

What do you conclude?

Now, staying below the melting point is one thing, but already at much lower temperature evaporation becomes significant. Let us have a look. The evaporation rate R_e of a solid surface is given by Eq. (13.1). Figure **??** shows the vapour pressure as function of temperature for a few relevant materials

(d) Combine the formula for the evaporation rate with the vapour pressure and the specific mass of the material, and produce a plot of 'loss rate of material' (i.e. thickness, in m/s) versus temperature.

(e) If we require that the loss rate of the wall material in the divertor is less than 1 cm in 1000 discharges of 1000 s each, what are acceptable values of T (considering only evaporation — not erosion by incoming particles)?

13.3 Active cooling.

Assume that a plasma facing material is mounted on a 'heat sink', usually a copper body that is water-cooled.

(a) Just to get an impression: if the temperature of the cooling water is raised by 100 K when it passes through a heat sink with a surface of 10x10 cm^2 that is subjected to a power density of 10 MW/m^2, how much water (liters/second) do you need to flow through the heat sink?

(b) If the plasma facing material takes a heat flux of 10 MW/m^2, and supposing the surface must stay below 3000 K , how thick a layer of plasma facing material can you apply in the case of tungsten and carbon? (look up the relevant numbers for those materials on Wikipedia).

Consider a target that receives a power flux density of 10 MW/m^2, which consists of a heat sink that is actively cooled and kept at 100 °C, on which a layer of 2 cm of CFC (carbon fibre composite) is mounted. Assume that the thermal conductivity of CFC is 200 W/mK, independent of temperature.

(c) Calculate the surface temperature of the CFC layer, under the assumption that the full 10 MW/m^2 is conducted to the heat sink.

(d) Using the graph in problem 13.2, calculate the evaporation rate.

(e) Calculate the time it takes to evaporate 1 mm of CFC. (CFC density $= 1.9 \times 10^3$ kg/m^3).

(f) Under the influence of the neutron flux in a fusion reactor, the CFC will degrade. Suppose this results in a 3 times lower thermal conductivity. Calculate the time it takes to evaporate 1 mm of CFC in otherwise the same conditions as above.

13.4 What if wall material ends up in the plasma?

Suppose we cause a mishap in which a small part of the tungsten divertor plate gets too much heat, partially melts and injects a droplet of 1 mm^3 into the plasma.

(a) If this droplet were to be evaporated and ionized, how many electrons would it produce? (consider for a minute if the plasma temperature is high enough to fully ionize the tungsten. If not, make a rough estimate of the number of electrons that come off the tungsten atom)

(b) Compare this total number of electrons to the total number of electrons in the plasma.

(c) Compare the total energy consumed by the ionizations to the total energy in the plasma

(d) Now repeat these calculations, if instead of 1 mm^3 of tungsten, 1 mm^3 of carbon falls into the plasma.

13.5 Hydrocarbon dust and the fuel cycle.

In the lecture notes it is stated that a single ITER pulse of 10 minutes could possibly produce 1 kg of hydrocarbon dust. Let us estimate the consequences for the fuel cycle.

(a) Compute the number of H atoms (deuterium or tritium) in the 1 kg of hydrocarbon dust (assume something for the composition) .

(b) Compare this to the total number of D and T atoms in the ITER plasma.

(c) If the particle confinement time is 10 times longer than the energy confinement time, i.e. some 10 s, which fraction of the exhausted D and T ends up in dust (instead of going into the pump duct and on to the gas handling system)? Hence, what is the probability that a tritium atom gets trapped in dust and is effectively lost per cycle through the machine?

The burn-up fraction of the exhaust gas is necessarily low — otherwise the reaction chokes on its self-produced helium. Suppose that the probability that a tritium undergoes a fusion reaction between the time it is injected into the tokamak and when it leaves the tokamak (either into the exhaust system, or by being trapped in dust), is 1%. Now, the tritium breeding blankets —if they work very well— can effectively reach a yield of 1.1 tritium atoms per fusion neutron (breeding ratio 1.1).

(d) Deduce how large the loss fraction of tritium may be on a single trip through the tokamak.

(e) What do you conclude?

14 Materials for fusion reactors

14.1 Introduction

If all the daunting physics challenges – stable magnetic confinement, turbulence control, ELM control, disruption avoidance and mitigation, runaway electron production avoidance, burn control, plasma wall interaction – if these are all solved, we can start to think about actually building a working, power producing reactor. But the success of this reactor will critically depend on our ability to develop materials that can deal with the extraordinary conditions in a fusion reactor. There are several material issues that are specific to fusion and we will give some attention to those in this course. The most important are:

- neutron-hard material – especially for the 'first wall'. The 14 MeV neutrons that are produced in the fusion reaction are a good thing for several reasons, but they also cause severe material challenges. They are a good thing, first, because they carry most of the energy freed in the reaction; second, because they will be absorbed in the blanket and thanks to their great penetration depth, will deposit their energy in a large volume rather than on a surface; and third, because they are essential for the generation of new tritium. However, the neutrons will do damage, too. There will always be a separation between the inside of the blanket and the hot plasma, and this separation is called the 'first wall'. The material this wall is made of will swell and become hard and brittle under the influence of the neutrons, and if the material is not optimised for this application it will not survive very long. Moreover, the material may not contain any elements that are activated by neutrons, such as nickel. And if at all possible, we would like the material to not be magnetic, so as to prevent distortion of the magnetic field in the plasma.

- materials for the fuel cycle, i.e. the tritium breeding blanket. Here the challenge is first of all to achieve the required tritium breeding ratio. But even just removing the heat from the blanket is a formidable task, which is intimately related to the materials that can be used, including the coolants. A hot blanket – think 800 -1000 C – is preferred from the perspective of thermodynamic efficiency of the conversion of heat into electricity. And to help materials survive the neutron damage, by slowly but continuously annealing, so to speak. A high temperature also helps the release of the tritium that is generated in the blanket. But at the same time, at such a high temperature the structural materials – in particular the steels that make up the first wall – will lose their strength and start to show creep. So, plenty of conflicting demands that need to be met.

- materials for the plasma facing components – as we already saw, the plasma facing components are subjected to intense fluxes of energy and the highly corrosive hydrogen radicals, on top of the neutron flux already mentioned above. Moreover, they have to deal with repetitive thermal shocks due to the ELMs and, in case the reactor is a pulsed tokamak, thermal cycling. They must be strong but not brittle, have a high melting point, a high heat conductivity and in case of erosion not lead to plasma contamination, and under neutron irradiation not produce long-lived radioactive isotopes.

- superconducting magnets. The magnets used in ITER are, at >5 T and a typical dimension of 30 m, the largest high field magnets in the world. (The magnets in the detectors in CERN are even larger, but operate at much lower field). The design of the ITER magnets therefore called for extensive development work, from the choice of superconductor to the design of the complex build-up of the cable from which the coils are would. In a DEMO reactor, magnets are foreseen that are even larger, while it would be very advantageous if they could operate at yet higher field than those in ITER.

14.2 neutron-hard materials: swelling, hardening, embrittlement and transmutation

When a material, especially a metal, is subjected to a neutron fluence, its properties will change in three different ways: it swells (i.e. its dimensions increase - with all the consequences for the integrity of the construction you can imagine *problem-what happens if materials swells*), it becomes harder (i.e. it will start to show plastic deformation at a higher stress) and it becomes more brittle (i.e. less energy is needed to break it, and the temperature below which the material becomes brittle goes up), so rather than deforming a little under stress, it is more likely to break. Figure 14.1 illustrates these three effects, which are caused by the direct lattice damage due to incoming neutrons and the formation of gas due to nuclear reactions induced by the neutrons. On top of these three effects, the material will gradually change composition due to transmutations, which in itself can completely change the material properties.

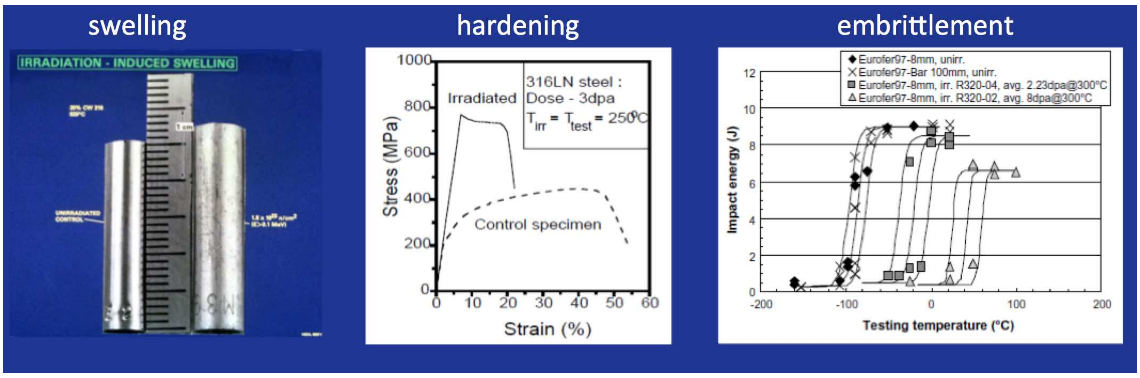

Figure 14.1: Three effects of neutron irradiation: swelling, hardening and embrittlement. (source: swelling: F. A. Garner, PNNL; others A. Fedorov)

14.2.1 Lattice damage by direct neutron impact

When a high energy neutron hits a material, several processes can take place: the neutron can transfer energy to the lattice in elastic collisions, or in inelastic collisions while emitting gamma photons, or it can induce a nuclear reaction, i.e. join or fission a lattice nucleus, usually with the production of hydrogen or helium as the result. We'll review these processes briefly here. Firstly, the interaction with the nuclei in the lattice of the material can be elastic, resulting in energy transfer to the nucleus which in turn starts a series of collisions with other nuclei. This is called a 'cascade', and takes less than a picosecond. The cascade knocks nuclei out of their lattice positions, leaving an empty spot (a 'vacancy') while the displaced nucleus ends up lodged between other lattice atoms, as an 'interstitial'. The vacancy and interstitial together are called a 'Frenkel pair'. (See figure 14.2). On its way through the material, the neutron, while gradually transferring its energy to the lattice atoms, leaves a trail of cascade branches and Frenkel pairs. Figure 14.3 shows an example of a computer simulation of such a cascade.

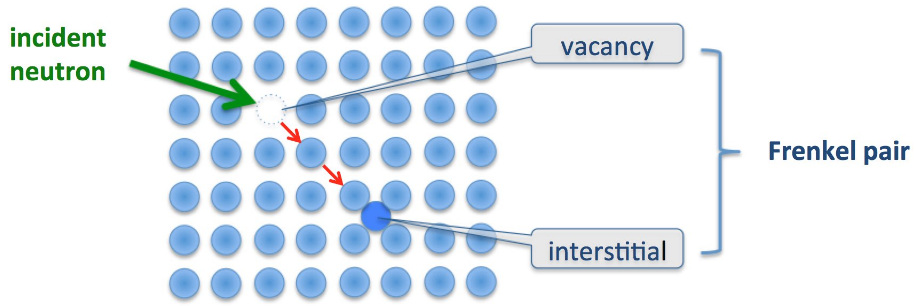

Figure 14.2: Cartoon of a primary knock-on event, resulting in a vacancy and an interstitial, together forming a Frenkel pair

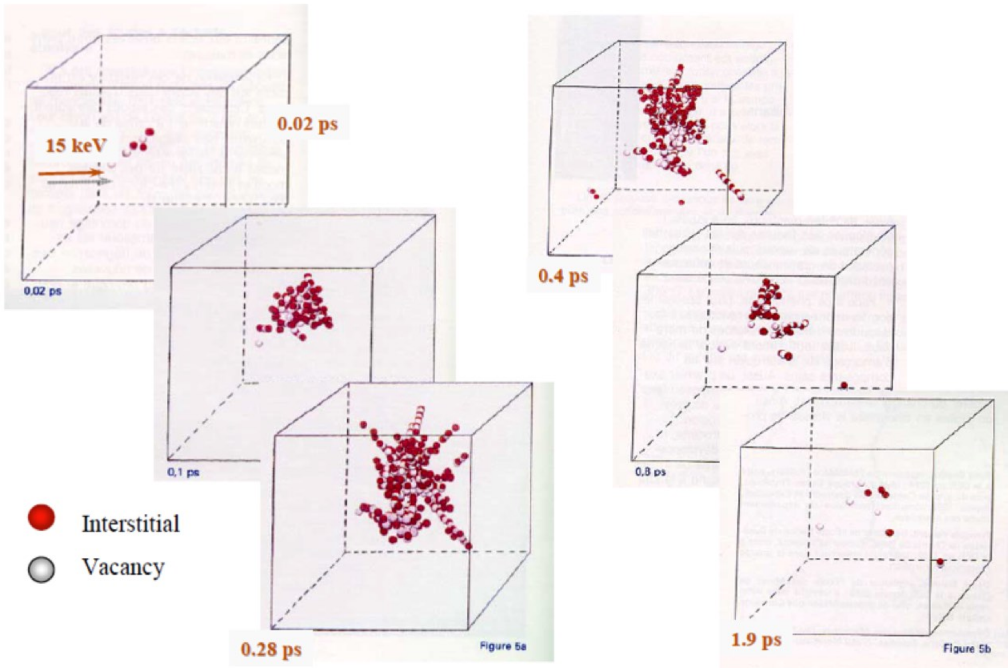

Figure 14.3: Computer simulation of a cascade, showing the branches of with trails of Frenkel pairs at different time slices and the fast annealing of nearly all damage (Source: C. Lemaignan, M. Guttmann, EDF DER 1994)

The energy transfer per collision depends on the mass of the lattice atom of course, and on the energy of the neutron and therefore changes during the cascade. This can be modelled numerically, while there are also approximate formulas, and thus the number of atoms that are knocked out of their lattice spot can be determined. In this way a quantity is calculated that is commonly used to characterise the damage due to neutron irradiation: 'displacements per atom' or 'dpa'. Note that this is a theoretical quantity, that cannot be measured. In fact, as the computer simulations show, a large fraction of the displaced atoms return to a lattice spot during or shortly after the cascade, i.e. while the average energy of the lattice atoms in the path of the neutron is still high, in what could be called instant annealing. So if a neutron irradiation dose is stated as 1 dpa, this does not actually mean that the lattice consists entirely of vacancies and interstitials –

a nonsensical notion of course. On the other hand, 1 dpa is a level of irradiation that is typically associated with severe damage to a material. To put this in perspective: the materials that are used in ITER should be able to survive a neutron irradiation of 1-2 dpa, and such materials are readily available. For a fusion power plant, however, because of the greater power, CW operation and much higher availability, the requirement is about 30 dpa per year, or 150 dpa for a first wall or blanket that is replaced every five years. And that is a level of neutron hardness not yet demonstrated for any material. Generally, a higher resistance to neutron damage requires higher operation temperature of the material, or rather, the higher mobility of the atoms in the lattice at high temperature results in a continuous annealing which limits the lattice damage. Figure **??** gives an impression of the developments. You'll find the conceptual neutron-hard materials at high working temperature - but this comes at the cost of them being brittle at lower temperature, or too soft at the working temperature of course. Not a solved problem, this.

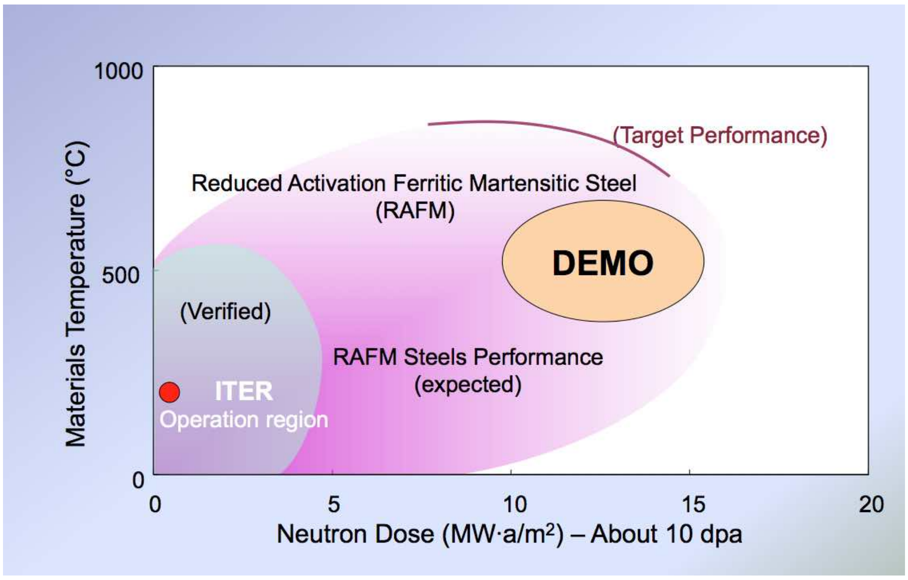

Figure 14.4: The new, high neutron resilience, materials under development typically need elevated operating temperatures (Source: P. Garin, 2007)

It also begs the question: how do you test a material for such high neutron doses, in view of the fact that the fusion reactor will be the strongest neutron source around with a wide margin, and one with a higher energy spectrum (peak at 14 MeV) than fission reactor to boot. Evidently, dedicated high flux neutron sources will be needed to test and qualify materials for fusion reactors, hence the plans for the International Fusion Materials Irradiation Facility IFMIF (see section IFMIF).

After the cascade other processes start to take place on a much slower scale, with the diffusion of interstitials and vacancies as the slowest, long term adaptation of the material taking place on a time scale of seconds to days or weeks. Table 14.5 shows the typical time scales of the processes that take place after the neutron hits the first lattice atom.

Note that the different of vacancies and interstitials may result in the formation of clustered vacancies - voids - inside the material, while interstitial atoms can end up at the edge of the material, or more likely, at the grain boundaries. The net result is swelling and embrittlement of the material.

Time	Energy
Cascade creation $10^{-15} – 10^{-12}$ s	Neutron or Proton $10^5 – 10^7$ eV
Unstable matrix 10^{-11} s	Primary Knockon Atom $10^3 – 10^5$ eV
Interstitial diffusion 10^{-6} s	Displaced secondaries 1 – 1000 eV
Vacancy diffusion >0.1 s	Unstable matrix 1 eV
Microstructural evolution $>10^3$ s	Thermal diffusion kT

Figure 14.5: Table of the processes that happen after the impact of the neutron and their time and energy scales (Source: A. Fedorov)

14.2.2 Gas formation

So much for the damage by elastic collisions. There is also the possibility of gamma-producing inelastic collisions. These have a similar effect as the cascades described above, but a different cross-section, especially at high neutron energy such as found in the fusion reactor. For an accurate assessment of the neutron damage – and of the stopping power of the blanket– these must therefore be accounted for separately. The (inelastic) nuclear reactions can give rise to the formation of gas, in particular hydrogen or helium. These will collect in vacancies, voids, and at grain boundaries where they build up pressure and lead to the swelling and cracking of materials. Figure 14.6 shows a photograph of gas bubbles at grain boundaries.

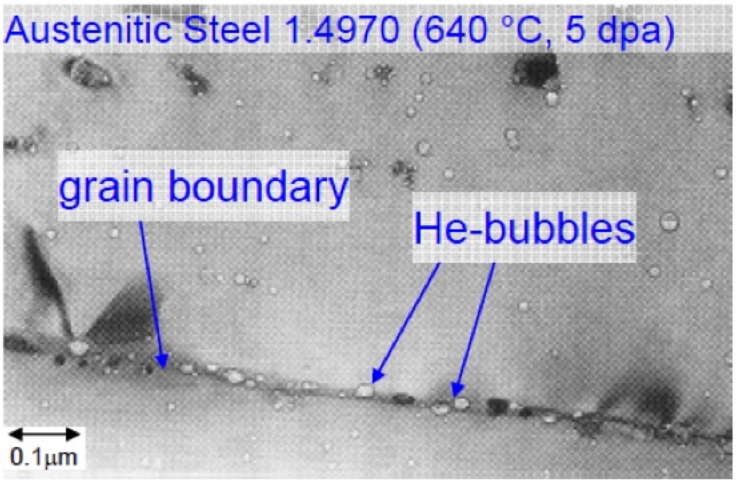

Figure 14.6: Formation of gas bubbles at grain boundaries (Source: A. Fedorov)

It is important to realise that while the gas formation occurs in proportion to the dpa dose, the relative importance depends on the neutron energy spectrum. This is seen in Figure **??**, which shows the typical neutron energy spectra of the high neutron flux fission reactor (HFR) in Petten (the Netherlands) in comparison to that of a fusion reactor (note the high 14 MeV peak – it's a log scale!), as well as the cross sections for the reactions that produce gas. Gas formation is 1 to 2 orders of magnitude stronger for the 14 MeV fusion neutron spectrum than for a the fission spectrum.

Since in practice neutron damage to materials can only be assessed by irradiation in fission plants, tricks must be applied to mimic the effect of gas production in the material, e.g. by seeding impurity atoms in the lattice that have a high gas production under neutron irradiation. In irradiation experiments it was found that the amount of swelling strongly depends on the lattice structure, i.e. basically the same material can have very different swelling behaviour depending on the geometrical arrangement of the atoms in the lattice.

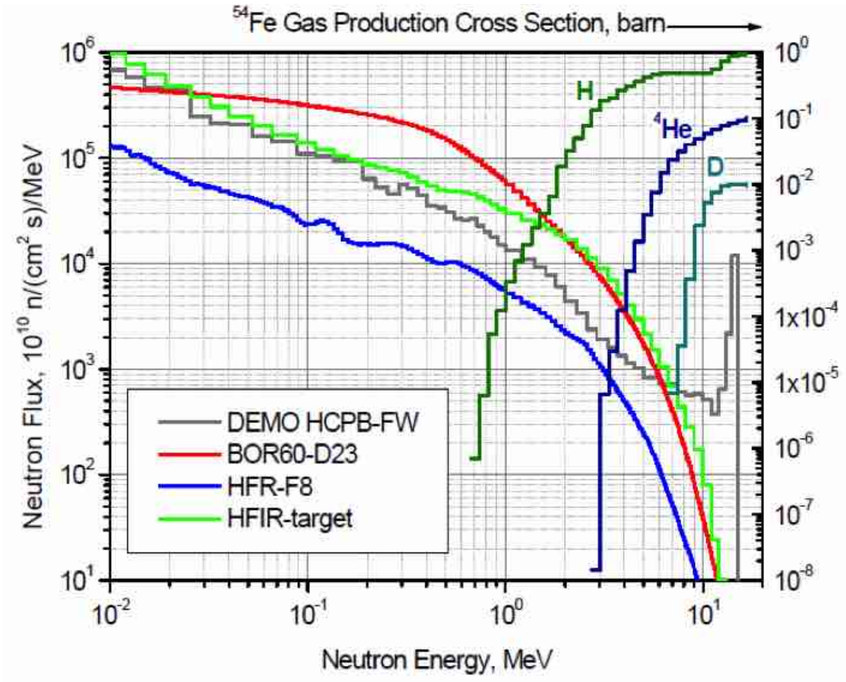

Figure 14.7: Energy spectra of the HFR (Petten) and a fusion reactor, and the cross-sections of the gas producing reactions. The fusion spectrum gives rise to 1-2 orders of magnitude more gas production than the fission spectrum. (Source: A. Fedorov)

In particular, the so-called bcc-structure is much less prone to swelling than the fcc-structure (if you don't know what fcc and bcc look like, look it up on wikipedia). Figure **??** shows how dramatic this difference is. Since the lattice structure is one of the material properties that can be manipulated by choosing the right composition of the alloy and additions, this is something the materials scientists can work with in their effort to develop neutron-hard materials.

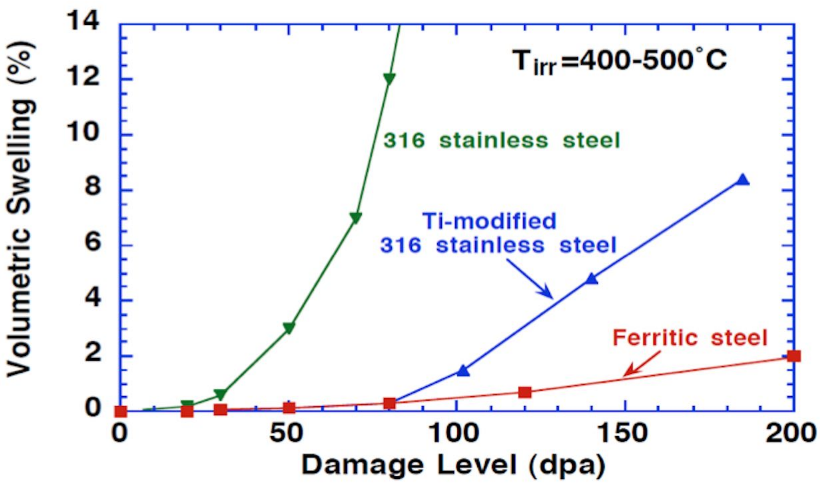

Figure 14.8: Swelling is much lower in body-centered cubic than in face-centered cubic lattices. (Source: A. Fedorov)

14.2.3 Transmutation.

Finally, and important only after prolonged irradiation, the same nuclear reactions that give rise to the gas production also mean that the composition of the material changes through transmutation. For example, a pure tungsten wall element will slowly turn into an alloy containing rhenium and osmium, which - depending on the gradually changing composition - will have properties that are very different from those of pure tungsten: lower melting point, reduced strength and hardness, lower heat conductivity. In other words: much worse than tungsten as plasma-facing material.

14.2.4 The first wall and blanket shielding

Most of the neutron damage will occur in the first wall, the steel wall that separates that inside of the blanket from the vacuum and will literally be the first material wall 'seen' by the neutrons from the fusion reactions. After passing through the first wall, the neutrons will be stopped in the blanket (see section), so that at the rear side of the blanket the remaining neutron flux is very small. Everything behind the blanket, in particular the cryostat with the superconducting magnets, should receive negligible amounts of neutron irradiation, and we will not consider neutron damage in those parts. (Note, however, that apart from the neutron damage, heating due to the residual neutron flux could still be a critical issue for the cryostat, even if the neutron flux is suppressed by 6 orders of magnitude ***Problem: nuclear heating in the magnets***). Figure 14.9, a computation of the neutron flux at different parts of the ITER reactor, shows that the ITER blanket is only marginally thick enough to provide a 4 order suppression of the neutron flux.

Should the first wall need to be protected against plasma effects such as erosion, then the protective layer - be it a thin coating or heat resistant tiles - will also have to withstand the full neutron fluence. And finally, the divertor gets all of the problems combined: (pulsed) heat load, erosion and neutrons. Figure 14.10 shows how this works out for a reactor: the divertor is expected to be replaced every 2 years, the first wall including its protection every 5 years or so, while all other parts of the reactor – in particular the vacuum vessel and the cryostat and super-conducting magnets – are meant to be permanent.

14.2.5 Eurofer, a neutron-hard steel

So, let us consider the material for the first wall. For this, a steel is thought to be the material of choice, as it needs to be strong and makable. But ordinary stainless steel is not acceptable because some of its constituents, in particular nickel, will activate under neutron irradiation. Therefore, in the 1980's alternative alloy compositions were explored, with the 'chromium steels' as the favoured outcome: a stainless steel consisting of iron, about 9% of chromium and a little bit of carbon ($<$1%). This alloy, named Eurofer, has become the standard for first wall materials. In general it does quite well with respect to neutron damage. It has been subjected to prolonged neutron fluxes - building up to tens of dpa - and these tests are still ongoing, and the results are reassuring. But an important issue remains: Eurofer starts to lose its strength when its temperature exceeds 500-550 C. And this is a severe problem, as it rules out the use of high temperature coolants - which are strongly preferred because they give access to much higher thermodynamic conversion efficiency in the electricity generation, whereas the high wall temperature is also highly desirable to reduce neutron damage to the material. There is a development to reinforce Eurofer by means of oxide dispersion strengthening (ODS), a method well known to fortify ordinary stainless steel. This involves the dispersion of nano clusters of ytrium-oxide, less than 1% of the volume, in the Eurofer bulk. This might raise the temperature limit by about 100 C, to some 650 C, which is an improvement but still not enough - ideally the

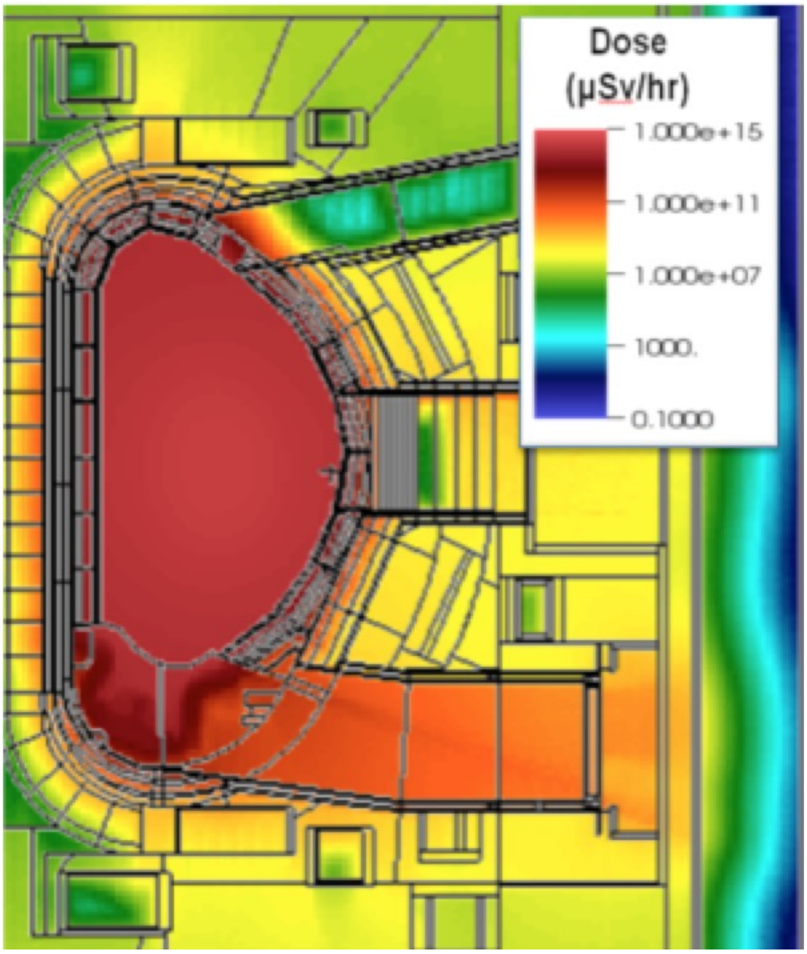

Figure 14.9: Computer simulation of the neutron flux density in ITER, showing that the blanket shields the outer reactor parts but reduces the flux by less than 4 orders of magnitude. (Source: $http - //fti.neep.wisc.edu/ncoe/lib/img/iter_doserate$)

wall should be operated in the range 800 - 1000 C. The dispersion of nano-particles of oxides does, however, turn out to have an interesting side effect: around the oxide particles there is a lot of space in the material where the gas that is produced in the transmutation reactions can gather without doing damage. In other words, the ODS-Eurofer shows much reduced swelling. To end with the cons, however: it is very expensive, and the welds are a potential problem (*problem: why?*).

14.3 the blanket and tritium breeding

The d-t reaction needs deuterium –which occurs in water in practically unlimited amounts – and tritium, which does not occur in nature at all because it decays with a half life of 12.3 years. Therefore, tritium must be made. Fortunately, this can be done as part of the fuel cycle, by letting the neutrons that are produced in the d-t fusion reaction react with lithium. Natural lithium, of which the reserves are plentiful (*problem: look up the reserves and calculate what this means for fusion*), consists of 92.5% lithium-7 and 7.5 % lithium-6. Figure 14.11 gives the two relevant reactions with a neutron :

Note that the reaction with 6Li is exothermal, producing a substantial 4.8 MeV additional energy in addition to the 17.6 MeV of the d-t fusion reaction. The n(7Li,He)n reaction, which has the nice property of producing

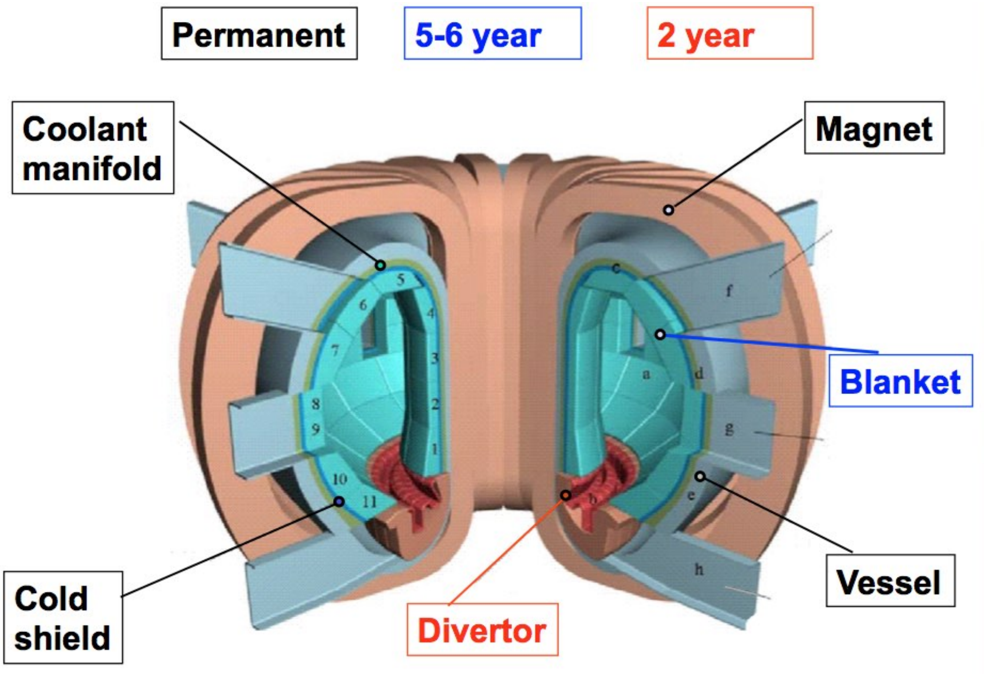

Figure 14.10: Overview of the reactor: the divertor is subjected to both neutrons and intense plasma fluxes and needs to be replaced often, the blanket survives the neutron flux about 5 years, the rest is permanent. (Source: Power Plant Conceptual Study, EFDA report 2005)

$$^{6}\text{Li} + \text{n} \rightarrow {}^{3}\text{H} + {}^{4}\text{He} + 4.8\ \text{MeV}$$
$$^{7}\text{Li} + \text{n} \rightarrow {}^{3}\text{H} + {}^{4}\text{He} + \text{n} - 2.8\ \text{MeV}$$

Figure 14.11: The tritium breeding reactions

a secondary neutron along with a tritium, requires 2.8 MeV and therefore has a threshold energy, which limits its applicability to the high en of the neutron spectrum. Moreover, the cross-section for the reaction with 7Li is very small (*problem: compute the fraction of 14 MeV neutrons that would be stopped by a pure 7Li blanket with 1 m thickness – see exam 2016*). (See Figure 14.12 for the cross sections of both reactions).

In principle, breeding tritium at the same rate it is burned seems simple: just have a blanket of lithium-6 around the plasma in all directions and absorb the neutrons in that blanket, so that every d-t reaction produces a new tritium through the neutron it emits. But obviously this scheme cannot work like that for a multitude of reasons. The blanket has a finite thickness, especially on the central column, so there will be a fraction of the neutrons that are lost, and there is a trade-off between the blanket thickness (which is costly, since it makes the reactor larger) and its tritium-breeding efficiency. Moreover, the blanket will have holes in it, for the heating, diagnostic and pumping ports. Then, the divertor area will not be covered by the blanket - and as there is also the need to spread the heat over a large divertor area, there is als a trade-off between acceptable heat load on the divertor and the effective tritium breeding efficiency of the blanket. Next, a fraction of the neutrons will be absorbed in the first walls, or in the support and cooling structures in the blanket. So, not all fusion neutrons will react with lithium and to achieve tritium self-sufficiency, a trick must be applied: so-called neutron multiplication. The most efficient neutron multipliers are beryllium and lead, with the following reactions:

$^{9}\text{Be} + \text{n} \rightarrow 2\ \alpha + 2\text{n} \quad E(n) > 2.5 MeV$

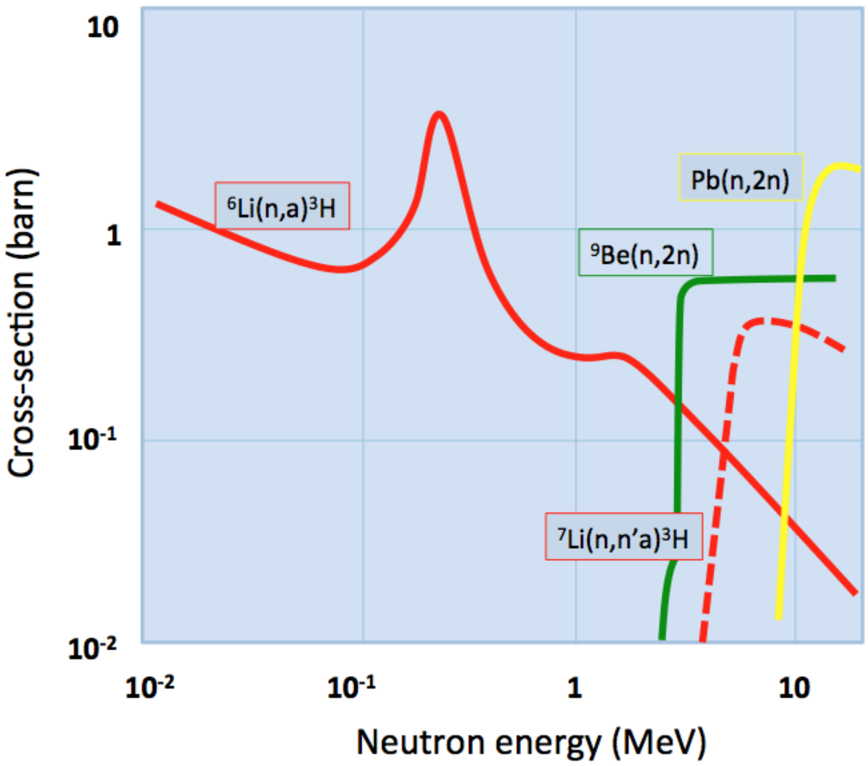

Figure 14.12: Cross-sections of the tritium-breeding reactions as well as the neutron multiplication reactions

$^{208}\mathrm{Pb} + \mathrm{n} \rightarrow \; ^{207}\mathrm{Pb} + 2\mathrm{n} \quad E(n) > 7.4 MeV$

Note that e.g. beryllium will be used up in this reaction, i.e. it is part of the fuel and must be replenished regularly. By clever design of the blanket and optimised distribution of the neutron multipliers and lithium isotopes over the blanket - taking into consideration that the neutron multiplication is most effective in the early stage of the neutron stopping, and that the average neutron energy will go down as it travels through the blanket, so that 6Li should be the dominant isotope in the rear part, if not everywhere - the number of tritium generated per incident neutron can be raised significantly above unity. If that is combined with the effective coverage of the blanket, its finite thickness and the absorption of neutrons by structural materials and coolants the total 'Tritium Breeding Ratio (TBR)' of the blanket can be brought just a bit above unity, TBR=1.15 typically being quoted as the maximum achievable.
The question is how large the TBR needs to be to ensure tritium self-sufficiency for the reactor. You may think TBR=1.0 is precisely what you need. But there are three factors that call for a somewhat larger TBR. First, through the natural decay of tritium, you lose about 5.5% of the inventory per year. This is one good reason to keep the inventory as small as possible, a fraction of the annual fuel use. Second, the burn-up fraction of the fuel in the reactor chamber is necessarily low (***why?***), which implies that the fuel must be cycled many time through the tokamak, the pumps, the gas cleaning plant, and the cryogenic system where fuel pellets are made. If even a small fraction of the tritium gets stuck somewhere in these systems, this can amount to a significant effective loss. Even if this tritium is still somewhere in the machine, not lost to the environment, it is not available as fuel any more and must be compensated by the TBR. Thirdly, to achieve exponential growth of the number of fusion plants during the first decennia of deployment, each reactor must overbreed in order to provide enough tritium to start up the next reactor. Taking these three factors into

account, we find that a TBR =1.1.5 may only be just enough to ensure tritium self-sufficiency. Which makes tritium breeding one of the critical technologies of the fusion development programme. ITER will not breed its own tritium, but it will be used to test various blanket concepts, in the so-called Test Blanket Modules. Figure 14.19 gives an overview of the blanket. Note that the blanket has three distinct functions, namely 1. to breed tritium; 2. to harvest the energy of the incoming neutrons; and 3. to shield the reactor parts behind the blanket from the neutron irradiation.

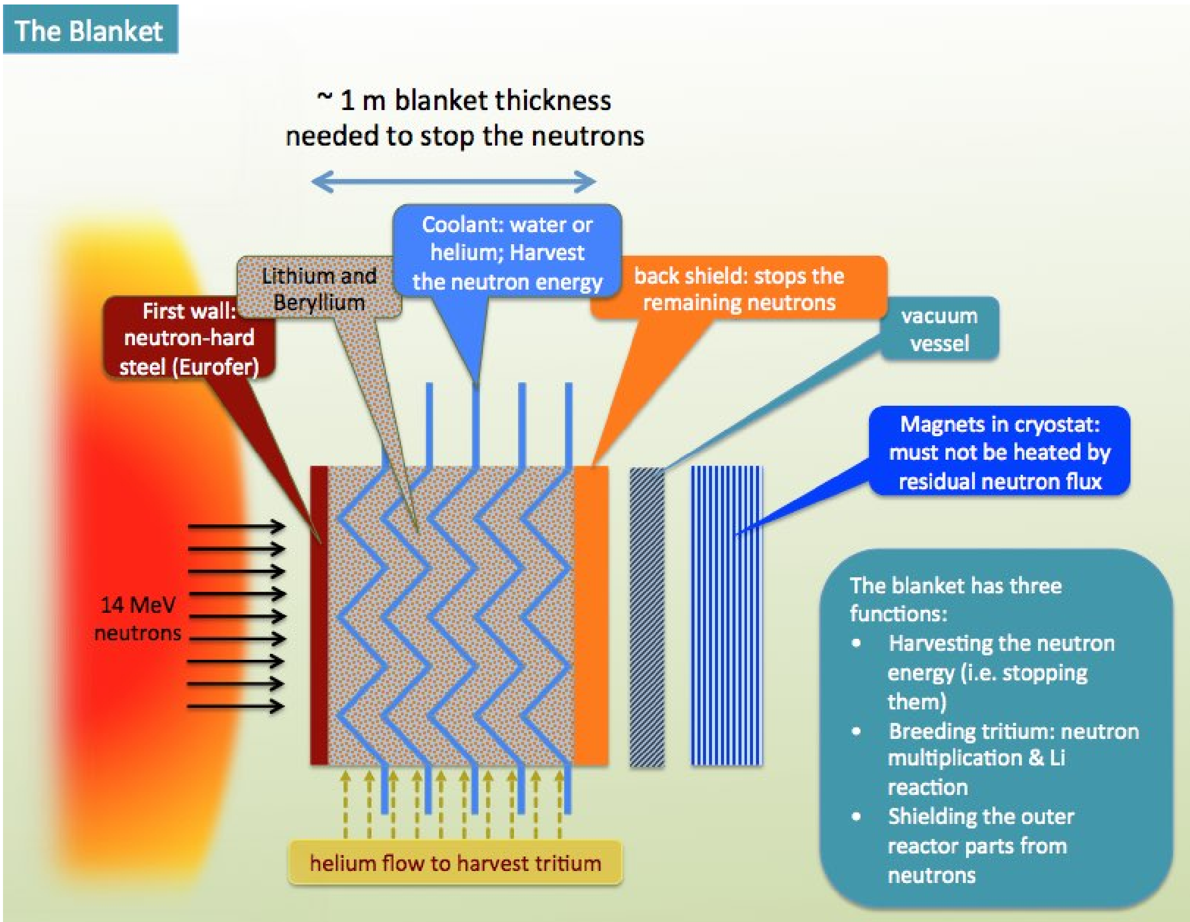

Figure 14.13: Schematic of the tritium breeding blanket

14.4 IFMIF

Since the fusion reactor will be the strongest neutron source around, certainly when it comes to 14 MeV neutrons, the question is how materials can be tested under a similar neutron irradiation. That such tests must be conducted is a certainty. Not only is it important to be sure about the performance of the materials in the reactor in order to avoid bad surprises, a fusion reactor is a nuclear installation and the regulators will demand that only materials will be used that are fully characterised in the relevant conditions. Testing with a lower flux density is really not an option if you want to show that a material will behave well during 5 years of irradiation: the test would take much longer than 5 years, and therefore cannot be done on a reasonable time schedule. To resolve this issue, the fusion community has worked out plans for IFMIF, the International Fusion Irradiation Facility. This is a facility that does produce the relevant neutron flux density and spectrum. It is necessarily a big and expensive (1 Billion Euro investment) facility, which is one of the reasons to do it jointly with many countries (hence the 'International'). The proposed technology is to direct two high energy (40 MeV) high intensity (2x125 mA) deuterium beams on a target that consists of a jet of liquid lithium. The nuclear reaction of the deuteron with the lithium results in the ejection of one or more neutrons. Hence high power accelerators are part of the design, as well as an installation that produces the fast streaming liquid lithium jet. Figure 14.14 gives an artists impression of the lay-out of IFMIF. The deuteron energy has been designed such that the energy spectrum of the neutrons matches the d-t fusion spectrum as well as possible. The flux density of the emitted neutron beam is high, corresponding to up to 20 dpa per year - so relevant tests can be done. But the beam is quite small: the test volume with the highest neutron flux density is only half a litre! IFMIF does not exist yet. It is a design, and prototypes of the some of the technology have been manufactured and tested. If you want more information about IFMIF, you may want to visit the IFMIF website: www.ifmif.org.

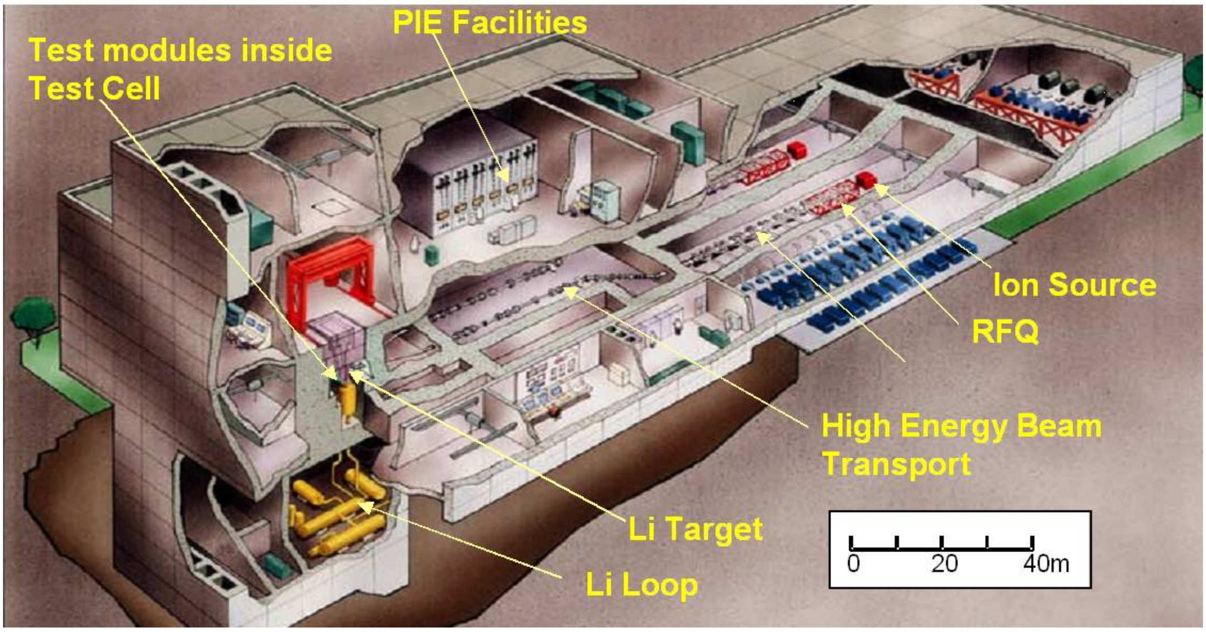

Figure 14.14: Artist's impression of IFMIF (source: IFMIF)

14.5 Plasma-Facing Materials

The plasma-facing materials have to endure the exposure to the plasma as well as the neutrons. So they are subjected to extreme fluxes of heat, hydrogen radicals, and neutrons, and to make things worse, the power and particle loads are pulsed due to the ELMs. Most of the relevant physics and plasma-wall interaction was already discussed in chapter 13. If we concentrate on the materials aspects here, these are the requirements that must be met: high melting point, high thermal conductivity, good neutron compatibility, good tritium compatibility (low retention). And all of that under the additional condition that the material is compatible with plasma operation. I.e., to the extent that wall material does enter the plasma, this should not lead to dilution, loss of performance, or in the worst case, disruption. In practice, this means that if we restrict ourselves to solid materials, only tungsten qualifies as plasma-facing material. ITER does have beryllium on the inside of the vacuum chamber outside the divertor, in places where the power flux density is not so large (beryllium has a relatively low melting point). For a power plant, however, this does not appear to be a solution as beryllium is too easily eroded or melted to last years of uninterrupted operation.

14.5.1 The tungsten monoblock design for the ITER divertor

The typical concept for a water-cooled tungsten divertor element is the so-called monoblock: a block of tungsten mounted on a cooling pipe made of a copper alloy. Figure 14.15 shows such a monoblock. This design has been manufactured and tested, and appears to be capable of handling heat fluxes well in excess of the required $10MW/m^2$. Yet, even so there may be an issue in the actual geometry of ITER, where care must be taken that no edges are hit by the incoming plasma.

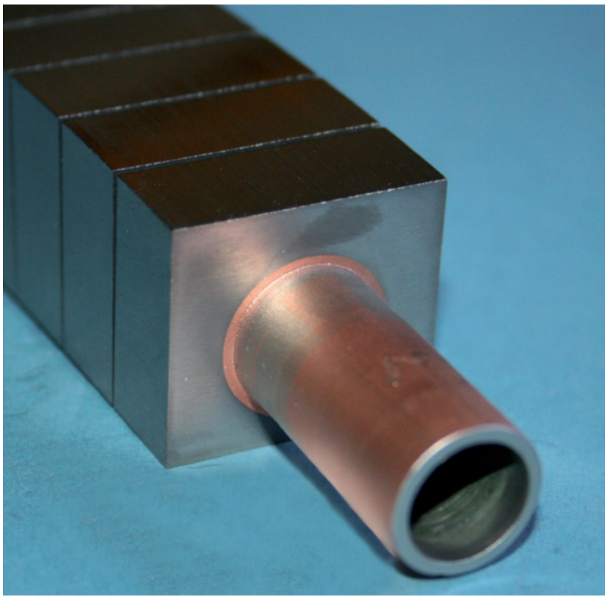

Figure 14.15: The tungsten monoblock divertor component: blocks of solid tungsten welded on a copper alloy cooling pipe. (Source: CEA)

For tungsten the main issue is how it will behave under neutron irradiation. The expectation is that its neutron resilience is good when it is operated at high temperature, 800 – 1000 C, but a plasma-facing component typically has a surface temperature much higher than that, whereas the cooled back end has the temperature of the coolant. As long as that is water, 300 C is the maximum achievable temperature. Tritium retention in tungsten is small, especially when it is at its elevated operation temperature. Another issue is the transmutation already mentioned above. Under neutron irradiation, tungsten gradually transmutes to rhenium and osmium, and thus an alloy is formed that is much softer and has a lower melting point than pure tungsten. This problem should only come to prominence after years of operation, though.

14.5.2 The ITER-like Wall project in JET

In order to provide a solid experimental basis for the operation of ITER, JET was equipped with the same composition of wall covering: beryllium on the vessel wall, tungsten in the divertor. This 'ITER-like wall' project started operation in 2011 and is still ongoing. Among other things, it showed that confinement and achievable β are somewhat better than with the original carbon wall, and intriguingly, the degradation of confinement with heating power is much less pronounced (see e.g. Challis et al, 2015). Generally, the experiments with a tungsten wall in JET and other machines have emphasised how important the wall is for the core plasma behaviour, through the recycling at the edge and the radiative power loss distribution. Figure 14.16 shows how the inside of JET was modified to mimic the situation foreseen for ITER.

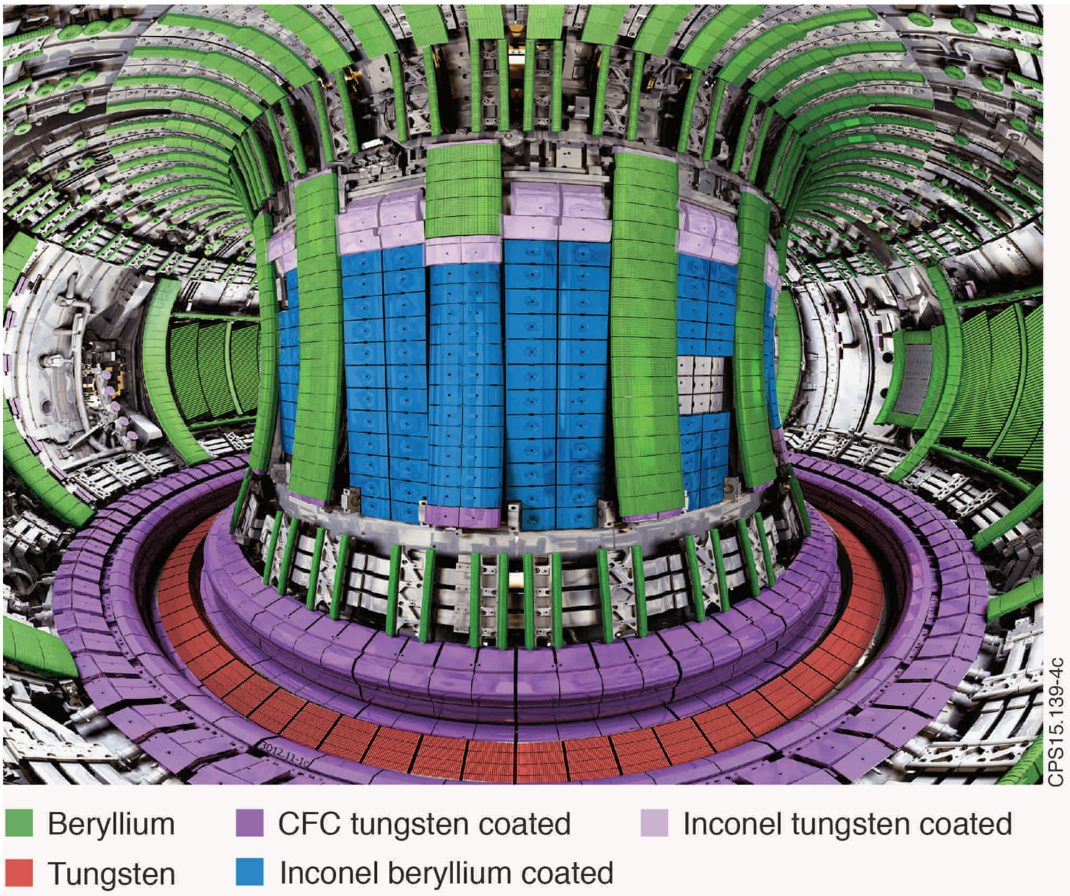

Figure 14.16: Overview of the ITER-Like Wall in JET, showing how the various parts of the JET interior were either replaced or coated by beryllium and tungsten (source: EUROfusion)

14.5.3 liquid metal wall concepts

Contrary to what was said earlier about the wall material needing to be strong and have a high melting point, liquid metals may offer a way out. Surprising, huh? Well, think of it in this way. With a liquid metal you don't have to worry about it melting, because it is already melted. And you don't have to worry about neutron damage to the lattice because there is no lattice. And you don't have to worry about structural damage, because it doesn't have a structure. And if it erodes or evaporates, you just flow in some new liquid to replenish. And if it absorbs some tritium, you flow it to a place where you can retrieve the tritium, after which you recycle the liquid metal. And this isn't the end of the list of wonders. Where pulsed heat loads are the bane of the existence of solid metals, a liquid metal handles them smilingly: it evaporates some and the vapour will radiate away power and shield the component from the incoming heat pulse. Oh, and - at least in the case of liquid lithium - it turns out that it's great for plasma performance, too. With a liquid lithium wall the recycling is strongly suppressed (from $>90\%$ to a very low percentage) and impurities are effectively gettered. The plasma becomes extremely clean, there is excellent density control, and the discharge can be operated with a high temperature close to the wall, which does wonders for confinement while we don't have to worry about the sputtering of the wall - since it is auto-repairing and self-replacing anyway. All of this is relatively unexplored, there is certainly no accepted complete theory of plasma wall interaction with liquid lithium or why it has a beneficial influence on plasma performance, but the experimental results are promising. Of course, you need power to keep the stuff liquid - but power we have, in the divertor. In spades. Enough to melt tungsten - let alone a low melting material such as lithium or tin. And, the liquid must be placed on top of a supporting structure, and this will still get the neutrons. True enough. But by separating the requirement for neutron hardness and structural integrity from the demands of the plasma surface interaction and retention issues, the integral problem becomes a lot more handleable. Yes, you'll object, that is all very nice, but how do you make a wall from a fluid? Excellent point. You still need a structure of some kind to keep the liquid where you want it to be. There are various concepts to deal with that, varying from tungsten sponge-like fabrics (so called 'capillary porous systems (CPS)', keeping the lithium in place by capillary forces), to wetted surfaces and even thick layers of flowing lithium (in the bottom of the tokamak). The idea dates back to the 1980's and has been revisited from time to time, but the practical application always seemed too daunting. However, in recent years there has been a revival. Without attempting to list all or even the most important activities and actors in this field, I draw attention to those with whom the TU/e fusion group has direct interactions through PhD and graduation projects: the Magnum-PSI experiment in DIFFER (check out this movie: https://www.youtube.com/watch?v=d3xJnHQTp5k) and the NSTX-U experiment at Princeton Plasma Physics Laboratory. Concerning the latter: TU/e student Peter Rindt made the design for a liquid lithium divertor module, to be tested in NSTX-U (see Figure 14.17. This design features a reservoir for the liquid lithium, from which it is sucked to the surface by capillary forces through narrow slits (wicks). The surface has a fine texturing to ensure fast and total wetting. So, this is one of the liquid lithium divertor concepts under test at the moment, but note that there are many other concepts, too.

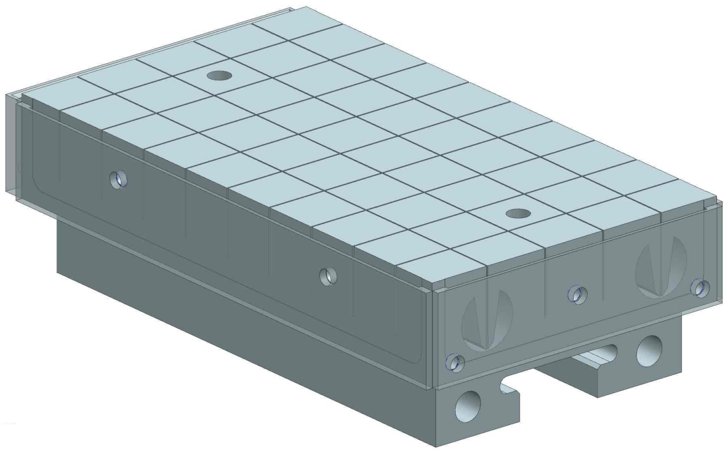

Figure 14.17: Design of a liquid lithium divertor component for the NSTX-U experiment in Princeton, made by TU/e student Peter Rindt. Note the reservoir for liquid lithium, the wicks (slits) and the textured surface that will be wetted by capillary action (source: Peter Rindt)

14.6 Magnet technology

As we have seen, the magnetic field is essential for confinement per se in the first place, and high performance in the second. The magnetic field coils of fusion experiments are among the largest and most powerful in the world, and typically cost about 30% of the total budget of the experiment! The older generation of tokamak experiments were equipped with (water-cooled) copper coils. However, there is a limit to their applicability: the dissipate a very large amount of power which implies that they heat up, and that no power supply can power them for very long. (***google question: what is the power of the toroidal field coils of JET, from which source are they powered and how long is the maximum pulse duration?***). The modern tokamaks and stellarators therefore have superconducting coils. (*** SC magnets don't have Ohmic dissipation, but that does not mean no power is needed to operate them. E.g. the ITER magnet system requires tens of MW to run. Why?***). We will not explain much about superconductivity here, but there are a few things you must know (and probably already know). First: the electrical resistivity of a superconductor is essentially zero. Not just very low, but zero. The disappearance of resistivity that occurs in particular materials when they are cooled to very low temperature is a quantum-mechanical effect that was only understood many decades after Kamerlingh Onnes had discovered the phenomenon in 1911. Then, there are 3 critical , material specific, quantities associated with superconductivities: the critical Temperature (Tc), above which the phenomenon does not exist; the critical Magnetic Field - again, above which superconductivity does not exist; and the critical current density - which is not strictly a material specific property as it depends on the preparation of the material. But is is still a very important limit if you want to make a SC magnet. These limits are not independent: close to the critical temperature the maximum B-field that can be tolerated is small, and vice versa. So for reliable operation, the work point has to be chosen at some margin of the critical boundaries.

There are many superconducting materials, but the work horse for magnets such as those used in MRI-devices in hospitals, is NiTi (Niobium-Titanium). This has the nice property that it is ductile , so it can be used in manufacturing processes without it breaking. Some ITER magnets (the poloidal field coils) use NiTi, too, but the ones that produce the highest fields (the toroidal field coils and the central solenoid) are based on the brittle Niobium-3-Tin, as this material offers higher critical field. However, the manufacturing of coils from Ni3Sn is much more complex and involves a heat treatment after the cables have been formed. Figure 14.18 shows the main magnet systems of ITER.

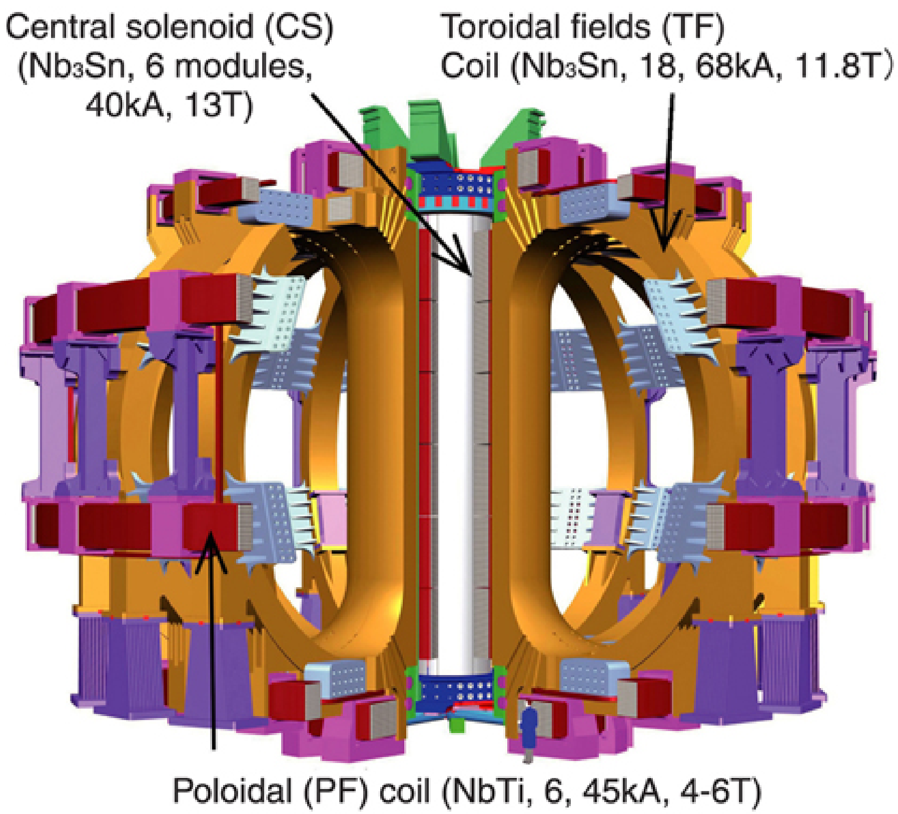

Figure 14.18: The ITER magnets are made of Niobium3Tin and NiobiumTitanium (source: ITER)

NiobiumTitatium and Niobium3Tin are classical superconductors, that operate at a temperature of about

4 K. This implies they must be cooled with liquid helium, which involves a complex and costly cryostat, heat shields, liquid helium leads and a power-intensive cryoplant where the energy collected by the helium is removed. For every Watt that needs to be removed at 4 K, some 400 Watts are needed in the cryoplant, so even small heat leaks or e.g. power deposition by a residual neutron flux lead to considerable power requirements in the cryoplant. The ITER cryoplant draws about 30 MW from the grid.

There is another class of superconductors that can operate at much higher temperature, even up to the temperature of liquid nitrogen. This is a big deal, as cooling at that temperature is two orders of magnitude cheaper (both money and power-wise) and the requirements on the cryostat are also much simpler. However, these - ceramic - materials are not yet ready for high field, high current-density applications. The developments are promising and the first commercial 'tapes' of high-Tc superconductors are being produced and also tested in small fusion experiments. So the next generation of fusion reactors may well be equipped with these materials, which would constitute a huge improvement. Not only do these materials exhibit a high T_c, they can also operate at higher magnetic field than the classical superconductors, and when cooled to about 20K (which is still a lot easier to cool than 4 K) they can also carry high current densities. Moreover, where classical superconductors need to be made from long length of uninterrupted cable (tens of kilometers!) it is possible to make joints in high-T_c superconductors. So, some innovative high-field tokamak concepts are based on the superior properties of the new high Tc superconductors, a hope-inspiring development.

There are a few more things I want you to realise in relation to superconducting magnets.

First, the elementary superconducting wire - a 'filament' - must be very thin, think 1 to 10 micron (the thickness of a hair). There are several reasons for this having to do with the stability of the current in the filament, but one obvious one is that current cannot easily penetrate in a superconductor (skin-effect) - so thick superconductors don't work. This means that a practical superconducting cable has a complex build-up, starting from strands that are combined and twisted to form wires that are again twisted and combined until finally a cable is obtained. Figure **??** shows the build-up of an ITER cable.

Second, if you look carefully at the picture (see Figure 14.20 you may notice that the cable contains a large fraction – 70% of so – of copper, rather than superconductor. Copper is indeed an essential component of any practical superconducting cable. There are two reasons for this. First, copper greatly increases the thermal stability of the superconductor. Imagine that for some reason a small amount of energy is deposited somewhere in a tiny volume element of a superconductor. This could e.g. be due to a small displacement of the conductor in its own magnetic field! Because the heat capacity of the conductor is extremely small (***why is that so?***) it will heat up and may exceed the critical temperature. If that happens, it suddenly has a very high resistivity and starts to dissipate power. If no measures are taken, this process grows exponentially will blow up the magnet. Copper effectively stabilises this process by a) providing thermal conduction to a much larger 'thermal bath' which can take up the heat without going up in temperature too much; and b) by providing a parallel electrical conduction path, so that the Ohmic dissipation, while finite, is still very small even when the temperature is above critical. So remember: without copper no superconducting magnets!

Third, despite the presence of copper, the runaway process describe above may still happen. This called a 'quench'. A quench cannot always be avoided, and therefore any superconducting magnet must be constructed in such a way that it can survive quenches. Without any special measures, a quench develops as a local hot spot where basically all of the magnetic energy of the magnet is dissipated. This literally blows up the magnet and causes unrecoverable damage: melting, deformation. To avoid this magnets are equipped with quench protection systems. In most cases, these detect the start of a quench in one place and as soon as this happens apply some heat to the rest of the magnet to bring it above Tc, so that the dissipation of the magnetic energy is evenly spread over the entire system. This will boil off all the helium in the magnet, which is costly, but

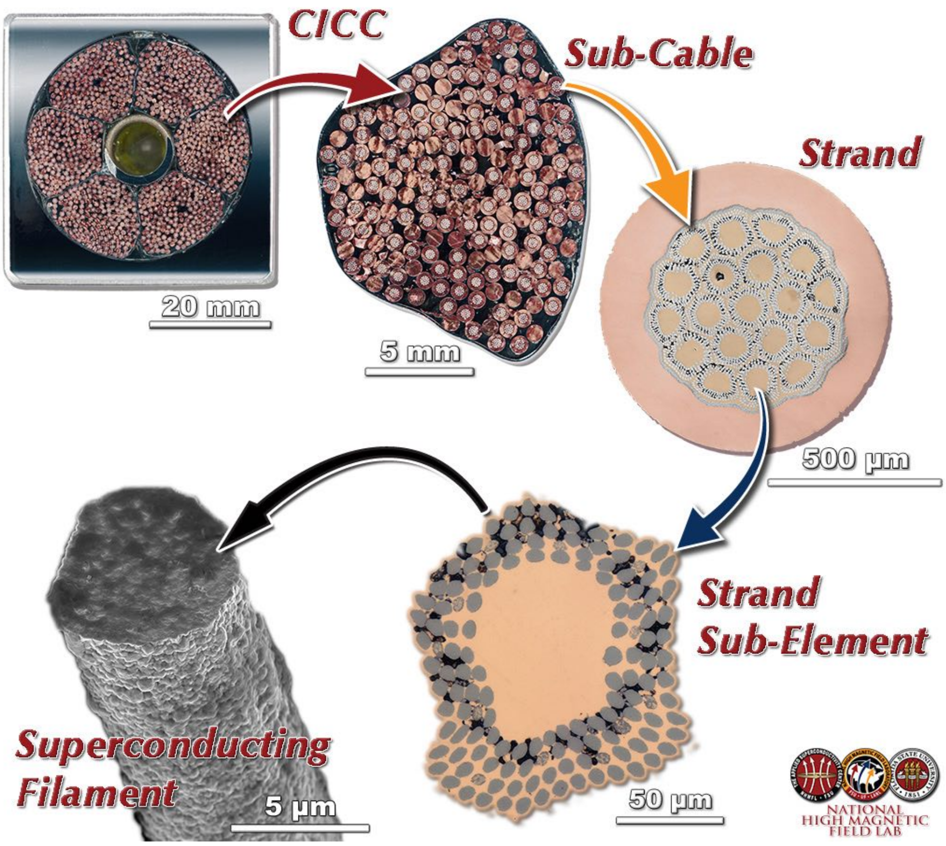

Figure 14.19: A superconducting cable from which an ITER magnet is wound consists of many stages, starting with the superconducting filament. (source: National High Magnetic Field Lab)

Figure 14.20: Cross-section of a superconducting cable for ITER, showing that a large fraction of the strands are in fact made of copper rather than superconductor (dark cross-section). The helium coolant flows in parallel direction between all the strands. (source: ITER)

does not do permanent damage. For the magnets of a fusion reactor, however, the recovery from such an event would take very long, because of the great mass that needs to be re-cooled. It would put the reactor out of operation for weeks or perhaps months, which is not acceptable. Therefore, machines like ITER have a quench protection system that dumps the magnetic energy of the coil in a (huge) external resistor. So, this process, too, starts with the detection of the onset of a quench, but then a fast switch is thrown that shunts the current to the external dump resistor, and the magnet remains cold. A good solution, but there is a price to pay: the voltages in the magnet will be quite high during a quench (***why?***), so the entire magnet must be implemented with high voltage insulation – yet another complication in an already demanding design.

Figure 14.21 shows the production of the skeleton of a toroidal field coil of ITER. This so-called 'radial plate' provides the strength of the magnet. The grooves will hold the superconducting cable. They have to be machined with great precision, because the cable will be subjected to large Lorentz forces, and must not move.

Figure 14.21: Preparation of the 'radial plate', the skeleton of the ITER toroidal field coil. (source: ITER)

Problems

14.1 The blanket

The plasma in a fusion reactor is surrounded by a blanket.

(a) The blanket has three distinct functions, each of them essential to the success of the fusion reactor. Which three?

(b) What is the typical thickness of the blanket?

(c) Comment on the role of the blanket in determining the minimum size of a fusion reactor

(d) The blanket in ITER does not fulfil all three functions. Which function(s) does it have?

(e) Which blanket function will be tested by the 'test blanket modules' in ITER?

(f) In a reactor, the blanket needs to be heavily cooled. Why?

(g) As coolant various options are being considered, including water, helium and liquid lead-lithium. Comment on the pros and cons of each of these.

(h) There is yet another possible coolant: FLiBe, a coolant also used in fission reactors. (Look up what it is). What is the advantage of FLiBe over lead-lithium? And what could be a drawback?

14.2 Materials under irradiation.

In a fusion reactor, most parts will be shielded from the neutron flux by the blanket, but the blanket itself, and especially the 'first wall' that separates the blanket from the plasma and vacuum, will be subjected to the neutron flux. Let's consider what happens to the material of the first wall.

(a) One of the elementary graphs that characterise a material is the engineering stress-strain curve. Sketch a typical stress-strain diagram and indicate in the graph: i) elastic modulus (YoungÕs modulus); ii) the yield stress; iii) the tensile strength; iii) the ductility.

(b) Now assume that the material is hardened by neutron irradiation. Draw in one stress-strain plot the curve for the same material before and after radiation hardening.

(c) Materials can best be used in a well-defined temperature window. Why is application of the material a problem at temperatures below this window? And why at temperatures above this window?

(d) In the Sharpy test, the energy E is measured that is needed to break the test specimen. Sketch the device used in the Sharpy test, indicate the parts, and explain the measurement procedure (you may want to google this).

(e) Sketch the typical curves of E as a function of the temperature of a steel test specimen before and after neutron irradiation. Identify the differences and comment on their importance for the application of materials in a fusion reactor.

14.3 Swelling

Metals swell under neutron irradiation.

(a) Why is swelling a problem?

(b) There are - at least - two fundamentally different causes of swelling. Which?

(c) The neutron energy spectrum from a fission reactor is different from that from a fusion reactor. What is the most prominent difference?

(d) How does that affect our ability to test materials for fusion reactors using fission reactors. Which is most different: the lattice damage (dpa) or the swelling? Why?

(e) It has been found that different lattice structures of otherwise similar materials show very different swelling. Which is more swelling-resistant: bcc or fcc?

14.4 Tritium breeding

Although a fusion reactor uses the deuterium-tritium fusion reaction, the fuel that is brought to the reactor is not tritium, which does not exist in nature, but lithium. So we need to turn lithium into tritium as part of the fuel cycle.

(a) What is the half-life of tritium? If you have a stock, which fraction do you lose per year?

(b) Give the reactions of a neutron with ^{6}Li and ^{7}Li, respectively. Which of the two is exothermic? Which has the highest cross-section?

(c) Pure lithium is not a very practical material to work with. Why not?

(d) Therefore, several other forms of lithium are being considered for application in the blanket. Give two examples.

(e) Clearly not all neutrons that are produced in a fusion reaction end up in the blanket. How is it possible to still have a net tritium breeding ratio larger than unity? Which materials can be used to achieve this?

(f) Give two reasons why the net tritium breeding ratio of the blanket has to be significantly larger than 1. What is - approximately - the value that is aimed for in blanket design?

(g) Explain how the burn-up fraction of the (burning) fusion reactor is related to the required breeding ratio. Would you rather have a high or low burn-up fraction, from the perspective of tritium management? And from the perspective of achieving burn?

14.5 Eurofer and ODS steel

Over the past decades a steel has been developed and tested that could be used for the first wall in fusion reactors. It is called EUROFER (as it was developed in Europe).

(a) What is the main element in the EUROFER alloy, after Iron? Which percentage, approximately?

(b) Which element that is abundant in normal stainless steel had to be removed from the alloy for EUROFER for application in fusion? And why?

(c) EUROFER has good radiation properties. Yet it has a severe limitation. What?

(d) An alteration of EUROFER has been proposed (and successfully tested) that remedies the above limitation: ODS. What is that?

(e) ODS steel turned out to have an additional benefit: it reduces swelling. How?

(f) So ODS steel has great promise. But there is no such thing as a free lunch. Mention two problems – of entirely different nature – of ODS steels.

14.6 IFMIF

There is a strong need to realise a test facility that could be used to test and validate materials for fusion reactors. This facility goes under the name 'International Fusion Materials Irradiation Facility' (IFMIF). IFMIF is in the conceptual phase. Some of the technology is being developed, but construction of the project hasn't started. In fact, it has not even been decided where it will be built.

(a) Why is it not sufficient to use fission reactors to test materials for fusion?

(b) Why can we not test the materials in large tokamaks like JET, when running them in d-t?

(c) Or yet another smart idea: use the fusor as a source of fusion neutrons. Cheap, simple, steady state, produces the correct fusion neutron spectrum ... so what's the snag?

(d) Explain the working of IFMIF: how does it produce the neutrons?

(e) How large is the test volume of IFMIF in which the highest neutron flux is achieved?

(f) IFMIF can only test small materials samples. To qualify materials or components for use in a fusion plant, a 'components test facility' would be needed. Can you propose a good candidate technology?

(last update Ch. 14: 161004)

15 The roadmap: ITER $\rightarrow$ DEMO $\rightarrow$ commercial power plants

15.1 introduction

Now that you have a basic knowledge and understanding of the open issues in fusion, as well as the relevant physics and technology, it is time to look ahead: which steps will take us to commercial fusion power and when may we expect to realise it?

The worldwide fusion development programme follows a somewhat organised path that is usually called a Roadmap. Note that unlike the roadmaps that exist for the deployment of e.g. photovoltaic energy, ours is a Research and Development roadmap, that leads to de demonstration power plant, commonly known as DEMO. However, you should realise that whereas ITER is a single global, joint effort, several parties are making their own DEMO plans. Which is a good thing, as it always better to explore different concepts in parallel than to put all your money on one horse. Countries with concrete DEMO-plans include S-Korea, China and Europe.

Europe - through its coordinating office EUROfusion - has explicated its roadmap in a document called European Research Roadmap to the Realisation of Fusion Energy. This document is available online at www.euro-fusion.org and in Canvas. It is summarised in Figure 15.1. It features basically two stepping stones, ITER and DEMO, while in parallel to that three development lines are pursued; i) development of reactor materials (in particular neutron-hard materials, for which the International Fusion Materials Irradiation Facility is planned); the stellarator as fusion plant (with W7X as the large experiment today — possibly to be followed up by a stellarator ITER-equivalent); and generic improvements and innovations that would allow more cost effective power plants.

Clearly, ITER is the big next step in all roadmaps. In ITER nearly all the open issues discussed in these lecture notes will be addressed and hopefully solved or at least taken a step forward. So, as a fusion student, you need to have a thorough knowledge of the targets and planning of ITER. What is its mission, what are the concrete goals in terms of physics, technology, materials, safety, etc? And how and when is it going to achieve these goals? We'll base this discussion on the official documents of the ITER project. Do keep in mind, always, that ITER is not an attempt at building a power plant. It is a scientific experiment and technology demonstrator. ITER will consume, not produce, electricity. It is not even equipped with generators that could turn the fusion power into electricity. Mid 2020 ITER officially started its assembly phase, which is really exciting. After decades of designing, negotiating, project management and high level governance, now the site is ready, the buildings are eager to receive the equipment, the big parts are being delivered and the Chinese puzzle can be put together. That will take a few years, but you will see progress from week to week! The planning is that first plasma will be achieved in 2025 - so those of you who have the ambition to do a PhD in fusion after obtaining the MSc diploma can go straight from the thesis defence to the ITER control room. Don't forget to subscribe to the ITER news line - and to check out the ITER site regularly.

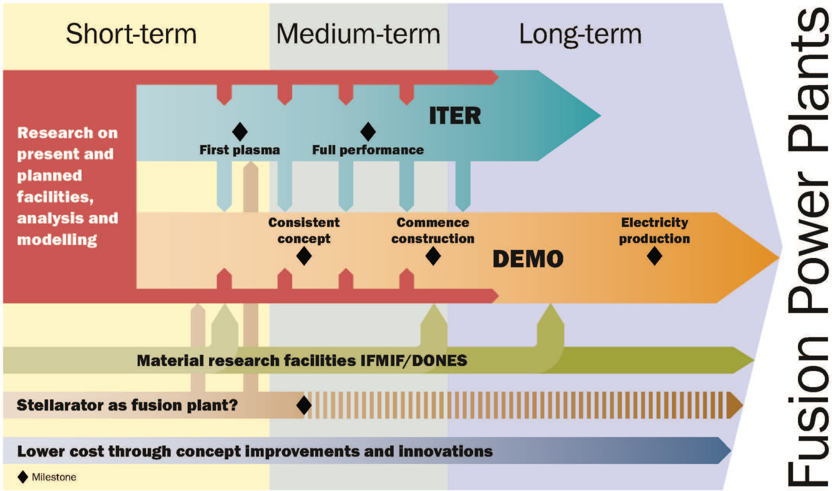

Figure 15.1: Schematic representation of the EUROfusion roadmap. Note that leads to, but does not include, the 'fusion power plants'. Also note the milestones, which show how the lessons learned in ITER feed into the design and construction of DEMO. This schematic does not give an explicit time scale, but full power (i.e. dt) operation of ITER is foreseen to start in the late 2030's, and construction + commissioning of DEMO will take 10-15 years. (source: EUROfusion)

At the same time that we are building ITER, we need to think seriously about the follow-up, DEMO. DEMO is meant to be a true demonstration power plant. It should produce electricity and demonstrate the proper functioning of all essential components. Thus, it should form the technology basis for the design and construction of the first 'commercial' power plants - commercial in quotation marks because the first series of a new product is often not a commercial success yet - it is the first step towards commercial deployment. Having said that, we should also stress that DEMO is not a commercial power plant. It will demonstrate all the required technology, but will not necessarily run 24/7 during the entire year, and it does not need to be productive for 40+ years. In fact, DEMO does not have to be productive at all. Selling electricity is not its goal, demonstrating the readiness of the technology is.
What would a full-blown, 10th of a kind, power-producing fusion power plant look like if we had to build it now, with present day technology or modest extrapolations of that? And what can we expect in terms of efficiency, cost of electricity if we assume that subsequent generations of power plants could avail of advanced technology, newly developed materials and improved plasma performance. Smaller, cheaper plants? The basic text we shall use here is the Power Plant Conceptual Study, carried out under the auspices of EFDA. And we briefly discuss alternative routes that are being explored.

Finally, assuming that the scientific and technological steps are made according to plan, the question remains how fast we can expect to deploy fusion power. Because, while it would already be a great achievement to realise a first generation of 10 fusion power plants, fusion only appears on the radar in terms of contribution to the world energy supply when the number of plants exceeds several 100, preferably 1000. There are recent papers on exactly this topic which we'll take as the basis for the discussion in this chapter.

15.2 ITER

15.2.1 a bit of history

When in 1985 president Reagan and General Secretary Gorbachev met for peace talks, in Geneva, one of the outcomes was an agreement to collaborate on a large nuclear fusion experiment (for peaceful purposes: power generation). This channeled the global effort in fusion into this one big experiment, which was to be ITER. The ongoing more local activities, such as the design for the European successor of JET, imaginatively called the Next European Torus (NET), were merged into the ITER project. Initially ITER was an acronym, for International Thermonuclear Experimental Reactor. But after a survey had shown that 3 of these words have a very bad connotation with the public (Thermonuclear in particular, but also Reactor (as in nuclear reactor) and Experimental (as in 'we don't know if it will work') ITER officially dropped that acronym and decided that ITER really means 'the Way' (in Latin), which makes sense if you think of a road map. While the agreement was initiated by the Russian and American presidents, Japan and Europe were from the start partners in the project.

Figure 15.2: In 1985 General Secretary Gorbachev and president Reagan met in Geneva in a superpower summit, where the ITER collaboration was initiated. (source: ITER)

ITER was originally designed with more ambitious design goals than the ITER we know today, but that earlier version, of which the design was ready in 1998, came out too expensive (about 10 Billion dollar) and a clear political signal was given that a new, smaller experiment had to be proposed for half the cost. The design report of that, smaller, 'ITER-FEAT' was approved by the ITER council in 2001. Then the question arose where it should be built, and how (and how much) the different parties were going to contribute. The negotiations started.

Note also the backdrop. In the 1990's the oil price fell dramatically, to pre-oil-crisis values. Investments in low carbon technologies all but stopped — that was true for renewables as well as clean fossil technology, and indeed, also the fusion budgets suffered. It was in this time that, in 1998,the USA actually pulled out of the ITER agreement. It was also a time of great change in Russia. So also there, ITER was not the highest priority for some time. Yet, that was also the time that scientists were become more and more vocal about climate change and the need to curb carbon emissions.

So three partners remained: Europe, Japan and Russia. And then, in 2003, China joined, not much later the USA rejoined, followed by S-Korea. India joined a few years later, in 2005. As the participating countries had vastly different economies, it was decided early on that each partner would contribute 'in-kind'. That is, to each component of ITER a value was assigned in 'ITER units of account', and in this way a fair distribution could be achieved. So, the ITER partners did not put the budget in one pot, under control of a director-general, but rather agreed to all deliver parts according to specifications. As many parties were interested in involving their domestic industry in these challenging, high-tech tasks, the 'procurement packages' were often spread over several partners. Several parties produce the superconducting cable for the magnets, and likewise the sectors of the vacuum vessel and the tiles for the heat shields are provided by more than one party. Figure 15.3 shows how the large procurement packages were shared. Note that this manner of working may seem inefficient, but it does result in the full commitment and involvement of all parties in the entire project. And this unique, global collaboration is a precious good and a truly inspiring aspect of the ITER project. As it has been put: 'ITER is a peace project, and for a peace project it is really cheap'.

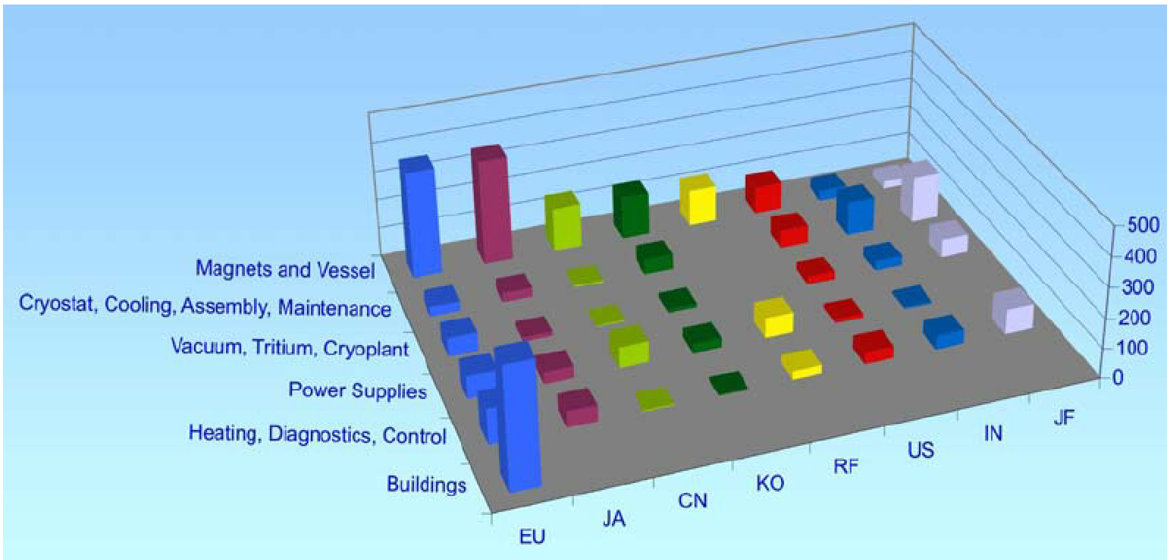

Figure 15.3: procurement packages (source: ITER)

And then there was the issue of the siting. Without going into details here, this was a long negotiation, where in the end Japan and Europe were the two contenders. It so happened that in the most difficult phase of the negotiations The Netherlands held the (rotating) chair of the European Union, so under auspices of the then minister of Culture, Education and Research Mrs van der Hoeven a deal was put together which saw ITER go to Cadarache (S-France) while Europe co-invested in a number of associated projects in Japan, most notably the satellite tokamak JT60-SA, the design of IFMIF and a computational centre for fusion studies. The whole package is known as the 'Broader Approach' and it is alive and kicking, having led to a very fruitful collaboration between Japan and Europe.

All said and done, the ITER agreement could be drawn up and it was signed in the Elysee, in Paris, in a ceremony that was hosted by the French President Mr. Chirac and the chairman of the European Commission, Mr. Barroso, on 21 November 2006. That was a great milestone! But of course, it was like deciding to put a man on the moon without having a NASA organisation to make it happen. The ITER organisation still had to be created, from scratch, starting with the nomination of the first director. And since ITER is put together from parts that are delivered by the parties, each party had to create an entity that could procure the components it had committed to. These organisations are called the 'Domestic Agencies'. The European Domestic Agency for ITER is 'Fusion for Energy' (F4E), and it has its offices in Barcelona. The domestic agencies work tightly together with the ITER team — and seeing that they often share the procurement of the same components — also with each other. So, ITER is a truly international, and intercultural, project.

Figure 15.4: The ITER agreement was signed on 21 November 2006 in the Elysee, Paris (source: ITER)

A very nice overview of all the critical moments in the history of ITER is actually presented on the ITER site, in the form of an animated time line: https://www.iter.org/proj/itermilestones.

A final bit of history. After the team had been formed, the first deed was to do a design review. After all, by the time the project started for real, the design was a decade old. And as long as the negotiations were ongoing, nothing could be changed! Also, during that period the cost of raw materials, most notably concrete and steel (which are both energy intensive) and the price of oil went up be factors of two to three. That, combined with the update of the design, which included some items that were not originally foreseen (such as ELM mitigation coils — science progressed there), led to a major shakeup of the budget. More political reconsiderations (fully justified!) and delays resulted. Gradually the milestone 'first plasma' shifted from 2016 to 2025. Just think of it: in the original planning, starting in 2006, ITER should have been purring for four years already! I include a schedule from that time so you can see for yourself. But you might say that it was naive to think that a project of the size of ITER, with the complex political make-up it has, could start start the day after the ITER agreement was signed. It is only natural that years were needed to set up the required organisations in all countries, starting with the ITER organisation itself. Today, the project appears to be in good shape, working with a lot of focus on the machine assembly in anticipation of first plasma in 2025. As said, you step into the ITER timeline at a very exciting time!

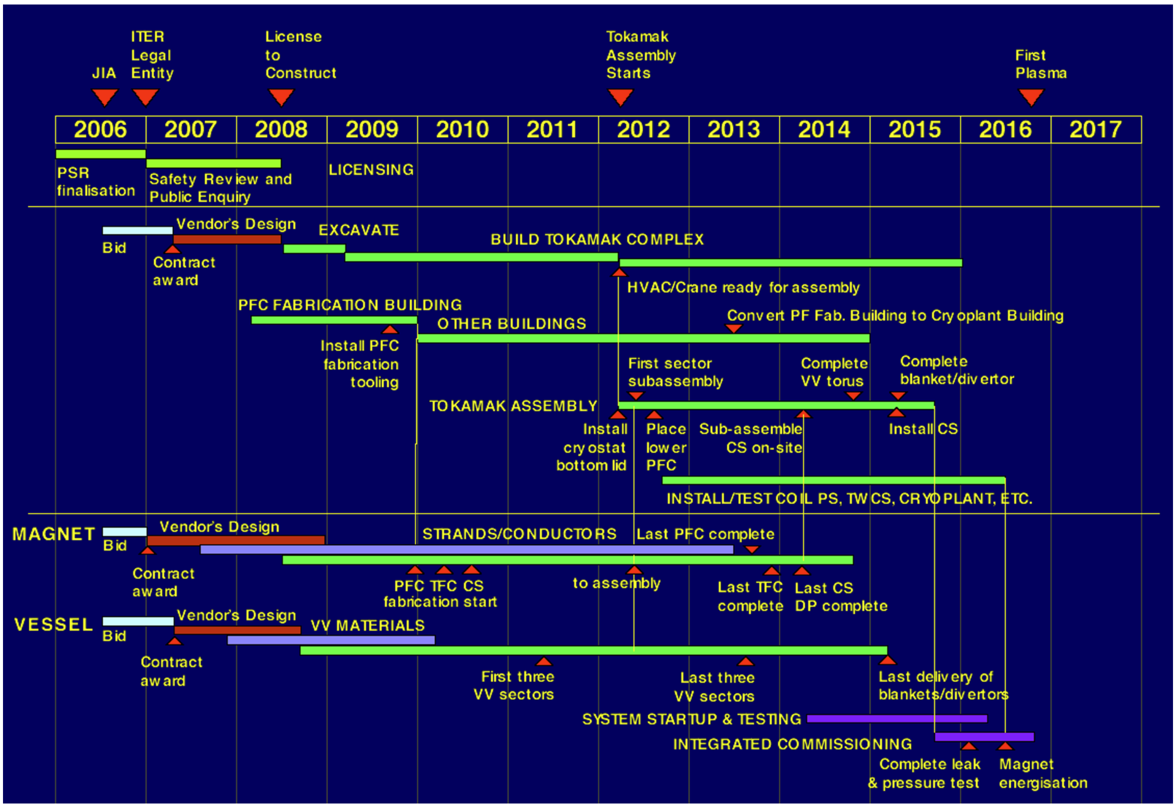

Figure 15.5: The ITER schedule as it was at the start of the project in 2006. Note, apart from the delays incurred in the initial phase of the project, the various major activity lines associated with the construction of the site and buildings and the manufacturing of the large components. In summer 2020 the Assembly phase started, first plasma is now scheduled for 2025. (source: ITER)

15.2.2 ITER in headlines

If I had only two minutes to talk about ITER, I would try to land the following messages: ITER is about twice JET in linear dimension (R= 6 vs 3 m) and has a significantly larger toroidal field (5.3 vs 3.5T). According to the scaling laws, that should allow ITER, when operated in a d-t mixture, to reach Q=10. That is: the fusion reactions are expected to generate 500 MW, ten times more than the power fed into the plasma for heating and current drive. Of those 500 MW, 100 MW is carried by the alpha-particles, which will transfer their energy to the bulk plasma. So, while the plasma does not reach full 'burn' (there is still some external heating power) the 'alpha-heating' dominates the power balance. And that is a big deal, really. It means that for the first time we'll have the opportunity to study a plasma that is governed by its own burn, rather than the external heating sources. I'll explain below why that is such an important step in terms of the physics.

It is important to realise that ITER has a dual mission. Partly — namely for the aspects where the extrapolation with respect to JET is modest — its mission is to confirm the design. Basically to show that yes, we can make a 15 keV plasma and keep it stably confined for ten minutes or more. Similarly we expect the technology, even if significant steps had to be made especially for the superconducting coils and the heating systems, to perform as foreseen. After all, these technologies were all extensively tested. But the more exciting mission of ITER is where the experiment will venture into uncharted waters. The plasma wall interaction — in terms of flux density but especially in terms of fluence — is orders of magnitude more intense than in any previous experiment. For the whole complex of ELM's — divertor technology — cooling concepts — ITER will be an invaluable laboratory. The neutron fluxes, too, will be orders of magnitude larger than in JET, the only experiment with d-t capability today. Which makes ITER a truly nuclear device, with all the consequences: licensing procedures, safety protocols, remote maintenance, ... At the same time, this neutron flux will allow Test Blanket Modules to be tested in ITER, which will be the first real-life experimental test of the viability of the fuel cycle.

15.2.3 New in ITER: burning plasma. The closing of internal feedback loops

Figure 15.6 shows the essential difference between a driven plasma (on the left) and a fully self-sustained burning plasma (on the right), with the ITER case — dominant alpha-heating but still supported by external heating, too — forms the transition between the two. In the driven plasma, the pressure, and thereby the pressure gradients (and temperature and density gradients) are the result of the balance between external input power and thermal losses. The latter are governed by turbulent transport, which itself is driven by the (pressure) gradient. Hence there is a feedback loop where the pressure drives turbulence whereas the turbulence limits the pressure. This defines a stable thermal equilibrium, of which the total pressure is given by the input power. In this case there is also a small population of energetic alpha particles — due to d-t fusion reactions, hence driven by the pressure — but their heating power is insignificant for the thermal equilibrium. That means that the operator has full control, he/she can vary the pressure at will given the available heating power and within the operational limits. When the plasma approaches the burn condition, say for $Q > 1$, a new feedback loop enters the dynamics. The alpha power, which itself is a product of the pressure, starts to be a factor of importance in the power balance. It is that in several ways. First and most obvious, the alpha heating increases the pressure. But the alpha particles — as a special example of a 'fast particle population' — can also have a direct effect on the turbulence, which depending on circumstances can be positive or negative. And in its turn, the turbulence can affect the confinement of the alpha particles. And if these are lost more rapidly from the plasma, they may not have time to pass on their energy with a reduction of the alpha heating power as result. In the case of full burn, in principle there is no longer external heating applied. That is good for the net efficiency of the reactor, but it does mean that the operator needs new tools, actuators, that can exert an influence on the plasma. Such tools could e.g. means to affect the turbulence (to be developed!) or the confinement of the alpha particles. But also the fuel mix (the relative amounts of tritium and deuterium) can be used, and possibly e.g. the helium content. But mostly, the active control of a burning plasma is an interesting challenge that lies ahead of us — always with the reassuring knowledge that the burn equilibrium cannot run away, of course. But some measure of control is still needed.

15.2.4 The parameters and objectives: excerpts from the official ITER documents

Figure **??** summarises the main parameters of ITER — dimensions, field and current, pulse duration and installed power — and below are the principal goals of the project. These are gleaned from the official ITER documents, and all of this can be found in more detail and glory on the ITER website. So this selection what I think you, as fusion master student, should know about ITER.

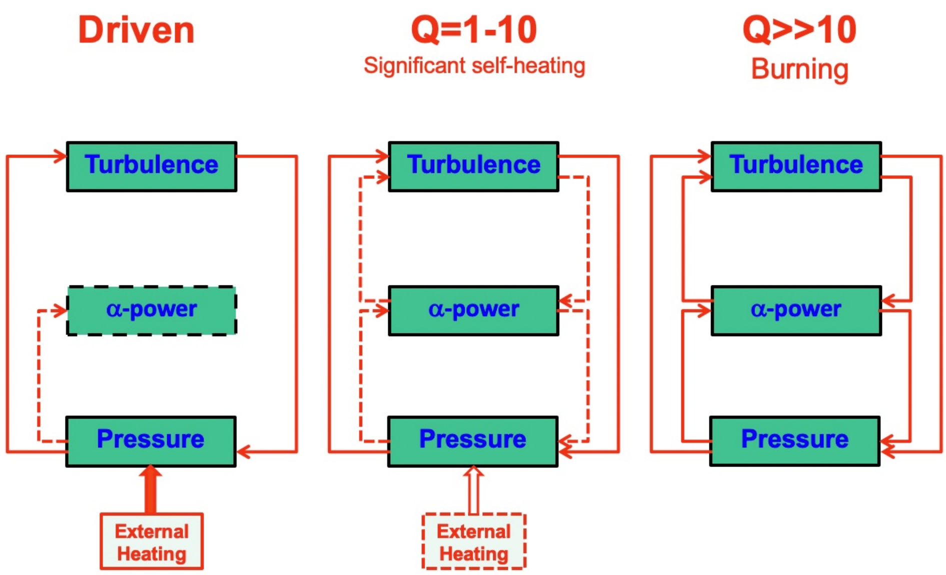

Figure 15.6: In the driven plasma (left) the pressure drives turbulence, which in turn limits the pressure. The operator is in control through the additional heating power. In a burning plasma, the alpha particles provide the heating. These depend on the pressure, can influence the turbulence and can also be influenced — i.e. be lost from the plasma — by turbulence. So this describes a complex set of feedback mechanisms, that leads the plasma to the stable burn. In the absence of external heating, the operator will need more subtle tools, e.g. to influence the turbulence or the alpha particle confinement, to exert any control.

total fusion power	500 MW
Q = fusion power/auxiliary heating power	≥10
plasma inductive burn time	≥ 300 s
plasma major radius	6.2 m
plasma minor radius	2.0 m
vertical elongation	1.7
plasma current	15 MA
toroidal field @ 6.2 m radius	5.3 T
safety factor	3.0
Installed auxiliary heating / current drive power	73 MW

Figure 15.7: The principal parameters of ITER. Note that the values of the safety factor (q) and the elongation refer to a flux surface just inside the separatrix, the so called 95% flux surface.

The box below summarises the objectives of the ITER project (summary version of the formal documents).

ITER objectives (abbreviated)

Programmatic objective

- Demonstrate the scientific and technological feasibility of fusion energy for peaceful purposes

Technical objectives

- Demonstrate extended burn of d-t plasmas,with steady state as the ultimate goal
- Integrate and test all essential fusion power reactor technologies and components)
- Demonstrate safety and environmental acceptability of fusion.

Performance and testing requirements

- Achieve Q $>$10 under stationary conditions on the timescales of plasma processes
- Aim at demonstrating steady-state operation with Q $>$ 5;
- Integrate the technologies essential for a fusion reactor (e.g. SC magnets, remote maintenance);
- Test components for a future reactor (e.g. divertor and torus vacuum pumps);
- Test tritium breeding module concepts for DEMO.

Design requirement

- The design permits advanced modes of plasma operation and a wide operating range
- Inductive flat top capability 300-500 s
- Operation up to a few 10,000 of pulses
- Able to support equilibria with high bootstrap fraction and plasma heating dominated by alpha particles.

The device is anticipated to operate for at least 20 years, *using externally supplied tritium.*

15.3 DEMO

To be written

15.4 Power plants: the Power Plant conceptual study

This subject is based on the report **A CONCEPTUAL STUDY OF COMMERCIAL FUSION POWER PLANTS**, downloadable from the EFDA-site: `www.efda.org` $\longrightarrow$ downloads $\longrightarrow$ EFDA reports and uploaded on OASE.

15.5 How fast can we introduce fusion energy (or any new energy technology)?

This section will be based on the paper 'Fusion Energy: expensive and taking forever?', which is uploaded on OASE.

15.6 epilogue

tbd

Problems

15.1 ITER

ITER is the Mondial Joint experiment in fusion, the essential next step towards a fusion power plant. A few things you ought to know about ITER.

(a) Which are the 7 parties that have signed the ITER agreement?

(b) What is the goal of ITER in terms of the 'power multiplication factor' Q and pulse duration? (give two distinct goals)

(c) When should these goals be achieved according to the present planning.

(d) What is the estimated cost of the experiment (the investment needed for the construction, not the running cost).

(e) ITER wil start by operating in hydrogen and/or helium, then plans to have campaign in which deuterium is used, to finally go to the deuterium-tritium mixture. Why this sequence?

(f) the mission of ITER is partly to confirm the predictions for confinement and performance. But in other areas ITER will enter entirely new territory. Give two areas where ITER explores uncharted waters and indicate why ITER is ground-breaking in these fields.

(g) the ITER parties contribute 'in-kind' to the project. Why did they not agree to simply creating a joint fund to build the reactor?

(h) during the ITER negotiations, many 'procurement packages' were split between several, in some cases all, partners. E.g. 2 segments of the vacuum vessel will be manufactured by S-Korea, the remaining 5 by Europe. Why, do you think, did the parties want to split such tasks?

15.2 Conceptual Power Plant Study

The Power Plant Conceptual Study considers 4 models (A-D)of increasing advancedness. Let us investigate what these advances are. You want to use Table 1 from the PPCS report at hand as a reference.

(a) Which is the largest model in terms of major radius: A or D?

(b) What is the trend in terms of net electric output power, going from model A to D

(c) What are the most striking aspects that lead to improvements in plasma performance (confinement) going from model A to D?

(d) What are the most striking improvements in the technology that lie at the basis of the improved plant performance?

(e) Comment on the relative amounts of power that are generated in the divertor and the blanket.

(f) Comparing the net electric output to the fusion power, factoring in efficiency of the electricity generation (Carnot sets an upper limit here) and the power used by the current drive system, it appears that the numbers don't add up. Which source of energy is implicit in these numbers?

(g) Why do these reactors need 'current drive'? This is important, since the current drive system is a major power consumer.

(h) For model A (near term technology) the power efficiency of the current drive system is supposed to be 60%. In present day experiments, the best current drive systems are the Neutral Beam injectors. What is the power efficiency of today's NBI systems?

(i) The power needed for the current drive system makes up a very large part of the 'recirculating power', i.e. the power needed to run the reactor itself. Why is this power so much lower in model D than model A?

(j) Safety is an important point in the PPCS study. Of particular concern in fission reactors is the 'residual decay heat'. Why is this a concern (under what circumstances) and what is the situation in a fusion reactor.

(k) In the formula for the Cost of Electricity, no performance parameter has a particularly strong influence (the power 0.4 being the strongest dependency). Even more striking, the influence of the obvious parameters Availability and Thermodynamic Efficiency (of the conversion cycle) also is around the square root dependence. How is that possible?

(last update Ch. 15: 201006)

16 All problems and suggested solutions

Problems Chapter 2: The energy problem and fusion

2.1 Fusion in the energy mix — the big picture.

(a) The energy demand will grow in the 21st century. Take as starting point that in 2000 the world population was 6 Billion, and that the 2 Billion in the rich countries made up 75% of the total energy demand. Calculate the factor by which the energy demand will increase by the end of this century, making reasonable assumptions on the increase of the population and energy use in the developing countries, and assuming that population and energy demand are stable in the rich countries.

Answer: Define: N(t)=population, P_C= energy use per capita. Starting point: $N_{rich}(2000) = 2B, N_{rest}(2000) = 4B$. $P_{R,rest} = P_{C,rich}/6$ Total demand $P_{tot} = N_{rich}P_{C,rich} + N_{rest}P_{C,rest}$; $P_{tot}(2000) = N_{rich}(2000)P_{C,rich}(2000) \times 4/3$ Assume (for instance) for 2100: Development of population: $N_{rich}(2100) = N_{rich}(2000) = 2B$. $N_{rest}(2100) = 8B$. Development of energy use: $P_{C,rest}(2100) = P_{C,rich}(2100) = P_{C,rich}(2000)$. (i.e. everybody reaches a level of prosperity/energy use comparable to the present rich countries). Then: $P_{tot}(2100) = 5N_{rich}(2000)P_{C,rich}(2000) = P_{tot}(2000) \times 3.7$ Upshot: Depending on the assumptions, the energy demand will increase by a factor of typically 3-4, mainly due to increasing population and standard of living in developing world. No need for complex modelling to arrive at a robust estimate!

(b) It is often said that there is plenty of coal — at least for 200 years at the present rate of consumption. Now, suppose we run out of oil and gas in the coming decades, and that we have to supply all the power calculated under a) by coal, clearly the 200 years reserve will last much shorter. Estimate how much shorter (or: what is the expected time the coal reserve will last) and make your estimates clear.

Answer: Coal presently accounts for 20% of the energy demand. Energy demand increases by factor (nearly) 4. So if we save all the coal while we use the oil and gas, and then must switch to 100% coal, the 200 year reserve is in fact a factor of 5×4=20 shorter, i.e. 10 years. That is very short! So keep in mind that oil and gas presently provide some 60% of energy, hence to replace them against a growing demand is a big challenge!

(c) Suppose that the rich countries manage to reduce the CO_2 emission to zero by 2100, and that the rest of the world manage a 50% reduction of CO_2-emission (per watt). What would that mean (approximately) for the total CO_2-emission by 2100, compared to now. Is that good enough?

Answer: In the estimation of a): $P_{rest}(2100) = P_{C,rich}(2000) \times 8B = P_{rich}(2000) \times 4 = P_{tot}(2000) \times 3$. Present emission: $E(2000) = E_W \times P_{tot}(2000)$ (E_W = emission per watt). By 2100: $E(2100) = 0 \times P_{C,rich}(2100) + 0.5 \times E_W \times P_{rest}(2100) = 1.5 \times E(2000)$. So even with these rather extreme assumptions the total world emission will still increase to 1.5 times the present level. To stabilise the CO_2 concentration at 550 ppm, the emission must be reduced to much less than the present level. So no, this reduction is by far not enough.

(d) What percentage of the energy generation is based on fossil fuel at present? What percentage of the energy consumption is in the form of electricity? What percentage of the energy demand is due to transportation?

Answer: Fossil 80%, electricity 18%, transportation 30-40%. Developing countries move up in electricity and transportation. Transportation as we know it needs 'fuels', while most new energy sources make electricity. So the energy transition is not only about clean ways to generate Joules, but also about changing over to a different infrastructure.

(e) Estimate how many person-hours of physical labour correspond —energy-wise— to 1 barrel of oil.

Answer: 20 thousand. A human can produce 75-100 Watt useful power during a few hours. One hour corresponds to 75-100 $\times$ 3600s = 270 - 360 kJ. A barrel (160 l) of oil (standard measure of energy content) is equivalent to about 6 GJ. So 20 thousand human labour hours to the barrel of oil. So, since a barrel of oil costs about 100$, and a person-hour (unschooled labour) 10$, it is no wonder why people are so keen on using oil.

(f) Estimate how much energy (express in litres of oil) is needed to produce 1 litre of milk and bring it to your table. (Borrowed from 'Energie Survival Gids')

Answer: Make a breakdown of the energy input: feeding the grass, feeding the cow (other than grass), heating the stable, milking the cow, cooling the milk, transporting the cooled milk from farm to factory, processing (heating, cooling) the milk, packaging, cooled transportation to supermarket, cooling in supermarket, transportation in private car to home. We get about 0.3 liter oil per liter milk.

(g) Domestic electricity use is about 7% of total energy consumption. Compare the statements: 'this wind park will power 1/3rd of all Dutch households' to 'a new nuclear power plant will reduce CO_2 emissions of the Netherlands by only 2%'. (Adapted from 'Sustainable energy without the hot air')

Answer: The two statements are equivalent in 'energy content' but clearly the emotional message is very different. Upshot: translate information in newspapers to your own standard to see what they are worth. (Apart from that: newspaper articles on energy are wrong most of the time. They mix up energy and electricity, and effective and nominal power all the time.)

(h) Calculate energy content of:

i. 1 litre of beer; 1 kg of oil; 1 kg of liquid hydrogen; (when burned)
ii. 1 kg of Li-ion battery (charged).
iii. 1 kg of uranium (in fission reaction, depending on process);
iv. 1 kg of D-T mixture (when fused).

(i) Calculate the energy in the oceans based on the d-d reaction. How many years of energy for humankind, if all energy consumption were to depend on it?

Answer: Sea: 70% of earth area, estimate the average depth (few km). Or google the mass of the oceans. Deuterium content: 0.03% of the mass of hydrogen. Hydrogen mass in H_2O: 2 out of 18. Energy from d-d fusion enough for 50 Billion years of total present energy demand, or some 5 Billion years if you factor in efficiencies and growing energy demand. In any case: longer than the sun will burn.

(j) Compare initial investment cost for 1 GW (year average output power) installation: wind, PV, solar thermal, fission, fusion. Compute the bottom price of 1 kWh, by assuming a life time and interest rate. Compare this to the price of 1 kWh from the grid (separate the kWh, the network cost and the taxes). And from an AAA-battery.

Answer: One example: fission plant investment $\approx$ 5 Euro/W. 1 GW plant: 5 Billion Euro. Annual interest at 5% interest rate: 250 Million Euro. Depreciation in 50 years: add 2% Hence the total cost of financing is some 350 Million Euro/year. Output: 1 GW = 10^6 kW, during 10^4 hours per year. Hence 10^{10} kWh. Hence the cost of financing is about $350 10^6 Euro/10^{10}$ kWh = 3.5 cents per kWh. So that is a very significant part of the energy price: the consumer price is some 25 cents/kWh, but most of that is tax and network charges. The cost of a kWh from a coal or gas plant is only a few cents. Oh, and an AAA battery contains about 1.5 Ah at 1.5 V, or about 2 10^{-3} kWh, and may be purchased for 20 cents or so, giving 100 Euro/kWh. So electricity from a battery is some 1000 times more expensive than from the mains.

2.2 Fusion in the energy mix – limits to growth

A new energy source can be said to be *available* when its total installed effective power is 10–100 MW. The total energy demand of the world is estimated to be some 20 TW in 2020. Assume for the moment that a new source goes through an exponential growth until it reaches 10% of its final, potential installed power. As the latter is typically 10–20% of the world demand, this 'materiality' point (as it is called by GJ Kramer and M Haig (Nature, 2009)) corresponds to 1–2% of the world demand.

(a) The exponential growth of new energy sources typically has a doubling time of 2–3 years. How long does it take to go from 'availability' to 'materiality'.

(b) The exponential growth is dominated by investment costs, which for many different technologies are found to be around 5 Euro per Watt of installed power by the time 'materiality' is reached. Estimate the annual investment at the end of the exponential growth. Compare to the world energy cost of approximately 10 thousand billion (!) Euro/year.

(c) The fusion roadmaps of different ITER parties foresee the operation of 3 DEMO plants by 2060, each with a fusion power of 5 GW. Taking that as your calibration and assuming the same doubling time as above, when will fusion reach 2% of the world energy demand (materiality), i.e. when can fusion enter the energy mix?

(d) One difference between fusion and some other technologies is that fusion appears to have more fundamental technological challenges. Give 2 major technological issues that need to be resolved before fusion can become commercial.

(e) After the exponential growth phase the development of a new source is expected to follow a linear growth.

(f) What could be a reason why the growth stops being exponential and becomes linear?

(g) How long does the linear growth take?

(h) Which two factors determine the growth rate of that linear phase?

Problems Chapter 3: Let's build a fusion reactor

3.1 Lawson criterion and basics of confinement

(a) Derive the Lawson criterion. (only the proportionalities, no constants asked in the derivation). Comment on the role of the efficiency of the conversion of fusion power to electricity.

Answer: see lecture notes

(b) The constant in the Lawson criterion has the dimension [Bar.s]. What is the typical value for the pressure (nT) and τ_E aimed for in a tokamak reactor like ITER.

Answer: A few bar and a few seconds

(c) The 'magnetic pressure' is expressed by $p_{\mathrm{magnetic}} = B^2/2\mu_0$, where B is the magnetic field in Tesla and $\mu_0 = 4\pi 10^{-7} NA^{-2}$ is the magnetic permeability in vacuum, giving p in Nm^{-2}. How large is the magnetic pressure of a 5 Tesla magnetic field?

Answer: about 100 Bar or 10^7 Nm^{-2}

(d) Which structure of the tokamak reactor contains this pressure?

Answer: The coils (Lorentz force due to crossed self-induced magnetic field and the current in the coil). High field coils must be strong structures, or they'll explode under the force of their own magnetic field.

(e) Estimate the pressure in the vessel of a fusion reactor before we even start filling it with the deuterium and tritium gas out of which the plasma is formed. Give a value (order of magnitude) + a reasoning. (yes, you can work this out with very little prior knowledge)

Answer: When operational the pressure is about 1 Bar, when the temperature is some 150 Million K. So when filled, at 300 K, the pressure of the working gas is $2.10^{-6} \times 1$ Bar $= 0.2$ Pascal. You may want to be precise and factor in the fact that we will het vessel with H_2 molecules - so then there is an extra factor 2 and the filling pressure is 0.1 Pa. The vacuum, before filling the vessel with the working gas, must be some 4 orders of magnitude lower, otherwise you get too much rest gas (=impurities) in your plasma: 10^{-5} Pa

(f) Calculate the central pressure in a fusion reactor of the size of ITER for central $T = 10 keV$ and $n = 10^{20} m^{-3}$. Then estimate energy content of the plasma in a fusion reactor for the above conditions. Compare to: a hot bath or a Mars bar (look up the energy content on the wrapper). Make some reasonable assumptions on the dimensions of the reactor.

Answer: Pressure (per species) $= nkT = 10^{24} eV/m^3 \approx 1.6 \times 10^5 J/m^3$. Estimate the total volume: Major radius R≈6 m, average (accounting for elongation) minor radius a≈3 m, volume = 1000 m^3. (the actual number for ITER is 840 m^3). Multiply by pressure (remember that pressure = energy density, and that we have to count both the electrons and the ions), and apply a form factor (pressure is lower at the edge) of e.g. 0.3, to find: $E_{tot} \approx 10^8$ J.

This is equivalent to precisely 100 Mars bars.

Or, the energy needed to heat a 100 liter bath from 10 C to 40 C: 100(liter) $\times 10^3$ (gram/liter)$\times$ (4.2 J/gram.Kelvin) $\times$ 30 (Kelvin) $\approx 10^7$ J. Enough energy for 10 baths.

(g) Calculate the energy confinement time for ITER, if it is operated with 70 MW input power and has plasma conditions as above.

Answer: As above, the stored energy is 10^8 J, divide by the input power to find $\tau_E \approx 1.4$ seconds.

(h) As we shall see later, there is a limit to the electron density in fusion reactor. Suppose (this is very reasonable) that a reactor runs close to that limit. Compare the generated fusion power for a plasma with pure d-t mixture, and one with 3% carbon impurity, in otherwise identical conditions.

Answer: Take the electron density in both cases the same, namely equal to the density limit:

n_e. The fusion power is proportional to $n_d \times n_t$.
Take $n_d = n_t$ for maximal output, hence $P_{fusion} = Const \times n_d^2$. Carbon is fully stripped in the plasma, so delivers 6 electrons. Pure d-t: $n_e = n_d + n_t$. $P_{fusion} = Const \times (0.5n_e)^2 = 0.25 Const \times n_e^2$
Carbon impurity: $n_e = n_C \times 6 + 2 \times n_d = 0.03 \times n_e \times 6 + 2 \times n_d$.
Hence $n_d = 0.41 n_e$ and $P_{fusion} = 0.17 Const \times {n_e}^2$ So, even a 3% carbon contamination reduces the fusion power by almost 40%!

(i) Further to this: the reaction produces helium, so it is unavoidable that the ideal d-t fuel mixture will be diluted by helium. How much reduction of the fusion power would result from a 10% helium content in the plasma? What does this mean for the burn-up fraction of the exhaust gas? And what are the consequences for the fuel cycle?
Answer: By the same calculation method, you'll find that 10% helium in the plasma corresponds to n_d=n_t=0.4 n_e assuming that there are no other impurities, resulting in a reduction of the fusion power by 36%. Not even considering the fact that if there was burn equilibrium without the helium, the gradual build up of the 'ash' will – due to the reduction of the fusion power – lead to a lower temperature and a further decrease of the fusion power.
The consequence is that a fusion reactor must be operated in such a way that the burn-up fraction is small, a few percent maximum. And that implies that a fuel atom must cycle some 30 to 50 through the reactor, the pumps, the gas refinery, the cryo system and finally the pellet injector before it takes part in a fusion reaction.

3.2 Historical context: the reactions that power the sun

The sun does not in fact use the deuterium-tritium reaction that we envisage for a fusion reactor, but a rather complex series of reactions known as the 'proton-proton cycle'.
Answer: I suggest you google this. Wikipedia has good coverage of this bit of science history

(a) Write down the 5 steps of the proton-proton cycle, or draw a diagram
(b) The names of Arthur Eddington and Hans Bethe are associated with the discovery of fusion as the energy source of the stars. What were their contributions
(c) Before fusion was proposed as the energy source of the stars, what could people possibly have thought of as the stellar power source?
Answer: here you want to look for the position of Lord Kelvin in this matter
(d) Lord Kelvin, in particular, put forward hypotheses concerning the power source of the sun. It brought him into a long and bitter conflict with Charles Darwin. What was the conflict about?
(e) Yet another famous name that is often associated with the energy released in fusion reactions: Albert Einstein. His famous formula $E = mc^2$ would explain how the mass difference before and after the reaction is converted into energy. Fair enough, but how about a chemical reaction, such as the oxidation of methane? Is there a mass deficit, too? In other words, is the mass of the reactants larger than that of the reaction products in an exothermic reaction?
Answer: Yes, of course. The laws of physics also apply to chemistry! You can actually measure the energy levels of an ion by very accurately measuring its mass (this is done by making it gyrate in a strong magnetic field and measure the change in cyclotron frequency upon exciting it with a laser pulse).

3.3 A fusion reactor fuelled by Helium-3 from the moon?

Refer to the graph of the reaction rate as function of the plasma temperature to answer the following questions (and if you are not familiar with reaction cross-sections, make sure you read up on this concept!):

(a) Express the fusion power density in terms of the reaction rate $<\sigma v>$ and the density of the reacting particles n (assume the reactants have the same density).

Answer: The fusion power density $P_{fusion} \propto <\sigma v> n^2 E_{fusion}$, where E_{fusion} denotes the energy released in a single fusion reaction

(b) At which burn temperature is P_{fusion} maximal, for the d-t reaction, for given pressure?

Answer: For given pressure n scales inversely with temperature, hence $P_{fusion} \propto <\sigma v> T^{-2} E_{fusion}$. So, in the plot of $<\sigma v>$ versus T you look for the spot where the slope of the curve equals 2 (decades per decade), i.e. where $<\sigma v>$ is proportional to T^2. For T higher than that, P_{fusion} is a decreasing function of T. For the d-t reaction P_{fusion} is maximal at T $\approx$ 15 keV.

(c) Now we use the same reactor to burn ^{3}He: d + ^{3}He $\rightarrow ^4$He + proton + 18.4 MeV. At which temperature is P_{fusion} maximal now?

Answer: By the same reasoning: $P_{fusion,3He}$ is maximal at T$\approx$ 50 keV

(d) What is the ratio of the maximum P_{fusion} for d-t and d-^{3}He (same reactor, same pressure, but each burning at its optimum temperature?)

Answer: Ok, this is somewhat tricky. There are 4 things to take into account. First, the energy per fusion reaction is approximately the same for both reactions (but this needn't have been the case, you had to check). Second: same pressure, temperature factor 3.3 higher means density factor 3.3 lower, hence reaction rate factor $(3.3)^2 \approx 10$ smaller. Third, at the optimum burn temperature, the value of $<\sigma v>$ is about a factor 4 lower for the ^{3}He reaction. Fourth: Helium has two electrons, so for the optimum fuel mix and sticking to the same density limit would imply a lower fuel density by a factor 2/3. However, we already assumed the burn was pressure limited and that led to a low density in the case of ^{3}He, which would therefore not be density limited. So we'll discard that (but had to give it some consideration). So, the power output of the same reactor is a factor 40 smaller when it burns ^3He rather than tritium and deuterium. The given in the question was hypothetical, because the ^{3}He reactor would definitely not reach ignition.

(e) What do you tell the government: go get ^{3}He from the moon?

Answer: No

3.4 Alternative fusion concepts: Beam on target concepts

You can make fusion reactions by shooting a beam of deuterium ions on a tritium target. The problem is that the probability of an elastic scattering event is much larger than that of a fusion reaction. Scattering leads to heating of the target, energy that is mostly lost. The graph below shows the cross sections for scattering and fusion, for deuterium incident on a tritium target.

(a) Estimate fusion-power versus losses as function of beam energy.

Answer: Power lost in scattering event = incident energy (E_i). Power gained in fusion dt event: $E_d t$= 17.6 MeV. So the energy gain factor is $Q = (\sigma_{dt} E_{dt}/\sigma_s E_i)$. For $E_i >$ 0.5 MeV or so, the ratio (σ_{dt}/σ_s) becomes constant at about a factor 10, hence for $E_i > (17.6 MeV/10) \approx 1.8 MeV$ clearly the losses dominate. For low E_i that ratio becomes very large and the losses dominate, too. (Sketch Q(E_i)!). So in the range 0.5 MeV $< E_i <$ 1.8 MeV there appears to be the possibility of having more fusion power than losses through scattering.

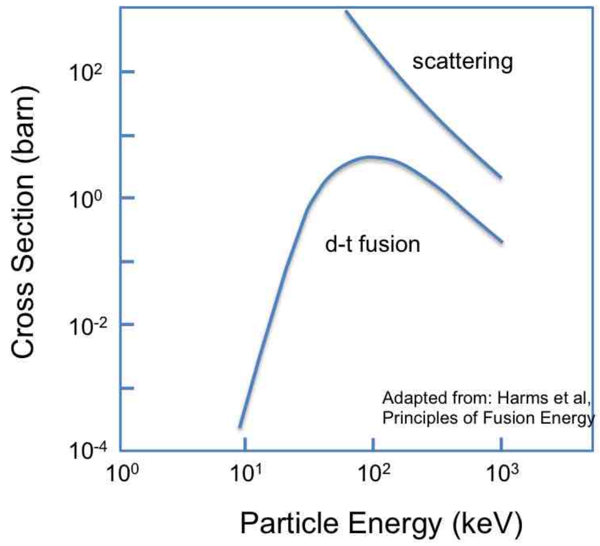

Figure 3.1: The cross section for the scattering of d on t is always much larger than that for fusion

(b) This estimate is still very optimistic. Why? (which other process(es) were neglected?)

Answer: However, the scattering with electrons (not in this picture) enhances the losses. And: the acceleration of the beam particles has a finite efficiency, as has the generation of electricity from the fusion-produced neutrons and heat. (on the plus side: the scattering-produced heat can also be used to generate electricity Ð but with a 40% efficiency or so at best).

(c) What does this mean for a viable fusion energy-generating scheme?

Answer: Conclusion: a beam-target scheme can never produce net energy generation. Therefore a form of confinement is necessary: it gives the fast d and t nuclei multiple chances to meet and fuse while keeping their energy in the system when they scatter without fusing.

3.5 Alternative fusion concepts: The fission-fusion hybrid scheme

In the so-called Fission-Fusion hybrid scheme, a neutron from a d-t fusion reaction is used to generate fuel (^{239}Pu) for a fission reactor.

(a) How much energy gain does this process yield? (this needs some googling)

Answer: 10 to 40

(b) Fusion makes many neutrons per MJ of produced energy (compared to fission). What would make the better business case: selling neutrons (e.g. to the owner of a fission reactor, or for the production of isotopes for medical purposes, or for material research) or selling energy? (in your considerations, take into account — apart from an obvious estimate of the market price of neutrons and joules — the different demands on operation of the device.)

Answer: This is really a discussion question. Realise for instance, that for fuel generation by neutron irradiation steady state operation is not a requirement. Also ignition is not a requirement, you could have a driven system and still have an overall energy gain. That can make the reactor design a lot simpler

(c) So what do you conclude: does it make sense to have a commercial neutron production programme based on fusion, and let the energy application grow out of that?

Answer: There actually are commercial fusion-based neutron sources, based on the fusor design. These are simple and cheap, and sell neutrons for special applications such as isotope production or luggage inspection, where either only few neutrons are needed or the added value of the product is very large. Commercial use of the enormously intens neutron flux produced by a tokamak reactor has not yet been proposed. But entrepreneurs among you: feel free to construct a business case.

3.6 Alternative fusion concepts: Muon-catalysed fusion

The muon-catalysed fusion scheme fails because it costs so much energy (5 GeV or more) to produce a muon. But we can get muons for free from the sky!

(a) if we could use the cosmic muon flux that reaches the earth, how much power per m^2 could we get from that if we use the muons to catalyse d-t fusion? (Assume a realistic number of catalysed reactions per muon.)

(b) Compare this to the $power/m^2$ from a solar plant.

(c) Conclusion: is the muon flux that impacts on the earth a viable potential energy source?
Answer: No, the muon flux from the sky is way too small: about 100 muons m^{-2} s^{-1}. If each catalyses 100 dt reactions, this corresponds to a power density of about 3 10^{-8} Wm^{-2}.
That is extremely little, compare for instance to PV or wind.

3.7 Alternative fusion concepts: Inertial Confinement fusion

In inertial confinement fusion an energetic laser pulse is used to compress a small amount of fusion fuel. The Lawson criterion is satisfied by the combination of a very high pressure and a very short confinement time. Check out on the internet (national ignition facility):

(a) what are typical numbers for the pressure and confinement time in an inertial fusion experiment? Compare to magnetic confinement fusion (MCF).
Answer: (These numbers you have to find on the internet or in a book, you are not asked to derive them from first principles). Pressure at ignition: about 10^{11} Bar, i.e. 100 times solid state density (MCF: 1-10 Bar). Temperature: about 10 keV (MCF: similar). Confinement time: the NIF laser pulse has a typical pulse duration of 1 ns, the confinement time of the fuel is a fraction of that. (MCF: seconds). Note that where in MCF the energy confinement time is determined by (diffusive) heat transfer, in ICF it is the time the compressed fuel needs to explode: the physics is completely different.

(b) how much energy does a single laser pulse of NIF carry (maximum)?
Answer: 1-2 MJ: a Mars bar

(c) What is the energy efficiency of the NIF-laser: how much energy into the laser building for 1 Joule of laser power at the exit?
Answer: The overall efficiency is only a fraction of a percent. For ICF to move forward this would have to become tens of percents. The developments in high power laser technology would already now allow NIF to build a much more compact and efficient laser with the same pulse energy

(d) What is the coupling efficiency of the laser energy to the pellet?
Answer: At least consider the fact that the implosion of the fuel is induced by blowing off the outer part of the pellet. On top of that, only a fraction of the laser energy that enters the Hohlraum is turned into photons that are absorbed by the pellet. A large fraction of the energy goes to the Hohlraum itself. Overall, the combined efficiency as a result of the 'burn fraction' (not all fuel burns up during the implosion and ignition), thermodynamic losses and losses due to the coupling of the laser energy to the pellet, is a few percent, typically.

(e) If ignition is achieved in a NIF inertial fusion experiment, how much fusion energy is released? Compare to a hand grenade (to get an impression of the sort of explosion we are dealing with). And to the energy in a Mars bar (any piece of chocolate).

Answer: At the end of the 'National Ignition Campaign' NIF came close to ignition. Yet the amount of fusion power released in such shots amounted only to tens of kJ. A sliver of chocolate. The aim was to achieve Q=1, which by definition would have meant about 1 MJ of fusion energy. Although you may argue about the definition here: is it the energy in the laser pulse, or the energy actually coupled to the pellet that should be used? As a reference: in MCF Q is computed using the power delivered to the plasma, not counting the losses between source and plasma.

(f) Explain how the 'hohlraum' is used to improve the homogeneity of the energy distribution over the surface of the target

Answer: The laser shines on the inside of the golden capsule and generates an intense and diffuse radiation field of X-rays, that in turn illuminate the pellet. However, it turned out in the experiment that even minute asymmetries in the Hohlraum, e.g. a tine hole use for diagnostic purposes, already gave rise to asymmetric, and therefore far less efficient, burn. So achieving symmetric implosion and burn remains a – probably the most fundamental – challenge in ICF.

3.8 Break-even, ignition and commercial operation

n a steady state fusion reactor, such as ITER, 'break-even' is achieved when the total fusion power generated in the reactor equals the power fed into the plasma in order to sustain it. The ratio (fusion power/external power) is called Q ('capital Q'). So for break-even Q=1.

(a) Explain why Q must be significant larger than 1 to achieve 'ignition'. (i.e. the plasma can sustain itself)

Answer: First of all: if we burn d-t, which is the only viable option, 80% of the power leaves the plasma with the neutrons. So at $Q = 5$, the fusion power that goes to the plasma via the alpha particles is only equal to the external heating power. If the plasma can truly sustain itself, i.e. no external power required, then Q goes to infinity, by definition. But remember that most likely there will always be some form of external heating (or current-drive, as we shall see) needed for control purposes.

(b) In ITER the target is to achieve Q=10. Explain why Q must be another factor larger in a commercial power plant. And if this extra factor is needed, why is ITER still a meaningful experiment?

Answer: Q is defined as a net ratio of fusion power to the power that is added to the plasma by external means, i.e. both powers are evaluated in the plasma volume itself. For a reactor to be commercial we need to consider the actual power used to produce that external heating power, which is a factor of 2-3 more. Moreover, there are other parts of the plant that consume power, such as the cryoplant that cools the magnets, or -ironically- the pumps that pump the cooling fluid around that removes the fusion power from the reactor in order to convert it to electricity. Finally, the fusion power is converted to electricity with a finite efficiency, too. So, to do those estimates for ITER, with $Q = 10$, 50 MW of net heating power for 500 MW of fusion power, the total electricity consumption of the plant would amount to some 200 MW, whereas if the 500 MW of fusion power were converted to electricity (which is not the case in ITER), perhaps 150 MW of electricity could be produced. Not enough to run the plant. Clear: Q must be much larger than 10 for a commercial plant. But ITER is not meant to be a commercial plant, it is a physics and technology development and demonstration project. And as such, it is absolutely essential, breaking new ground in a number of fields and thereby entering for the first time the realm of the 'burning plasma'. From physics and technology perspective, totally relevant to the reactor. But not yet commercial.

(c) In inertial fusion, because of the pulsed nature of the experiment, 'ignition' has a somewhat different meaning than in a quasi steady-state experiment. Comment on the relation between 'break even' and 'ignition' in the tokamak (steady state) versus inertial fusion. (also consider the efficiency of the laser)

Answer: In ICF a pellet can be brought to the point that significant burn-up occurs. This could be called 'ignition'. But the relation between teh released energy and the energy needed for the implosion is not so clear. In particular, the efficiency of the laser and the coupling of the laser power to the fuel need to be taken into account. Thus, NIF (correctly) claimed being close to ignition, while the fusion energy produced was only a few percent of the laser energy, which in turn had an efficiency of a fraction of a percent. The net energy multiplication factor would have been well below 0.01. Which, like in the case of ITER, does not mean that the experiment was meaningless, not at all.

Problems Chapter 4: Outstanding questions and challenges

4.1 Challenges

Before fusion can be rolled out as energy source several scientific and/or technical issues need to be resolved.

Answer: These questions may serve for you as a test, to see if you have absorbed and internalised the material in this chapter. You may need to use the internet, peruse the EFDA Roadmap, other documents that I have uploaded. No standardised 'correct' answers here, the value is in the study and discussion.

(a) List at least 4, essentially different, major issues that need to be resolved before we can have commercial fusion power

(b) Of each of these, explain why it is an issue.

(c) Of each problem, sketch the possible solution routes that are being considered and/or pursued (if there are any). And possibly your own solution, too.

(d) Discuss, for each issue, if it is generic for fusion devices or specific to e.g. the tokamak. Consider Stellarators, Inertial fusion and any other scheme you can find on the internet.

(e) Which of the issues must be addressed in ITER, which can be dealt with in other fusion devices, which don't need a fusion device such as ITER at all to be sorted out, and which need very specific, presently not existing facilities?

(f) Apart from the technical and scientific difficulties, there may be other issues that could slow down or hinder the development of fusion power. Mention at least 2.

(g) Of each of those, explain what the problem is, and a possible way to deal with it.

(h) Which, of this multitude of important and urgent problems, would you like to work on if you could do research in fusion?

4.2 Spin-off

Of the outstanding issues, some are not specific to fusion. Conversely, some of the solutions of issues encountered in fusion research may find application elsewhere, either as a scientific or technological advance or as commercial application.

Answer: Also for this assignment holds that the value is in the study and discussion, not in memorising the correct answer. The European commission and several countries have brought out brochures on spin-offs from fusion. You may want to check those.

(a) Discuss issues which the development of fusion shares with other fields.
Answer: Here you may think of: the superconducting magnets, high-T_c superconductors; neutron-hard materials; use of molten salts as high-temperature coolant; tungsten technology; intense plasma-surface interaction – erosion, deposition, plasma chemistry; application of liquid metals as plasma facing component; advanced control technology; advanced measurement systems; data handling and storage techniques; advanced remote handling technology; computational science and use of very powerful supercomputers, massively parallel computing etc; magnetohydrodynamics; management of extremely large and complex projects; management of international, highly political projects

(b) Discuss examples of scientific advance or technological development that was spurred by fusion but found application elsewhere in science or technology

(c) Discuss examples of commercial spin-outs of fusion research, and how they relate to fusion research and development.

Problems Chapter 5: Basics 1

5.1 Magnetic coils: Maxwell's laws in practice

(a) Give an expression for the voltage between the open ends (i.e. no current) of a single wire loop in a time varying magnetic field. And what is the voltage if instead of a single loop, the coil has N windings? (Don't look up expressions for this, derive the result starting from Maxwell's laws).
Answer: This is standard application of Maxwell's law that links the curl of E to the time derivative of B, and Stokes' theorem. In words: the surface integral of time derivative of B equals the line integral along the contour of that same surface of E, i.e. the voltage between the open ends of the wire loop. Not that the surface integral of B is also called the magnetic flux. N windings give N times higher voltage.

(b) How is such a loop used to measure the magnetic field? What is the complication? Why is the signal fed into an integrator? Would this integrator have a high or low impedance?
Answer: First: you don't measure the field but its time derivative. So you have to integrate the signal. Integrators always have a finite drift, so it is difficult to have an accurate determination of a field that is constant. With digital integration this is already a lot more accurate, but it remains a principal problem. The integrator should have a high impedance, you want to measure the voltage (the generated E-field along the wire), in the absence of a current. As to the coil: adding more windings (so that the lengths of the coil along the axis become larger than its diameter) will give the coil more directivity and sensitivity. So if you want to measure three components of the field, you use a set of three coils oriented in three orthogonal directions.

(c) Show that a long solenoid that itself forms a loop can be used to measure the enclosed current (this is called a Rogowski coil). Derive a formula by making use of Maxwell's laws and the theorems of

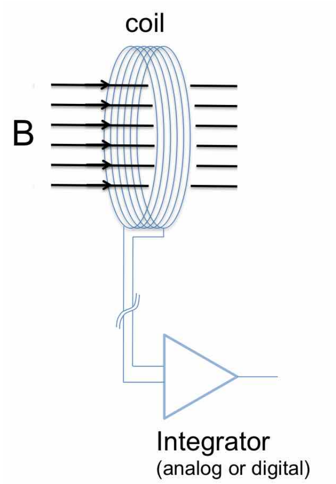

Figure 5.2: Magnetic loop

Gauss and/or Stokes.

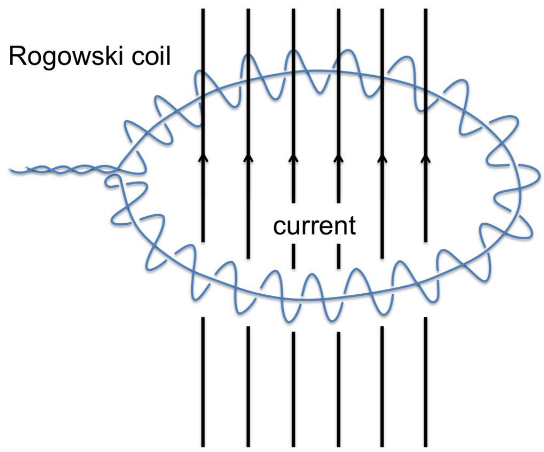

Figure 5.3: Rogowski coil

Answer: again straightforward application of Maxwell's laws and Stokes's theorem. This time we already know that a single loop of the solenoid measures the local time derivative of B. So for the Rogowski coil we reverse Stokes' theorem to find that the line integral along the loop of the time derivative of B equals the surface integral of the current density that intersects the surface enclosed by the loop.

(d) The Rogowski coil in the sketch is wound in such a way that the return wire follows the coil again (rather than going out the short way). Why is that?
Answer: otherwise the loop is also a loop in the plane of the paper and will measure a time varying field that is perpendicular to the paper, thus compromising the measurement of the field induced by the current.

(e) If you wanted to measure the total toroidal current in a tokamak, you could use such a Rogowski coil. Where would you place it? Why is this measurement insensitive for the position of the plasma? (this is what you want: you want the total plasma current, irrespective of its position or spatial distribution)

Answer: the coil is placed in the poloidal plane, i.e. encircling the plasma. Preferable inside the vacuum vessel (why?). As you have seen in the previous question, it measures the total current that crosses the plane defined by the loop

(f) If you also wanted to have information on the position of the plasma current (and thereby of the plasma), what could you do (still using magnetic coils).

Answer: If instead of one continuous Rogowski coil we would have a large number, say 2^m, of coils that together form a loop, we could chose to add all signals up and measure the total current. But clearly, if the plasma moved to one side, the coils on that side would pick up a larger signal and the ones on the opposite side a smaller one. So by using the differences of the signals of coils in opposite positions, i.e. top-bottom and in-out, the position of the current can be determined. Now this is an 'average' position, but the current density need not have a symmetric distribution over the cross-section of the plasma. So we can go on and try to determine higher moments of the current density distribution: elongation (and its orientation), triangularity ... All of these are in fact important in a fusion experiment. But you have to realise that the higher the moment of the magnetic field, the faster its field component falls off with distance. So: if your coils are at a large distance from the plasma you'll only be able to measure and localise the total current (this you can always measure), if you move closer you start to 'see' that the current channel is elongated, and upon moving closer you may see more and more details of the shape of the current channel. Later, you'll find that inside the plasma in a fusion reactor magnetic structures may form, so-called magnetic islands. So, with magnetic coils, these will be detectable if they live close to the edge of the plasma (close to your coils) and are large. But if they are located further to the centre of the plasma, further away from the coils, their magnetic field perturbations may be too small to measure.

So, Maxwell's laws and Stokes' Theorem are not just theory, they translate very directly into measuring devices and limits to our ability to figure out what goes on inside a fusion reactor.

5.2 Gyration and drifts.

We have seen how charged particles gyrate in a B-field, drift when the field has a gradient and/or curvature and how an electric field adds another drift effect. Now let us apply this in a physical situation. Consider a straight and infinitely long wire, that is surrounded by a plasma. The wire carries a current I, the magnetic field at a distance r from the wire is given by $B = \mu_0 I/(2\pi r)$. There are no currents in the plasma.

(a) Give the gyro frequency of the electrons at a distance of 10 cm of the wire, which carries a current of 1 MA.

Answer: Fill 1 MA and 0.1 m into the formula for B. One finds $B = 2\,\mathrm{T}$. Then apply $\omega_{ce} = eB/m_e$ or the practical formula $f_{ce} = 28$ GHz per tesla.

(b) Describe (a drawing will do) the orbit of an electron that only has a velocity component perpendicular to the magnetic field.

Answer: The orbit remains in the plane perpendicular to $\mathbf{B}$: gyration plus ∇B-drift opposite to the current direction

(c) Describe the orbit of an electron with velocity predominantly parallel to the magnetic field direction.

Answer: Orbit at constant distance R from the wire, consisting of: velocity v parallel to $\mathbf{B}$ (which requires centripetal force $m_e v^2/r$) plus a constant curvature drift opposite to the current direction (which provides the required Lorentz force).

(d) Now, answer b) and c) again if, in addition to carrying the current, the wire is electrically charged to a potential ϕ with respect to infinity. Assume that the electric force is small compared to the Lorentz force.

Answer: If ϕ is positive, the electric field points radially outward, and there's an additional $E \times B$ velocity in the current direction.

(e) Now, we replace the wire by a plasma column with radius a, for which we assume a uniform current density $j = I/\pi a^2$. (So this would be a 'Z-pinch'). Answer the questions b) and c) again, for an electron at a distance a from the wire. Consider the difference for an electron that is just 1 gyroradius inside a and one at exactly a.

Answer: Inside the plasma, at $r < a$, the field is generated by the current $I(r) = \pi r^2 j$ of the partial plasma column with radius r. Therefore $B \sim r^2/r = r$ inside the plasma so that there the ∇B-drift is opposite to the ∇B-drift outside the plasma, and opposite to the curvature drift (which does not change sign). Also the $E \times B$-drift remains unchanged.

An electron gyrating around a field line at $r = a$ spends half of its time inside the plasma and half of its time outside. The two contributions to the ∇B-drift very nearly cancel.

5.3 The magnetic mirror effect

In a magnetic mirror, particle trajectories are affected by the gradient of B parallel to $\mathbf{B}$. The constant of motion you want to consider in this case is the magnetic moment (μ), as well as the kinetic energy.

(a) The charged particle is subject to the Lorentz force, which accelerates it. Why is its kinetic energy conserved?

Answer: The Lorentz force $\mathbf{F}_{\text{Lorentz}} = q\mathbf{v} \times \mathbf{B}$ is always perpendicular to the velocity so $\mathbf{v} \cdot \mathbf{F}_{\text{Lorentz}} = 0$: this force can never do any work.

(b) Show that the magnetic moment is a constant of motion, by combining the equation of motion of a magnetic dipole in magnetic field with a parallel gradient, and the conservation of kinetic energy.

Answer: $\frac{d\mu}{dt} = \frac{d}{dt}\frac{mv_\perp^2}{2B} = \frac{1}{B}\frac{d}{dt}(\frac{1}{2}mv_\perp^2) - \frac{\mu}{B^2}\frac{dB}{dt}$. First term: apply kinetic energy conservation $\frac{d}{dt}(\frac{1}{2}mv_\perp^2) = -\frac{d}{dt}(\frac{1}{2}mv_\parallel^2) = -mv_\parallel\frac{dv_\parallel}{dt}$. Second term: use the fact that the magnetic field itself does not change in time. Only the field experienced by the particle changes as it moves through the gradient: $\frac{dB}{dt} = \mathbf{v} \cdot \nabla B = v_\parallel \nabla_\parallel B$. Using the formula for the force on a magnetic dipole, $m\frac{dv_\parallel}{dt} = -\mu\nabla_\parallel B$, one finds that both terms cancel.

(c) Consider a magnetic mirror in which the magnetic field is varied from B_{low} to B_{high}. Whether a particle is trapped or not depends on its pitch angle ($v_\perp/v_\parallel$) at $B = B_{\text{low}}$ and the mirror ratio $B_{\text{high}}/B_{\text{low}}$. Derive this dependence (using your result under b).

Answer: Using $\frac{1}{2}mv_\perp^2 = \mu B$ and setting the turning point $v_\parallel = 0$ at $B = B_{\text{high}}$, conservation of energy and magnetic moment yield $\frac{1}{2}mv_\parallel^2 + \mu B_{\text{low}} = \mu B_{\text{high}}$. Writing $\mu = \frac{1}{2}mv_\perp^2/B_{\text{low}}$ yields $v_\perp/v_\parallel = (B_{\text{high}}/B_{\text{low}} - 1)^{-1/2}$

(d) Calculate the fraction of trapped particles as function of the mirror ratio, assuming an isotropic velocity distribution.

Answer: The critical pitch angle is $\theta = \arctan(v_\perp/v_\parallel)$. The solid angle of the two halves of a cone with aperture 2θ equals $\Omega = 4\pi(1-\cos\theta)$. The fraction of trapped particles (the particles outside the cone) is therefore $f = 1 - \Omega/4\pi = \cos\theta = (1+\tan^2\theta)^{-1/2} = (1 - B_{\text{low}}/B_{\text{high}})^{1/2}$

(e) Calculate the mirror ratio of a toroidal magnetic field in a torus as function of the inverse aspect ratio $\epsilon =$ (minor radius/major radius).
Answer: Since $B(R) \propto \frac{1}{R}$ we have $B_{\text{low}}/B_{\text{high}} = \frac{B(R_0+r)}{B(R_0-r)} = \frac{R_0-r}{R_0+r} = \frac{1-\epsilon}{1+\epsilon}$.

The inverse aspect ratio ϵ is a 'small parameter': effects due to the curvature of the torus with respect to a cylinder can often be expressed as a correction on the cylindrical result with ϵ as the leading parameter.

(f) Which fraction of particles can in principle be trapped in a tokamak, as function of the inverse aspect ratio? Would you reckon this as a small correction on the cylindrical case, despite the fact that ϵ is much smaller than unity?
Answer: From (d,e) we derive $f = (1 - B_{\text{low}}/B_{\text{high}})^{1/2} = \sqrt{\frac{2\epsilon}{1+\epsilon}}$. This fraction decreases slowly with ϵ: $f \approx 0.43$ for $\epsilon = 0.1$.

(g) What do you think might be the effects of trapped particles in a tokamak?
Answer: Since the trapping also involves reversal of the toroidal velocity, there is a large fraction of particles that cannot contribute to a toroidal plasma current. Effectively, there is lower conductivity (proportional to the density of conducting electrons times collision time. Also, trapped particle orbits are quite sensitive to grad-B and curvature drift effects, and later it will be shown that this has severe consequences for heat conduction and phenomena such as the Ware pinch and the Bootstrap current.

Problems Chapter 6: Basics 2

6.1 Plasma physics: getting a feel for numbers

Much of this exercise is straightforward calculation. Its purpose is to give you a feeling for orders of magnitude of important quantities in a fusion plasma. It should help you develop a coherent picture, in which different effects have a place.

In the hot core of a fusion reactor the typical parameters are: $T_e = T_i = 10keV; n_e = 10^{20}m^{-3}$; $B = 5T$. Assume a pure hydrogen plasma (the ion charge $Z = 1$)

(a) Calculate the electron collision time and the ion collision time (use formulas below)
Answer: Back-of-the-envelope, approximate numbers: $\tau_e \approx 200\mu$ s; $\tau_i \approx 10$ ms

(b) Calculate the electron and ion mean free paths.
Answer: Thermal velocity electrons $\approx 3.10^7 m/s$, times τ_e gives the mean free path: $\sim 7km$. For the ions: the collision time is longer by the square root of the mass ratio, the velocity is lower by the same factor, hence: same mean free path. This means that in a toroidal confinement device with a typical circumference (in toroidal direction) of tens of meters, both types of particles can travel hundreds of times around the torus between collisions. Unless they are trapped in a banana orbit (see previous chapter), in that case they can bounce between the ends of the banana

hundreds of times. Also note that in the previous generation of smaller tokamaks, in which the temperature was a few rather than 10 keV, the mean free path was 1 to 2 orders of magnitude smaller - that makes an essential difference for the character of e.g. trapped particles.

(c) Calculate the ion and electron Larmor radii, and their gyro-frequencies.

Answer: Electrons: $\rho_{ce} \approx 0.04$ mm; $f_{ce} \approx 140$ GHz (this you can do by heart, with the practical formula 5.16); Ions (hydrogen): frequency smaller by the mass ratio (~ 2000 for estimations) : 70 MHz, Larmor radius larger by the square root of the mass ratio (larger mass, smaller velocity): $\rho_{ci} \approx$2 mm

(d) Compare the inverse gyro-frequency to the collision time for both species. What do you conclude for the 'magnetisation' of both species? Also compare the electron gyro-frequency to the plasma frequency.

Answer: We call a plasma species magnetised if the particles complete many (at least one!) gyrations between collisions: in that case the gyration is the dominant motion. For the electrons $\tau_e \; f_{ce} \approx 3.10^7$, so they are deeply magnetised, no doubt about it. For the ion the magnetisation parameter is smaller by the square root of the mass ratio (work this out), but that means that they, too, are completely magnetised. Of course, that was the point of magnetic confinement in the first place. Finally: it so happens that in a reactor-grade plasma the electron gyro-frequency and the plasma frequency are in the same range. There is nothing fundamental behind this, it is mostly a nuisance because it means the plasma frequency can be in the way when you want to apply heating at the electron gyro-frequency, or if you want to use the radiation for diagnostic purposes.

(e) Below also a formula is given for the ion-electron energy exchange time τ_{ex}. Give a useful definition of this quantity and give an expression for the energy density that is transferred from ions to electrons when their temperatures are not equal.

Answer: First, it important to realise that in a plasma the different species — electrons and ions — can have different temperatures. (In a TL-tube, the ions are even at room temperature, while the electrons are at some 10-thousand C). But since the ions and electrons are colliding, they are exchanging energy. So there will be an energy flow from the hotter to the colder species. The only thing to realise is that, due to the difference in mass, the energy transfer per collision is not very large — this is why the energy exchange time is a factor (mass ratio) larger than the electron collision time. So, the power source density (p) due to the temperature difference is given by $p = -n.(T_i - T_e)/\tau_{ex}$ for the ions, and the same with opposite sign for the electron population (n is the plasma density).

(f) The energy exchange time is a factor $(m_i/2m_e)$ longer than the electron collision time (τ_e is the collision time of electrons with ions). Why would this be so?

Answer: See above

(g) Calculate τ_{ex} for the fusion plasma and compare this to the energy confinement time of ITER. What does this mean for the difference of T_e and T_i in ITER?

Answer: The energy exchange time is an order of magnitude smaller than the energy confinement time. The result is that T_i and T_i will be almost equal in ITER: energy that is deposited in one population has two channels to flow into: to the other species and to the energy sink (the wall or divertor). The first option is faster (but gets less intensive when the temperatures are closer together). Note that where the confinement time is size-dependent, the energy exchange time is not. So, in smaller reactors it is easy to maintain a temperature difference between electrons and ions, whereas in ITER and larger machines (DEMO), the electron and ion temperatures cannot differ much. Wendelstein 7X is not a small machine, but one in which the plasma is heated exclusively with ECRH (pure electron heating). So here we see a discharge that starts with cold ions and hot electrons, and it takes a long time (order seconds) for the two to come together.

(h) Show – functional dependencies only – that the resistivity of a plasma does not depend on the density. Give a simple explanation, in words, not equations, why this is so.
Answer: In words: the mean free path, and thereby the velocity an electron reaches between two collisions, and hence the current carried by each individual electron, is inversely proportional to the density. But this is exactly compensated by the density: for higher density there are more electrons that carry current, but each single one carries less.

(i) Now, in a real fusion plasma there is always a finite amount of impurity present, if only the helium that was produced by the fusion reactions! So we need to consider how things change if Z is not equal to 1. Derive the collision frequency — dependencies only — while retaining the Z dependence.
Answer: This is given in the lecture notes, section 6.5

(j) Work out the Z-dependent resistivity. Find a logical definition of the 'effective ion charge' Zeff and show that this can be measured by measuring the resistivity of the plasma.
Answer: idem

(k) Calculate Z_{eff} for a plasma with a 3% content of carbon (i.e. $n_C = 0.03 n_{ions}$, where n_{ions} is the density of all ion species together), assuming that all carbon atoms are completely stripped.
Answer: 6 electrons per carbon atom, so (use eqn 6.13) $Z_{eff} \approx 1.8$

Formulas:
Electron collision time for singly charged ions: $\tau_e \approx 6.4x10^{14} T_e^{3/2} n^{-1}$ s (T_e in keV)
Ion collision time for singly charged ions : $\tau_i \approx 0.9(2m_i/m_e)^{0.5}\tau_e$ ($\approx 55\tau_e$ for protons) taking $T_i = T_e$.
Ion-electron energy exchange time: $\tau_{ex} \approx (m_i/2m_e)\tau_e$

Problems Chapter 7: Tokamak 1

7.1 The q-factor in cylindrical geometry

In cylindrical geometry it is easy to calculate the q-profile, i.e. q as function of the radius ($q(r)$), if the current density profile $j(r)$ is known. Consider a cylinder with radius a in which a total current I_0 runs in longitudinal direction, while there is also a uniform longitudinal B-field B_T, and use the definition $q = q_{cyl} = rB_T/RB_p$, where $B_p(r)$ is the field generated by the current:

(a) calculate $q(r)$ for the case that all current is concentrated on the axis of the cylinder (i.e. a delta-function)
Answer: In that case $B_p(r) = \mu_0 I_0/2\pi r$, so $q(r) = 2\pi r^2 B_T/\mu_0 I_0$. Hence the q-profile is a parabola which is zero on the axis and reaches $q(a) = 2\pi a^2 B_T/\mu_0 I_0$ at the edge of the cylinder. Important to note: the current can not be more concentrated than this. Any realistic j-profile will be broader than the delta function and will result in a q profile that lies above this parabola everywhere (except at r=a, where only the total current counts and the shape of the j-profile does not matter).

(b) calculate $q(r)$ for the case that $j(r)$ is uniform

Answer: now $B_p(r) \propto r$, so $q(r) = q(a)$ is uniform, too.

(c) calculate $q(r)$ for the case that $j(r) = 1 - (r/a)^2$

Answer: this should now be straightforward

(d) give an expression for $q(0)$ and $q(a)$ in terms of $j(0)$ and I_0

Answer: for vanishingly small r the current $I(r)$ within radius r is given by $\pi r^2 j(0)$, so $B_p(r) = \mu_0 r j(0)/2$. Work q(0) out from there. q(a) you have already found in the first question

(e) map a flux surface onto a rectangle with the toroidal and poloidal angels along the axes, and plot the trajectory of a field line on a flux surface for q=1, 2, and 3/2, respectively. (everything still in cylindrical geometry)

Answer: Just keep in mind that q=m/n means 'm/n toroidal turns for one poloidal turn', hence 'm toroidal turns for n poloidal turns'. The figure below shows the result, in cylindrical geometry the field lines are straight lines in this representation. For any (arbitrary) value of the toroidal angle you can check that you see the field line at m values of the poloidal angle, hence an island appears m times in the poloidal cross-section of a tokamak (this answers the next question).

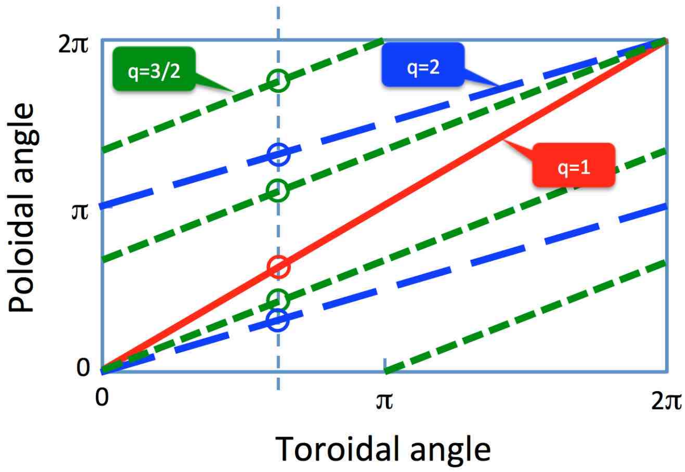

Figure 7.4: Map of a field line on a cylindrical flux surface for different values of q

(f) The magnetic axis of a magnetic island still has the same q-value as the original flux surface. With this, and the picture you just drew, in mind, how many 'islands' do you see in a poloidal cross-section of a tokamak, for q=1, 2, 3/2 as above.

7.2 Flux surfaces and magnetic islands

The topology of the magnetic field in a tokamak or stellarator is that of nested toroidal surfaces. Field lines lie in those surfaces.

(a) The surfaces are called flux surfaces. Explain why. Which flux is constant on a flux surface?

Answer: Field lines lie on the flux surface, so the number of field lines that stick through an area enclosed by the flux surface (e.g. the 'hole' in the donut) is always the same. Therefore the poloidal flux is constant on the flux surface.

(b) Explain why flux surfaces are isobaric as well as isothermal surfaces.

Answer: Isobaric because there is free flow of plasma along the field lines, so in an equilibrium there can be no pressure gradient; Isothermal because the thermal conductivity along the field lines is so large. If pressure and temperature are uniform on the flux surface, then so is the density. Now that is true in a steady state situation (why?). Of course, it is possible to perturb the density and temperature locally on a flux surface. That will set in motion transport processes parallel to the magnetic field that eventually will restore the isobaric and isothermal state.

(c) Which transport process will be faster: particle transport or heat transport. (we are running a little bit ahead of the chapter on transport here, but this is parallel transport, so the B-field does not play a role. Therefore, you can solve this without special knowledge, just common sense will do).

Answer: Particle transport parallel to the field is limited by the speed o the slowest particles, or if you think of shock waves, roughly by the speed sound, i.e. which is approximately the thermal speed of the ions (if $T_i = T_e$). The thermal conductivity is determined by the speed of the fastest particles, i.e. the electrons. So thermal transport is much faster.

(d) We can fuel the plasma by shooting a frozen fuel pellet into it. The pellet ablates (due to the impact of the high energy electrons) which provides a strongly localised source of particles on the flux surfaces it intersects. Supposing that the flux surface will stay intact, describe the sequence of events. (hint: the pellet is cold, so it carries particles, but very little thermal energy).

Answer: The pellet is a local particle source but does not carry energy (it is cold), so locally the density is strongly enhanced while the temperature plummets (no energy input, so the pressure remains almost the same). Then, the temperature on the flux surface equilibrates while the density is still peaked where the particles were dumped. So now there is a pressure bump: the symmetry is broken and the equilibrium is perturbed. Such a phenomenon, if it happens on a surface with a rational q value (q=m/n) will give rise to a perturbation of the temperature, hence the resistivity, which is localised in poloidal angle. Such a perturbation can easily give rise to the formation of a magnetic island.

(e) What is the value of the safety factor q at the flux surface where an m/n=2/1 island can form? What is the q factor of the field lines that live inside the island?

Answer: $q = 2$. Inside the island all field lines have q=2. But, one can define a q^* inside the island: this describes how many toroidal turns a field line has to make to complete a full poloidal turn around the island. This q^* goes to infinity at the edge of the island (the separatrix), but is value inside the island depends on the distribution of the current density in the island.

(f) In an island, there is transport along field lines (i.e. very fast) from one side to the other. Consider the separation between the magnetic island and the field around it. This separation is the imaginary surface called the separatrix. How many times does a field line on the separatrix have to go around the torus in toroidal direction to circle once around the island in poloidal direction?

Answer: As above: on the separatrix this number goes to infinity. So transport parallel to B does not spread heat or particles around the island on the separatrix. Of course, the separatrix is infinitely thin, but also close to it, the poloidal transport along field lines is significantly reduced because field lines have to go round the torus so often in order to get to the other side of the island.

(g) An island is formed by a perturbation of the poloidal magnetic field with respect to the ideal topology. This can be described as a surplus or deficiency of current density (along the field lines, so mainly toroidal) in the O-point of the island. Question: what is it – a surplus or deficiency? The answer depends on the radial dependency of q. Explain. (This is a tough question, but you can still solve it without any math, just imagine that you sit on a field line inside the island)

Answer: For a 'normal' q-profile, which is low (1) in the centre of the plasma and increases towards the edge (to typically 3-6), the O-point of an island corresponds to a deficiency of toroidal current density. Therefore, by heating the O-point and thereby reducing the resistivity, current is restored and the island shrinks. Also direct current drive (using EC waves) can be used to restore the missing current in the island and so shrink it. N.B. These methods are well developed and actually work. Eindhoven university and the DIFFER institute have played an important role in this development.

7.3 The sawtooth instability

The sawtooth instability is important for a number of reasons. Let's think it through.

(a) The sawtooth instability occurs when a $q = 1$ surface forms. Unless you take special measures, this automatically happens in a tokamak. Why?

Answer: Because the resistivity decreases as $T_e^{-3/2}$ the current concentrates in the hottest part of the plasma, which is the center. The more the current density peaks, the lower the central q becomes. Until a $q = 1$ surface is formed and the sawtooth instability prevents further peaking.

(b) The sawtooth instability periodically mixes the plasma within a radius known as the 'mixing radius'. Sketch the evolution of the temperature at 5 different radii in the plasma: a) the center; b) the mixing radius; c) the radius in between the center and the mixing radius which is known as the 'inversion radius' (and explain its name); d) just outside the mixing radius and e) further away from the mixing radius. Explain why you drew these time traces the way you did.

Answer: centre: slow rise, fast crash; between mixing and inversion radius: fast rise, slow fall; at in version radius: some wiggle during the crash, but otherwise very little response; outside the mixing radius: delayed response, smooth rise and fall. The further away from the mixing radius, i) the longer the delay, ii) the smaller the amplitude of the temperature excursion; iii) the less you see of the higher harmonics, hence, the smoother the shaper of the temperature excursion.

(c) What is the typical duration of the 'crash' of the sawtooth instability?

Answer: 10^{-4} s

(d) Make a reasonable assumption for the temperature profile just before and just after the sawtooth crash in order to estimate where the inversion radius is with respect to the mixing radius.

Answer: OK, so here you have to make some assumptions. For instance: assume that the density is uniform in the centre (this is a good assumption) and that the peak of the temperature profile is parabolic out to the mixing radius (also quite reasonable). The integrals of the thermal energy (remember to use cylindrical geometry) should be the same (no energy loss during the crash, just redistribution). You will find that the mixing radius is at $r_{mix}/\sqrt{2}$.

(e) The sawtooth instability has beneficial effects. Mention one.

Answer: In a burning plasma, most fusion reactions take place in the centre, where the 'ash', the helium, will pile up and after some time choke the reaction. The sawtooth helps to sweep the centre clean of helium and bring fresh fuel in.

7.4 The disruption

The disruption is one of the most important problems the tokamak has, and therefore you must familiarise yourself with its basics aspects.

(a) In a disruption, the plasma temperature drops very rapidly (the 'thermal quench'). How long does this typically take? And how does this give rise to the generation of strong electric fields in toroidal direction?

Answer: The thermal quench is very fast, taking only hundreds of microseconds. When the temperature drops, the resistivity soars (going as $T^{-3/2}$) while the toroidal current is still running, hence the high toroidal electric field. Just to get an impression: if the average temperature drops from 10 keV to 10 eV, the loopvoltage goes up from typically 0.1 V to 3 kV.

(b) One consequence can be the generation of a population of runaway electrons (remember those from the section on collisions?). When these lose confinement, that can be very damaging for the machine. Why – and why is that worse than dumping the same amount of energy on the wall in the form of thermal particles? What can be done to prevent the damage of runaway electrons?

Answer: The runaway electrons acquire a high energy, tens or even hundreds of MeV. That gives them a high penetrating power, so they can punch holes in the water cooling leads. Moreover, they tend to be lost in very concentrated, localise spots. To prevent the damage we must prevent the birth of runaway electrons. So-called 'killer pellets', injecting massive amounts of gas that stop the runaway production, are considered for that purpose. Not elegant but probably effective.

(c) Another consequence, especially if during the disruption the vertical position control is lost and the plasma flies off, can be the generation of electrical currents in the vacuum vessel. Why is this a problem?

Answer: Electrical currents in the vacuum vessel that run at an angle with the toroidal magnetic field will cause large forces between the TF coils and the vacuum vessel. Two heavy weights that shake each other. A disruption in JET was registered as earthquake, because of this. The energy and forces released in a disruption scale unfavourable with the machine size, so in ITER it will be worse than in JET, and in DEMO worse than in ITER. The long and short of it is that we cannot tolerate disruptions in ITER. But we are not 100% sure how they can be avoided.

(d) How do the thermal energy content of the plasma and the magnetic energy of the poloidal field scale with the linear dimension of the reactor? Is the 'disruption issue' more or less severe for larger machines?

Answer: These energies scale with volume. Moreover, ITER has a higher magnetic field than JET.

(e) Stellarators don't have disruptions, at least not the violent ones we know in the tokamak. Why not?

Answer: no current, no worries!

Problems Chapter 8: Heating, fuelling and current drive

8.1 Ohmic heating

Even if no other heating is applied, a tokamak plasma is heated by Ohmic dissipation. Straightforward, but it does have some interesting aspects.

(a) What is the cause of the so-called 'Ohmic heating' in a tokamak? Explain why it is not sufficient to reach the required burn temperature of some 15 keV. Would a stellarator exhibit Ohmic heating?

Answer: The current running in a tokamak loses energy to resistance at a rate $P = I^2\Omega$ with Ω the resistance. This energy is converted into heat. The resistivity of the plasma drops with increasing temperature as $T^{-3/2}$, leading to less efficient heating at higher temperatures. The current cannot be increased indefinitely because of the q-limit. The balance comes out so that Ohmic heating can bring a large tokamak like JET to some 2 keV. In a stellarator there is no plasma current and this heating method does not work.

Consider two tokamaks, one small ($R = 0.3$ m) and one large ($R = 3$ m). Both have the same toroidal magnetic field B_T and aspect ratio R/a and have cylindrical cross sections. Both are operated with a current I_p such that the cylindrical safety factor $q_{cyl} = aB_T/RB_p$ is the same. Assume that the heat conduction coefficient κ is uniform, independent of temperature and the same in both devices. Show, considering only functional dependences, not the numerical values of constants, by which factor:

(b) the temperature in the large device is higher (or lower) than that in the small device

(c) the loop voltage (needed to induce the current) is higher (or lower) in the smaller device compared to the larger.

(This is back-of-the-envelope: you may think you do not have enough information to answer this, but you do).

Answer: Equilibrium requires $P_{ohmic} = P_{loss}$. P_{loss} is due to diffusive heat loss from the centre to the edge: $P_{loss} \propto \kappa(T_0/a)R\ a$ (i.e. heat flux density times surface) $= \kappa T_0 R$.
Now estimate $P_{ohmic} = I^2\Omega \propto I^2(T_0^{-3/2}R/a^2)$. Using $q_{cyl} \propto a^2 B_T R^{-1} I^{-1}$ we can estimate $P_{ohmic} \propto a^2 B_T^2 R^{-1} q_{cyl}^{-2} T_0^{-3/2}$.
Requiring $P_{ohmic} = P_{loss}$ we obtain $T_0^{5/2} = \text{constant} \times B_T^2 (a/R)^2 q_{cyl}^{-}2\kappa{-}1$. If q_{cyl} and B_T and (a/R) and κ are the same in the two tokamaks, the central temperature is the same too. There is no size-scaling here. If κ depends on the temperature gradient — which it does in fusion plasmas — then the larger tokamak gets the advantage.
The loop voltage $V_L = I\Omega \propto I(T_0^{-3/2}R/a^2) \propto a^2 B_T R^{-1} q_{cyl}^{-1} \times (T_0^{-3/2}R/a^2) \propto B_T q_{cyl}^{-1} T_0^{-3/2}$ is the same, too.

8.2 Neutral Beam Injection

Neutral Beam Injection is applied to virtually every tokamak. But it gets harder when going to larger machines!

(a) Explain the processes of Neutral Beam Injection (NBI) heating that result in a population of fast ions in the plasma.

Answer: Fast neutral particles are injected into the plasma, and are subsequently ionised by collisions with electrons. These collisions do not change the velocity of the particles much, leading to fast ions.

(b) How is the energy of this population transferred to the bulk plasma?

Answer: Through collisions. The process is called ‘slowing down’ and involves many collisions until the energetic particles are thermalised.

(c) Which species gets most of the power of NBI: electrons or bulk ions? Explain your answer.

Answer: That depends on E_{beam}/T_e. Although the ‘hot’ ions transfer energy more efficiently to ions in an ion-ion collision than to electrons in an ion-electron collision, there are many more of the latter because the collision frequency depends on the relative velocity v^3. The hot ions have a large velocity difference with the cold ions, whereas there are plenty of electrons with similar speed.

(d) The NBI system at JET injects the particles with an energy of 140 keV. In ITER a much higher energy (500 — 1000 kV) will be used. Why?

Answer: Because the beams would otherwise not penetrate deep enough.

(e) The JET NBI system is based on the acceleration of positive ions (D^+ for instance). Why can this principle no longer be applied in reactors the size of ITER or larger?

Answer: Because the cross-section for charge exchange drops dramatically for ion energies in the hundreds of keV, so the neutralising gas cell doesn't work anymore.

(f) Tangential NBI heats the plasma but also transfers momentum — which is useful if you want to make the plasma rotate in toroidal direction. For maximum momentum transfer at given heating power, would you maximise or minimise the injection energy (within the limits set by other considerations)?

Answer: Since the momentum of a particle goes as v, and the energy as v^2, you get more momentum for the same power at lower beam energy.

(g) The neutral beam used for heating can also play a role in diagnosing the plasma. Which measurement is enabled by the neutral beams? (running ahead of the chapter on diagnostic techniques here — use your imagination).

Answer: Charge exchange recombination spectroscopy. (And also neutral particle analysis: the neutralised plasma ions can leave the plasma and be analysed — but in large devices this does not work anyway, because they get reionised on the way out).

8.3 Electron Cyclotron (Resonance) Heating

If heating is applied to simply increase the central plasma temperature, the deposition profile is not that important, as long as it is central. However, if heating is applied with the aim to manipulate — optimise — the temperature profile, it is essential that the heat deposition can be concentrated in a narrow region. The same holds *a fortiori* for current drive.

(a) Electron Cyclotron Heating (ECH) is a resonant process, which in principle yields strongly localised power deposition. Two mechanisms broaden the deposition (in radial direction). Which are those two mechanisms? Give a third reason — not a mechanism, really — why the power deposition profile is finite.

Answer: See lecture notes.

(b) Why is Electron Cyclotron Heating used for local heat deposition, whereas Ion Cyclotron Heating is much less suited for that?

Answer: Ion cyclotron waves have lower frequency (by the mass ratio), therefore longer wavelength: meters rather than mm. Therefore, ECH can be focussed to a much smaller spot.

(c) In a large, hot reactor plasma (in this context this means: the *optical depth* is very high), would ECH waves launched into the plasma in the mid-plane from low-field side be absorbed by electrons that have a relatively high energy (compared to the average in the distribution function), or just the opposite (relatively low energy)?

Answer: Low — the higher energy electrons sit behind the slower ones, and the latter absorb all the power.

(d) Same question, but now in an optically thin plasma?

Answer: Then it is more evenly spread.

(e) Same question for the situation that the waves are injected from the high-field side?

Answer: In that case the fast electrons get most of the power.

(f) Calculate — approximately — the broadening (i.e. give a length) of the power deposition profile due to the finite toroidal width of the antenna pattern, taking this to be 0.1 m, with $T_e = 10$ keV, and major radius $R = 6$m.

Answer: $v_{\|} = 3 \cdot 10^7$ m/s, $\Delta s = 0.1m$, so the inverse transit time $\approx 3 \times 10^8$ s^{-1}. Heating at 170 GHz means that the transiting electron sees roughly 500 ECH cycles. It cannot determine the frequency to an accuracy better than 1 cycle in 500, so ≈ 0.2 %. The broadening is then $R_0/500 \approx 1$ cm.

(g) Why is ECH mostly used at the second harmonic frequency (as opposed to the first harmonic)?

Answer: Higher frequency allows application at higher density – must stay above the plasma frequency

(h) In a tokamak plasma with 'normal' magnetic shear (i.e. q increases with radius) the O-point of a magnetic island corresponds to a region with slightly reduced current density. How can ECH be used to suppress the island? Explain where and when the power must be applied, and why this suppresses the island.

Answer: Heat the O-point so that the resistivity will go down and more current will be attracted.

(i) How, in your island suppression scheme, do you make sure that the ECH power is absorbed where you want it (i.e. you have described the actor, but we need a sensor to steer it).

Answer: The island sits at a specific radius, related to the q-profile, and therefore does usually not sit at the resonant position of the ECH beam. This is solved with steering mirrors. We do not have tuneable sources in this frequency and power range. But we need a measurement to steer the mirror. For this, ECE can be used at frequencies adjacent to the ECH frequency: this will allow you to detect the island in the first place, then to find the position of the O-point, and finally to monitor whether the application of ECH is having the desired effect. This works best – see the dissertation of Hans Oosterbeek – if the same antenna is used that also injects the ECH. In that case your are absolutely sure that you are steering the ECH to right position, even if you don't have an absolute localisation. The difficulty: you have to detect ECE signals at the nana-Watt level using the same antenna that launches a mega-Watt, at nearly the same frequency. A challenge!

(j) In general, the plasma and therefore the island will rotate in toroidal direction. How does this influence the island suppression scheme you described just now.

Answer: In principle you want measure when the island O-point is in front of the ECH-antenna and only heat then. But because the confinement in the island is better than in the ambient plasma, it is also effective if you just heat at the right radius, the island will be heated every time it passes the beam.

Problems Chapter 9: How to take measurements in a hot plasma?

9.1 Interferometry

Every tokamak or stellarator needs to have some form of density measurement, if only to know if the density limit is not approached. The most robust density measurement is performed with an interferometer.

(a) Give a schematic of the basic interferometer setup.

(b) The wavelength chosen for the laser in an interferometry system determines the maximum and minimum density that can be measured. For a tokamak plasmas the densities usually range from 10^{19} to $10^{21} m^{-3}$. What determines the maximum density you can measure?

Answer: The plasma frequency $f_p \approx 10 n_e^{0.5}$. If f_p exceeds the laser frequency f_{laser}, the refractive index $N^2 = 1 - (f_p/f_{laser})$ becomes imaginary and the laser wave is evanescent in the plasma (i.e. it does not propagate). This defines the maximum density $n_{e,max}$ that can be measured.

(c) What determines the minimum density?

Answer: The minimum phase change that your system can detect. The density corresponding to one fringe (i.e. a phase shift of 2π or the distance between two maxima) can be calculated from the refractive index: $\Delta\phi = L.(N-1)\frac{\omega}{c} \approx 0.5 \frac{n_e}{n_{e,max}} \frac{L}{c} 2\pi f_{laser}$, hence $n_e = 4\pi n_{e,max} \Delta\phi \lambda_{laser}/L$. You can measure a fraction of a fringe, depending on the quality of your system.

(d) What are these values for a laser in the visible wavelength (say $\lambda = 500$ nm)? What do you conclude from that?

Answer: $\lambda = 500nm$, $f = c/\lambda = 3 \cdot 10^8 / 5 \cdot 10^{-7} = 0.6 \cdot 10^{15}$ Hz. No problem with the plasma frequency.

(e) At several tokamak experiments an HCN interferometer ($\lambda = 337\mu m$) is used. What are the limiting densities for this diagnostic?

Answer: See above for the generic answer.

(f) How many 'fringes' are measured for a plasma with a (line-averaged) density of $1 \times 10^{20} m^{-3}$?

Answer: See above for the calculation of $\Delta\phi$.

9.2 Thomson scattering

We consider the measurement of the electron temperature using Thomson scattering.

(a) Estimate the electron temperature obtained from Thomson Scattering in the example below.

Answer: In the non-relativistic limit (valid, since the spectrum is only slightly asymmetric), the width of the spectrum (relative to the central wavelength) corresponds to the thermal velocity of the electrons (relative to the speed of light c, as this ratio determines the Doppler effect). The electron temperature then follows from $T = 0.5mv^2$.

(b) The spectrum has two gaps (where the points have artificially been placed on the x-axis). Why would that be (Give separate explanations for each of the gaps)?

Answer: You have to use your common sense here. The points lie on the x-axis, not because there is no signal from the plasma, but because the signal is suppressed in the output. Why? Because the signal in those wavelength regions is strongly corrupted. Around the central laser wavelength (694 nm) there is always a bit of scattering from the laser light into the detector other than by Thomson scattering, so this part must not be used. Around 660 nm there is a strong plasma emission line: the H_α light. Also this must be discarded.

(c) The curve that is fitted to the measurements is asymmetric, where perhaps you had expected to see a — symmetric — Gaussian. Which phenomenon causes this asymmetry? Explain qualitatively how the asymmetry arises.

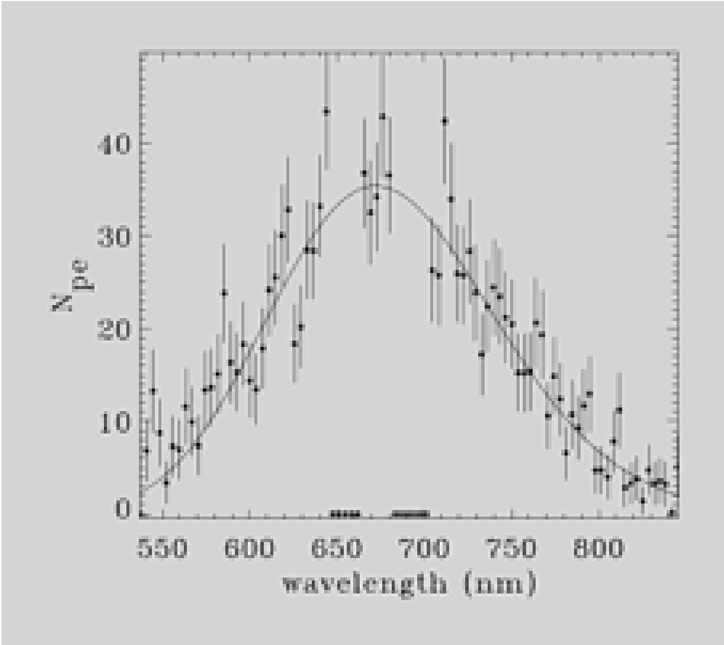

Figure 9.5: Scattered spectrum measured by a ruby laser TS system in which the viewing line is perpendicular to the laser beam. The vertical axis shows the signal strength (number of detected photons). The points are measurements, the line is a theoretical curve fitted to the measurements. The ruby laser has a wavelength of 694 nm

Answer: The asymmetry is caused by the relativistic effect: electrons that move towards the detector emit in a narrow cone and give an enhanced signal, the opposite holds for the electrons that move away from the observer. Therefore the blue wing is lifted and the red wing is suppressed.

9.3 Spectroscopic measurement of the ion temperature

In principle, the ion temperature in a fusion plasma can be obtained from the broadening of a spectral line emitted by hydrogen.

(a) How?

Answer: Line broadening due to Doppler effect, similar to Thomson scattering. But now itâĂŹs the ion speed we see, so the broadening is much smaller and we need higher resolution spectroscopy.

(b) However, the hot core of the plasma in a fusion reactor does not emit much line radiation. Why not?

Answer: Too hot, all hydrogen atoms are fully ionized, so there are no electronic transitions left.

(c) What can be done to induce line radiation from the hot core of the plasma, in order to perform spectroscopy?

Answer: Inject a neutral beam. Charge exchange will lead to — generally excited — neutral hydrogen atoms that radiate until they become ionized.

(d) Apart from the ion temperature also the toroidal plasma rotation speed can be measured by this method. How? And what does this mean for the geometry of the measurement?

Answer: Again using the Doppler effect, but now you look at the shift, not the broadening of the line. As the rotation speed is normally significantly lower than the sound speed, the shift is much smaller than the width of the line. Of course you only see the component of the velocity in the direction of the observer, so for toroidal plasma rotation you want a tangential sight line.

9.4 Electron Cyclotron Emission spectroscopy

The hot core of the plasma does emit Electron Cyclotron Emission (ECE). This is an alternative possibility to measure the electron temperature T_e.

(a) How do you deduce the electron temperature from a measurement of the ECE intensity?
Answer: Planck's black-body radiation law and the long wavelength approximation (Raleigh-Jeans) show that the radiation intensity is proportional to temperature at given wavelength. If we measure the ECE intensity, at a given wavelength (frequency), this corresponds to a location via $B(R) \propto 1/R$ and the gyrofrequency $\omega_{ce} = eB/m_e$.

(b) Which condition must be fulfilled for this method to be applicable?
Answer: The plasma must be a black body radiator at the ECE frequency, i.e. must have sufficiently high temperature and density. Not a problem for present day tokamaks.

(c) How can ECE be used to obtain a temperature profile, i.e. a measurement of T_e as function of the location in the plasma?
Answer: See a.

(d) Draw a temperature profile (T_e as function of major radius) and indicate where the ECE measurement is difficult or impossible because of 'harmonic overlap' (and indicate which harmonics are overlapping).
Answer: The temperature profile maps 1-1 on the spectrum of the first and second harmonic, higher harmonics are usually not optically thick but still have a similar profile. But since the width of the profile is a fraction of the central frequency of the harmonic, and the harmonics are evenly spaced, the profile is increasingly broad at higher harmonics and more and more overlap occurs. The practical limitations are mainly: measurement of the 2nd harmonic at the high field side overlaps with 3rd harmonic from the low field side.

(e) It can happen that during a tokamak experiment in which the density is increased, e.g. because a pellet is injected, the ECE signal suddenly disappears. Explain how this can happen.
Answer: The plasma frequency increases and blocks ECE.

9.5 Neutron measurements

One might say that in a power-producing fusion reactor the only diagnostic you really need is one that measures the neutron production — after all the neutron production is what we really care about.

(a) Since neutrons are not charged, they can only be detected in an indirect manner, i.e. by measuring the effect of their interaction with other particles. Give three examples of these interactions.
Answer: Scattering (elastic and inelastic) and absorption.

(b) Why is the neutron rate such a sensitive measurement for the ion temperature? Give the advantages and disadvantages of this diagnostic.
Answer: The fusion rate depends on the ion temperature, and this relation is well-known. The neutron production rate is linear in the fusion rate (absent any neutron multipliers). The fusion rate increases very rapidly with temperature, and is therefore localized in the center.

(c) Not only from the neutron rate, but also from the neutron spectrum the ion temperature can be obtained. Explain how.

Answer: Using the doppler broadening of the neutron energies (which are well-defined after a fusion reaction).

(d) Give two examples of techniques to measure the neutron energy.

Answer: Time of flight measurements can yield the neutron energy, as well as measurements using Magnetic Proton Recoil (MPR). Both are quite cumbersome.

Problems Chapter 10: Engineer's view

10.1 The principal components of a tokamak

Let's review what you need to build a tokamak.

(a) What are the four principal (set of) items you need to make a tokamak? Just a tokamak to create a plasma in, nothing fancy, no fusion power or anything: the principle, not the technology. So think as a theoretical physicist would.

Answer: 1. toroidal vacuum vessel; 2. coils to produce the toroidal magnetic field; 3. means to induce the toroidal plasma current, i.e. transformer or central solenoid; 4. coils to produce the vertical magnetic field

(b) Of each of the four items, describe their principal function

Answer: 1. you need a vacuum vessel because the plasma has a very low number density, even if its pressure reaches a bar when heated. Before the plasma is created and the fuel is introduced, the base pressure in the vessel must therefore be 10^{-5} Pa or so: a pretty good vacuum; 2. the toroidal field has at least two functions: reducing the cross-field transport, and stabilising the magnetic confinement against kink and ballooning modes; 3. the toroidal plasma current induces the poloidal magnetic field, essential to produce the 'rotational transform', while the poloidal field is also an important factor in the magnetic equilibrium; 4. whereas we can do without any shaping or positioning coils, the one additional field that is indispensable is the vertical field: crossed with the plasma current it produces the force that balances the hoop force due to the kinetic and magnetic pressure in the plasma.

(c) Now, thinking as an engineer, your shopping list is much longer. Give a handful of indispensable bits of hardware the theoretical physicist may have forgotten, and explain your choice.

Answer: I would propose: power supplies to drive the currents in the coils; vacuum pumps to evacuate the vessel and a gas system to fill the vessel with hydrogen; basic operational diagnostics, at least to measure the currents in the coils, the plasma current, and the horizontal position of the plasma (so as to feed back the vertical field); preferably also at least a single channel interferometer to measure the density.

(d) In the picture below there are quite a few magnetic coils that go round in toroidal direction, the so-called PF coils. What is their function? Why so many?

Answer: The PF coils are used to shape and position the plasma. There are so many to properly control the shape: elongation, triangularity; and in particular the formation and precise positioning of the X-point near the divertor.

(e) Apart from the PF coils that you see in the picture, there is another very important coil that encircles the central hole in the torus. What is that and what is its function?

Answer: The Central Solenoid (CS) is the coil that induces the flux swing through the 'hole in the donut', i.e. this is what drives the plasma current. In older tokamaks sometimes a transformer

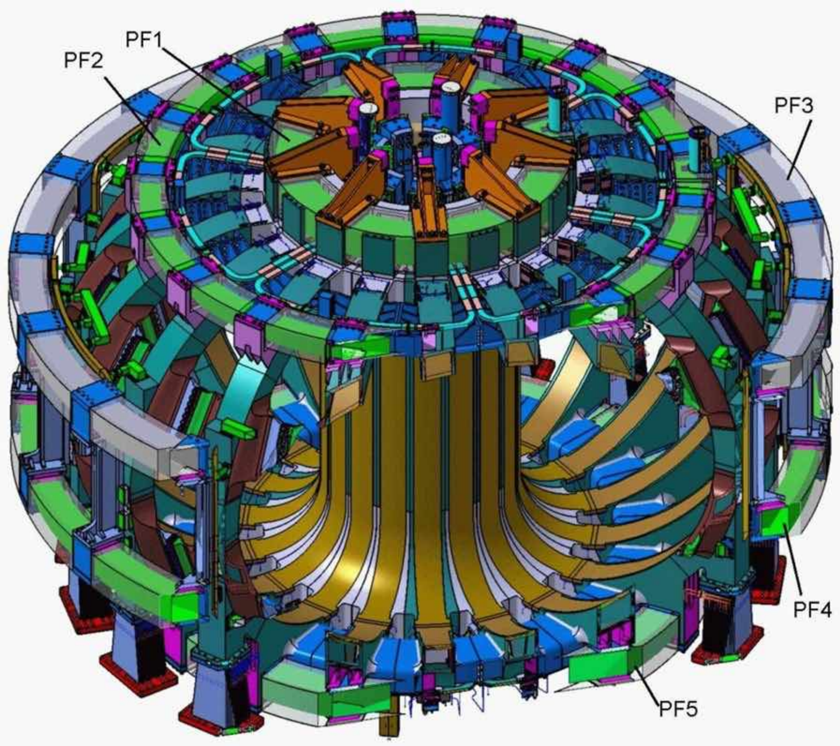

Figure 10.6: The poloidal field coils of ITER

yoke was used which had the primary winding on an outside leg, but it is much better to have the primary winding in the center of the torus: a central solenoid. It does, however, pose problems with respect to the shielding against neutron irradiation, and more generally, there is very little space in the centre of the torus, so the design of the central solenoid is heavily constrained. For ITER, the CS is among the most difficult coils to make.

10.2 Vertical field

The vertical field is not very strong compared to the toroidal field, but without it tokamak operation is not possible.

(a) Explain the function of the vertical field

Answer: see above: balance the hoop force

(b) Derive an expression for the vertical field by approximating the torus by a cylinder if you want to compute the thermal and magnetic energy – while retaining the characteristic of the torus that gave rise to the need for the vertical field in the first place. Make sure to express the vertical field in terms of the dimensionless parameters β_{pol} and l_i, plus all the dimensional parameters you need. (This way you will get a simplified version of the equation given in the lecture notes)

Answer: The total energy contained in the plasma torus is given by the volume integral of the kinetic and magnetic pressure. This pressure tries to expand the torus. The radial expansion (in minor radius) is kept in check by the confining magnetic field and need not be considered here. But upon an increase of the major radius (R), i.e. the circumference, this pressure does work: $Work = p.Area.2\pi R$. By equating this Work to the change of total energy you'll find the expression. I find: $B_V = \frac{\mu_0 I}{8\pi R}(\beta_{pol} + l_i)$. Note that this is very similar to the correct, toroidal expression given in the lecture notes, except that the dependence on β_{pol} is twice stronger (why do you think that is?)

(c) Rework your expression to find the B_V as function of the toroidal field (so, apart from that only dimensionless parameters).

Answer: Use the expression for q_a to find $B_V = B_{pol}\frac{\epsilon}{4}(\beta_{pol} + l_i) = B_T\frac{\epsilon^2}{4q_a}(\beta_{pol} + l_i)$

(d) Give a typical value for the ratio of the three principal B-field components in a tokamak: $B_T : B_{pol} : B_v$

Answer: So, using typical values $\epsilon \approx 0.3$ and $q_a \approx 3$, we find typically $B_T : B_{pol} : B_v = 100{:}10{:}1$

10.3 Scaling Laws

Scaling laws are commonly used in fusion research to find empirical expressions that relate e.g. the energy confinement time of a tokamak to its design parameters (size, shape, B-field) and operational parameters (plasma current, heating power, density). This is the engineering approach. By recasting the engineering scaling law into a form that uses physics parameters and dimensionless numbers, we may hope to get a hint of the physics behind the scaling law, and even the processes by which the confined plasma loses energy. The (simplified) L-mode scaling law given in the lecture notes reads:

$$\tau_E \propto I^{0.85}B^{0.2}P^{-0.5}n^{0.1}a^{0.3}R^{1.2} \tag{10.1}$$

(a) Bring this into a form that uses as much as possible the dimensionless numbers q, β, and ϵ, while eliminating P.

Answer: There are different way to do this. First replace τ_E by its definition: (stored energy)/P. To eliminate P, multiply both sides by $P^{0.5}$, then multiply both sides with (stored energy)$^{-0.5}$, so that the LHS becomes ${\tau_E}^{0.5}$. Then take both sides to the power two. The rest is straightforward substitution of the definitions of q, β, and ϵ. I get: $\tau_E \propto a^2q^{-1.7}\beta^{-1}\epsilon^{0.3}(n^{0.2}a^{0.3})$. Or, leaving out the factors with small exponents: $\tau_E \propto a^2q^{-1.7}\beta^{-1}$, as in the text of the lecture notes.

(b) Now multiply by nT: what do you conclude for the quantity we really have to optimise, the triple product that we know from the Lawson criterion?

Answer: For this L-mode scaling, the triple product turns out to be proportional B^2, and the $q^{-1.7}$ as before (but no room to optimize there)

(c) Repeat the exercise for the H-mode scaling, which you may simplify by forgetting about A (atomic mass) and κ (elongation), to:

$$\tau_E \propto I^{0.93}B^{0.15}P^{-0.69}n^{0.41}a^{0.58}R^{1.39} \tag{10.2}$$

(d) Comment on your result. Compare to L-mode scaling.

(e) Do you note anything peculiar about the density dependence in your result for the Lawson criterion, i.e. for the condition required for ignition?

Answer: You can see, even from the original form, that when you use the H-mode scaling in the Lawson criterion the density drops out. This is very strange, we don't quite understand what to make of this. It suggests that a tokamak - provided it reached H-mode in the first place - can ignite at very low density and correspondingly low fusion power. This strange property is the result of the intrinsic coupling of density, temperature and power in a burning plasma, when the power is the fusion power and not some externally controlled parameter. It is probably a spurious result of the particular form of the scaling law, we don't really think that a tokamak can ignite at arbitrarily low density.

10.4 The safety factor in an elongated plasma

You may wonder why tokamak plasmas are always designed to have a large elongation. Let's find out by doing a thought experiment.

(a) Consider a tokamak in which rather than one, two identical plasmas with circular cross-section sit on top of each other, i.e. they have the same major axis but one is at a different vertical position than the other. They see the same toroidal B-field, carry the same toroidal plasma current, hence they have the same – cylindrical – q_a, for which we'll arbitrarily assume $q_a = 2.5$. Now, we move the plasma rings together, so that they merge into a single plasma ring with elongated cross-section ($\kappa = 2$). What is the q-value of this newly formed plasma?

(b) What are the values of the plasma current and the Toroidal B-field in this new plasma ring?

Answer: trivial: B_T is unchanged, the plasma current is the sum of the individual currents

(c) What is the maximum achievable value of β in the new situation, compared to that of the separate plasma rings?

Answer: Now it gets interesting: the β-limit depends on the plasma current, the toroidal field and the minor radius. But since q_a went up by a factor of two, you are allowed to still double the plasma current! So in total, if your run at the same q_a, you have 4 times the plasma current, for unchanged minor radius and toroidal field. Hence you can achieve four times higher β!

(d) What do you conclude for the κ-dependence of the β-limit?

Answer: β_{max} scales as κ squared

10.5 The Density limit

The density limit implies that we need to have control over the density when running a tokamak. For control you need actuators and sensors.

(a) Mention and briefly describe a sensor for the electron density that could be used in a control loop.

Answer: a single or multichannel interferometer (see lecture notes for a description) is normally used for this purpose

(b) Give three different actuators that can be used to increase the density

Answer: 1. NBI; 2. Pellet injection; 3. gas puff

(c) But given that the density limit is an upper limit, perhaps we should be more concerned with actuators that decrease the density. Discuss the possibilities for such actuators.

Answer: To exhaust the plasma, we can only put pumps at some place quite remote from the plasma. So it is of utmost importance that the recycling isn't too high, or worse, that the walls act as a particle source during the discharge (walls have particles attached to their surface, they can easily come off when the walls warm up, or are bombarded by plasma particles). So wall-conditioning is essential: 'baking' them to remove adsorbed molecules, and coating them with a material that acts as a 'getter', such as titanium or boron. Well conditioned walls act as a pump, binding particles to their surface, but only for a finite amount of time. The divertor provides a good means of actually channeling the plasma exhaust towards a pump, but as said, the recycling coefficient tends to remain quite high which reduces the pumping efficiency. In short: density control is not trivial, in particular not if the plasma is fuelled, e.g. by NBI

(d) In the famous world-record d-t discharge in JET, which produced 16 MW of fusion power, the high performance lasted only for about a second. Why – do you think – could this not be sustained

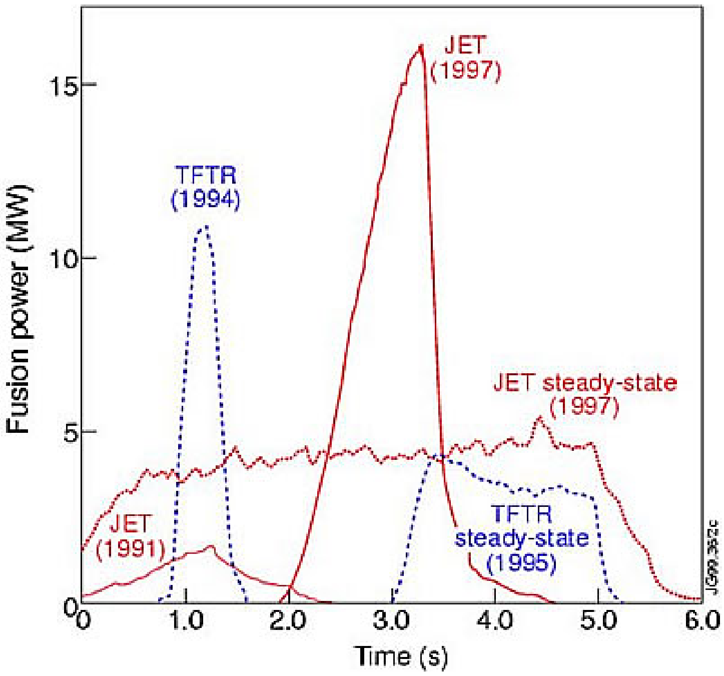

Figure 10.7: In 1997 the JET experiment achieved the highest fusion power in a magnetic confinement device ever: 16 MW. To achieve this, about 25 MW of input power was needed, so Q=0.64 – not quite unity but exceeding the design value of JET

longer? (In the same graph you see that a sustained fusion power could be achieved at a level of about 4 MW.)

Answer: the strong NBI heating also implied strong fuelling. In an H-mode discharge, which has excellent particle confinement, this tends to lead the discharge towards the density limit (and/or the β-limit), as was the case in the high performance d-t discharges in JET.

Problems Chapter 11: Transport

11.1 Classical transport and confinement

The purpose of this exercise is to let you discover how heating, ohmic dissipation, and transport through classical diffusion hang together in a fusion plasma. You'll also practice the art of working with proportionalities — very useful if all you have is the back of an envelope to write on. My answers are one-liners. Now grab an envelope and turn it over.

Let us assume that the cross-field thermal transport is solely due to classical diffusion. (i.e. no neoclassical effects, no turbulence). The collision times for ions and electrons are given by the following formulas:

Electron collision time for singly charged ions: $\tau_e \propto T_e^{3/2} n^{-1}$
Ion collision time for singly charged ions: $\tau_i \approx (2m_i/m_e)^{0.5}\tau_e$.

(a) give the generic expression for a thermal conductivity χ, expressed in a mean free path λ and a collision time τ.

Answer: $\chi = \lambda^2/\tau$ (from dimensional analysis, units must be m^2/s).

(b) Which species (electrons or ions), still in the classical case, give the dominant contribution to heat conduction? (Prove your answer)

Answer: The mean free path perpendicular to the B-field, in the case of classical transport, is given by $\lambda_{i,e} = \rho_{i,e} = m_{i,e}v_{i,e}/eB = \sqrt{2m_{i,e}T}/eB$ where the subscript denotes ions or electrons, and $T_e = T_i = T$ for simplicity. Filling in this mean free path in the answer of a yields: $\chi_e =$

$2m_e T(eB)^{-2}/\tau_e$ and $\chi_i = 2m_i T(eB)^{-2}/\sqrt{2m_i/m_e}\tau_e$. Their ratio is $\chi_i/\chi_e = \sqrt{m_i/2m_e}$. So, the ion diffusivity is larger than the electron diffusivity by a factor of about the square root of the mass ratio, i.e. about 60 for deuterium. Expressed in the variables T, n and B (the rest are constants): $\chi \propto nT^{-0.5}B^{-2}$.

(c) Derive a scaling law for the global confinement time τ_E based on classical diffusion only. Consider only the dependence on the minor and major radius a and R, the magnetic field B, and the plasma parameters n and T. And as always: only the proportionalities, not the constants.
Answer: The energy confinement time scales as $\tau_e \propto a^2/\chi$, where we can take only the ion diffusivity since it is much larger. Filling in the answer of exercise b yields: $\tau_e \propto a^2 B^2 T^{0.5} n^{-1}$.

(d) Now consider a tokamak that is purely heated by the Ohmic dissipation of the toroidal plasma current. Express the heating power P in terms of the plasma current I, the temperature T, the density n and the geometrical parameters a and R (again: only the proportionalities.)
Answer: Let Ω denote the resistance of the plasma for a current that runs in toroidal direction. Then $\Omega \propto T^{-3/2}R/a^2$, and the Ohmically dissipated power $P_\Omega = I^2\Omega \propto I^2 T^{-3/2}R/a^2$

(e) This power has to be conducted by the plasma. Express (only the proportionalities) the thermal conductivity χ in terms of T, n, B and the dimensionless numbers $\epsilon = a/R$ and the safety factor q.
Answer: Ok, so now you have an expression for the Ohmic dissipation and for the confinement time (based on classical diffusivity), everything expressed in plasma parameters. In this case you everything is known and solvable, you can express whatever you want. For instance, the temperature is found by substituting the Ohmic power in the definition of the confinement time, to find $T \propto \epsilon B^2/(nq)$

11.2 Determining the heat conductivity: Power Balance analysis

For a power balance analysis we need to evaluate the heat sources and sinks in the plasma. Heat sources are: Ohmic heating and auxiliary heating. Heat sinks are: Radiative losses. Further, the exchange of energy (through collisions) between ions and electrons constitutes a heat sink for the hotter species, and an equally large heat source for the colder species. Consider a tokamak plasma which is heated – in addition to the Ohmic heating – by Neutral Beam Injection, which you may assume to have the center of its deposition profile at half radius, and ECH which is resonant at $x = r/a = 0.7$ (with r the radial coordinate, running from 0 at the magnetic axis to a at the edge of the plasma). Assume that the tokamak is large, e.g. the size of JET or ITER.

(a) Sketch, as function of the normalised minor radius of the torus (i.e. as function of x) the power density of the sources and sinks, for the electrons and ions. Plot the sinks as negative sources, i.e. negative values of the power density.
Answer: power sources: Ohmic is centralised, NBI has its center at $x = 0.5$ and is quite broad around that value; ECH is a narrow peak at $x = 0.7$; the only power loss is due to radiation and is concentrated close to $x = 1$.

(b) Sketch, again as function of x, the total radial heat flux, i.e. the integral of the sources and sinks. (make sure that you get the right values at x=1).
Answer: Take care to take the volume effect do to the cylindrical geometry into consideration; the integral must start as a linear function departing from the origin. It reaches the total externally

applied power (i.e. the sum of Ohmic heating , ECH and NBI) at $x = 1$; The losses add up to a value that must certainly not exceed the total input power, but may be a good fraction of that.

(c) How would these graphs change if you consider a much smaller tokamak?

Answer: the relative width of especially NBI becomes much larger compared to the minor radius; The relative width of the ECH deposition increases, too, but can still be a small fraction of the minor radius

(d) To complete the power balance analysis, you also need to find the local temperature gradient. You know how to measure the local temperature (confirm this for yourself). Explain why the measurement of the temperature gradient is so much more difficult, that is: its statistical error is much larger than that of the temperature measurement itself and it is sensitive to systematic errors, too. How does the accuracy of the temperature gradient evaluation depend on the spatial resolution with which you want to determine it?

Answer: You measure the temperature profile e.g. by Thomson scattering or ECE. To determine the gradient you need to take the temperature difference between two points. For a local determination of the gradient you want these to be close together, but the closer these are together the smaller the temperature difference. The absolute error on the difference is the (quadratic) sum of the individual errors on each of the two measurements, but the relative error, because of the small difference, can become very large. Moreover, there is an error on the distance between the two measuring points, too. Again: the smaller the distance, the larger the relative error. Finally, any error in the calibration, e.g. in teh relative calibration of the ECE channels, will translate into an error in the gradient.

11.3 Determining the heat conductivity of a deserted island

You are marooned on a deserted island. The classical tiny white beach island with a palmtree in the middle. Or a modern variation (Figure 11.8).

Figure 11.8: marooned but not bored!

Hot days, cold nights, the works. You are bored stiff, but fortunately you find a thermometer in your pocked and you realise that you can use that to measure the heat conductivity of the sand of the beach. Write down the relevant equations (in the sand), describe your measurement procedure and how you derive the heat conductivity of the island from these measurements.

Answer: You use the 24-hour variation of the surface temperature. You measure the temperature as function of time of a point close to the surface and at at least one point at greater depth. You plot the measurements (in the sand) and derive the formulas for perturbative transport (as in the lecture notes – make sure you can do these derivations). You can then determine the heat conductivity either from the phase velocity (no calibration needed) or the decay length (no absolute calibration needed)

11.4 Determining the heat conductivity: Perturbative methods

You have two measurements of the electron temperature in a tokamak as function of time, taken at different values of the radius r: r_1 and r_2. Think for instance of ECE as the diagnostic to take these measurements. The distance Δr between the two points is small compared to the minor radius of the plasma (a) and the region in between is source-free, i.e. there is no volume heat source there. Because someone is applying modulated ECH in the centre of the plasma (i.e. at $r = 0$), the temperature measured at r_1 shows a harmonic oscillation with amplitude ΔT and frequency f_{mod} around its average value (T_0). Assume that the propagation of the temperature perturbation in the region $[r_1, r_2]$ can be considered in slab geometry. So now you look at a slab of plasma with a measurement of $T(t)$ at either end.

(a) Sketch the $T(t)$ signals at r_1 and r_2, indicating relative amplitude, frequency and phase.

Answer: T at r_1 is an harmonic oscillation, as given. At some distance, the temperature shows a harmonic oscillation with the same frequency but lower amplitude and a phase lag

(b) Derive (i.e. do not look up) the formula that allows you to calculate χ from the measured phase of the temperature oscillation.

Answer: now you can look up the formulas

(c) What is the advantage of using the phase information to arrive at χ, compared to using the amplitude information?

Answer: Phase information does not require either absolute or even just relative calibration.

(d) How would you choose the modulation frequency f_{mod}? (To which quantity would you have to compare it?). What happens to the phase and amplitude measurement if one modulates too fast, or too slow, respectively?

Answer: You want to compare the period of the modulation to the energy confinement time. If you go much lower than that, the entire plasma heats up and cools down, the phase shift is small compared to the period and the decay in amplitude becomes very small, too. Moreover, since the plasma pulse duration is typically only so many energy confinement times, you get but few cycles of modulation. So the measurement is not accurate and certainly not very local. If you modulate too fast, on the other hand, the perturbation decays so fast that only a narrow region outside the deposition zone is affected. In that limit a reliable measurement is difficult, too. (Although you may get many cycles, and if you do manage to get a good measurement, it is highly localised). In practice, the period is chosen close to the energy confinement time or a bit shorter.

(e) And here is a really tricky one: because of the modulated heating, the total kinetic energy – hence the poloidal β – of the plasma will be modulated too, and this will lead to a small in-out oscillation of the plasma. How would this affect the measurements?

Answer: Due to the modulation of the pressure, the plasma will make an in-out movement approximately in phase with the central temperature modulation. Temperature measurements taken in the gradient of the temperature profile, will register a modulation solely because of this movement, and with that phase, independent of the propagated temperature perturbation due to transport. So this effect contaminates the measurement. How large it is depends on the local temperature gradient and on the amplitude of the position modulation, which also depends on the speed of the position feedback system. A correction can be constructed if the temperature perturbation is measured both on the inboard and outboard side (where the effect is opposite).

(f) Sometimes the ECH power is modulated in order to determine where the power is deposited. ECE in that region is used as $T(r,t)$ diagnostic. How would you choose the modulation frequency in this case: higher or lower than for the perturbative transport measurement? Give considerations.
Answer: To measure the deposition profile you want to use a very high frequency, much higher than the inverse energy confinement time. The perturbation will decay in a short distance (high frequency does that), so you only see the modulation in the deposition region

(g) Can you predict — in this case — the phase of the temperature modulation in the deposition region, with respect to the phase of the power modulation?
Answer: The phase delay goes to $\pi/2$ (For instance, for a block wave the temperature perturbation is maximal at the end of the heating block and minimal at the end of the 'off' period)

Problems Chapter 12: Aspects of theory

12.1 Making friends with the Maxwellian velocity distribution

This may seem to be an elementary exercise in integration rather than a fusion problem. Yet, by doing this you will discover somewhat unexpected and very important properties of the well-known and omnipresent Maxwellian velocity distribution.

(a) Consider a 1-dimensional Maxwellian velocity distribution $f(v) = Av^2e^{-\frac{mv^2}{2kT}}$, centered around $v = 0$: Calculate the following five quantities:

- the constant, considering that integrating f over all v must yield the density n.
  Answer: $\int_{-\infty}^{\infty} Av^2e^{-\frac{mv^2}{2kT}} = A\sqrt{2\pi}\left(\frac{m}{kT}\right)^{-3/2} = n$ so that $A = \frac{n}{\sqrt{2\pi}}\left(\frac{m}{kT}\right)^{3/2}$.
- the average speed and the average absolute speed
  Answer: $< v >= A\int_{-\infty}^{\infty} v^3e^{-\frac{mv^2}{2kT}} = 0$ (because it is an odd integral).
  $< |v| >= 2A\int_0^{\infty} v^3e^{-\frac{mv^2}{2kT}} = \frac{4n}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}}$.
- the most common absolute speed
  Answer: This is found at the maximum of the distribution function. Taking the derivative of $f(v)$ to be zero we have $2ve^{-\frac{mv^2}{2kT}} + v^2e^{-\frac{mv^2}{2kT}}\frac{-2mv}{2kT} = 0$. Since the exponential factor is nonzero for $T > 0$ and $v > 0$, we can divide these out and obtain $2 - \frac{mv^2}{kT} = 0$. This works out to $|v_{mc}| = \sqrt{\frac{2kT}{m}}$.
- the absolute speed for which the kinetic energy equals kT
  Answer: From $E = \frac{mv^2}{2} = kT$ we have $v = \sqrt{\frac{2kT}{m}}$. This is the most common sped as well.
- the average energy of the particles
  Answer: The average energy of the particles is $< \frac{mv^2}{2} >= Am\int_{-\infty}^{\infty} v^4e^{-\frac{mv^2}{2kT}} = \frac{3nkT}{2}$.

(b) repeat in 2 dimensions and plot $f(|v|)$

(c) repeat in 3 dimensions and plot $f(|v|)$

Problems Chapter 13

13.1 Heat flux in the SOL (scrape-off layer).

The power generated in a fusion reactor comes in two forms: (i) the neutrons from the fusion reaction (80% of the power) are not confined by the magnetic field and are absorbed in the wall (in specially designed wall elements called the 'blanket modules'), and (ii) the α-particles from the fusion reaction (20% of the power) are confined and transfer their energy to the plasma. This power sustains the burn temperature. But it is transported to the plasma boundary by conduction. There, it enters the SOL, where it is transported along the field lines to the divertor.

(a) For a reactor with major radius of 8 m, compute (roughly) the connection length, i.e. the length a particle has to travel along a field line from the moment it crosses the separatrix until it hits the wall in the divertor. Consider the ratio of the poloidal and toroidal magnetic field.
Answer: $L \approx \pi R q = 100\,\mathrm{m}$ (take $q = 4$).

Measurements show that the thickness d_{SOL} of the SOL is about 1–2 cm, rather independent of the machine size. In the lecture notes an estimation of the thickness of the SOL is given based on the parallel and perpendicular conductivities, which results in a number that is compatible with the experimental finding.

(b) Demonstrate that if d_{SOL} does not depend on machine size the power density on the part of the wall that takes the heat flux that leaves the plasma by conduction scales as R^2 (major radius), whereas the power itself scales as R^3. Conclusion?
Answer: Surface area that takes the heat: $A \propto 2\pi R\, d_{\mathrm{SOL}}$. The power, for given n and T, scales as volume ($\propto R^3$). So:bigger machine, much bigger problem.

(c) Assuming that the total power produced in this reactor is 5 GW, calculate the heat flux density that would hit a surface if it were inserted in the SOL perpendicular to the field lines
Answer: 1 GW thermal through a surface area $A = 2\pi R\, d_{\mathrm{SOL}}$ (or half of that, considering that it can flow in two directions, towards the inner and outer divertor plates).

(d) What is done in a tokamak to reduce this heat flux before it hits the target plates in the divertor? Give two different methods. Can you think of a third method?
Answer: (i) Small angle of incidence between field lines and target plate; (ii) Radiation. Each can reduce the power flux density by a factor $\approx 10 \Longrightarrow$ reduction of the average flux density to $10\,\mathrm{MW/m^2}$. (iii) Third method: flux expansion (see lecture notes).

(e) What is the disadvantage of placing the divertor tiles under a very small angle with the incoming field lines? Explain. Also explain the role of the toroidal field in this matter.
Answer: A small misalignment, a ridge that sticks out, or even a tiny bump or dent in the tile, will lead to locally strongly enhanced heat deposition $\Longrightarrow$ temperature difference $\Longrightarrow$ uneven erosion $\Longrightarrow$ hole digging!

13.2 What is hot? Erosion and evaporation. Get a feel for the numbers.
Passive cooling (by radiation).

Whereas in the plasma we are often content with estimates or calculations that are correct within an order of magnitude, when it comes to materials absolute numbers become crucial. Material properties are highly nonlinear in the temperature, even a few hundred degrees can move a material out of its operational temperature window. This exercise will provide a coarse calibration of your intuition.

(a) Give (find on the web) the expression of the power radiated by a 'black' surface as function of temperature. (i.e. assume that the surface is a black body radiator).

(b) The time-averaged power density to the divertor is estimated to be 10 MW/m^2 in ITER. Assuming that all of this must be radiated away, calculate the surface temperature.

(c) Compare this to the melting temperature of the following materials:

i. Tungsten
Answer: 3700 K

ii. Iron
Answer: 1800 K

iii. Carbon
Answer: 3900 K (sublimates)

iv. Beryllium
Answer: 1600 K

What do you conclude?
Answer: Passive cooling will not do the job $\Longrightarrow$ need active cooling.

Now, staying below the melting point is one thing, but already at much lower temperature evaporation becomes significant. Let us have a look. The evaporation rate R_e of a solid surface is given by Eq. (13.1). Figure **??** shows the vapour pressure as function of temperature for a few relevant materials

(d) Combine the formula for the evaporation rate with the vapour pressure and the specific mass of the material, and produce a plot of 'loss rate of material' (i.e. thickness, in m/s) versus temperature.

(e) If we require that the loss rate of the wall material in the divertor is less than 1 cm in 1000 discharges of 1000 s each, what are acceptable values of T (considering only evaporation — not erosion by incoming particles)?
Answer: All straightforward calculation

13.3 Active cooling.

Assume that a plasma facing material is mounted on a 'heat sink', usually a copper body that is water-cooled.

(a) Just to get an impression: if the temperature of the cooling water is raised by 100 K when it passes through a heat sink with a surface of 10x10 cm^2 that is subjected to a power density of 10 MW/m^2, how much water (liters/second) do you need to flow through the heat sink?
Answer: 1 l/s, 100 K increase $\Longrightarrow 10^3 \times 10^2 \times 4.2$ J/s $= 4 \times 10^5$ W. Needed: 4×10^5 W. In a pipe with 1 cm^2 cross-section $\Longrightarrow v = 10$ m/s.

(b) If the plasma facing material takes a heat flux of 10 MW/m^2, and supposing the surface must stay below 3000 K , how thick a layer of plasma facing material can you apply in the case of tungsten and carbon? (look up the relevant numbers for those materials on Wikipedia).
Answer: Tungsten: $T_m = 2700$ K, heat conductivity $= 100$ W/K/m.
$\Longrightarrow P = \text{flux} \times \text{area} = (100\,\text{W/m/K})(\text{area}/d)\delta T$.
$\Longrightarrow d = (100\,\text{W/m/K})\delta T/\text{flux} = (100\,\text{W/m/K})(3000\,\text{K})/(10^7\,\text{W/m}^3) = 3 \times 10^{-2}$ m.
So tiles mounted on top of the heat sink can be a few cm thick, not more.

Consider a target that receives a power flux density of $10\,\mathrm{MW/m^2}$, which consists of a heat sink that is actively cooled and kept at $100\,^\circ\mathrm{C}$, on which a layer of 2 cm of CFC (carbon fibre composite) is mounted. Assume that the thermal conductivity of CFC is $200\,\mathrm{W/mK}$, independent of temperature.

(c) Calculate the surface temperature of the CFC layer, under the assumption that the full $10\,\mathrm{MW/m^2}$ is conducted to the heat sink.

(d) Using the graph in problem 13.2, calculate the evaporation rate.

(e) Calculate the time it takes to evaporate 1 mm of CFC. (CFC density $= 1.9 \times 10^3\,\mathrm{kg/m^3}$).

(f) Under the influence of the neutron flux in a fusion reactor, the CFC will degrade. Suppose this results in a 3 times lower thermal conductivity. Calculate the time it takes to evaporate 1 mm of CFC in otherwise the same conditions as above.

Answer: (all straightforward calculation)

Important to note: if the thermal conductivity degrades, the surface temperature goes up so much that the tile will basically evaporate quickly until the surface temperature is back in the acceptable range. Then it has lost approximately $2/3^{\mathrm{rds}}$ of its thickness.

13.4 What if wall material ends up in the plasma?

Suppose we cause a mishap in which a small part of the tungsten divertor plate gets too much heat, partially melts and injects a droplet of $1\,\mathrm{mm^3}$ into the plasma.

(a) If this droplet were to be evaporated and ionized, how many electrons would it produce? (consider for a minute if the plasma temperature is high enough to fully ionize the tungsten. If not, make a rough estimate of the number of electrons that come off the tungsten atom)

(b) Compare this total number of electrons to the total number of electrons in the plasma.

(c) Compare the total energy consumed by the ionizations to the total energy in the plasma

(d) Now repeat these calculations, if instead of $1\,\mathrm{mm^3}$ of tungsten, $1\,\mathrm{mm^3}$ of carbon falls into the plasma.

Answer: straightforward calculation, meant to give you a feel for numbers.

13.5 Hydrocarbon dust and the fuel cycle.

In the lecture notes it is stated that a single ITER pulse of 10 minutes could possibly produce 1 kg of hydrocarbon dust. Let us estimate the consequences for the fuel cycle.

(a) Compute the number of H atoms (deuterium or tritium) in the 1 kg of hydrocarbon dust (assume something for the composition)

Answer: Experimental studies (google) have shown that the hydrogen or deuteriu concentration (atomic ratio H or D to C) in carbon co-deposits in JET and other devices is typically 0.5 hydrogen atom per carbon atom. For deuterium, with mass 2 vs. 14 for C, that means: $14N_{\mathrm{C}} + 2N_{\mathrm{D}})m_p = 30N_{\mathrm{D}}m_p = 1\,\mathrm{kg} \Longrightarrow N_{\mathrm{D,dust}} = 1/30/(1.7 \times 10^{-27}) = 2 \times 10^{25}$.

(b) Compare this to the total number of D and T atoms in the ITER plasma.

Answer: Roughly, $N_{\mathrm{D,plasma}} = 500\,\mathrm{m^3} \times 10^{20}\,\mathrm{m^{-3}} = 5 \times 10^{22}$.

(c) If the particle confinement time is 10 times longer than the energy confinement time, i.e. some 10 s, which fraction of the exhausted D and T ends up in dust (instead of going into the pump duct and on to the gas handling system)? Hence, what is the probability that a tritium atom gets trapped in dust and is effectively lost per cycle through the machine?

Answer: The pulse duration is 600 s, the particle confinement time is 10 s $\Longrightarrow$ 60 're-fills'. Of the total of $60(5 \times 10^{22}) = 3 \times 10^{24}$ D and T atoms that are put into the vessel, 2×10^{25} end up in the dust! This does not add up, clearly, probably because of the somewhat rough assumptions. But it shows: if ITER produces 1 kg of dust in a day, then the dust is a very efficient D and T capturer.

The burn-up fraction of the exhaust gas is necessarily low — otherwise the reaction chokes on its self-produced helium. Suppose that the probability that a tritium undergoes a fusion reaction between the time it is injected into the tokamak and when it leaves the tokamak (either into the exhaust system, or by being trapped in dust), is 1%. Now, the tritium breeding blankets —if they work very well— can effectively reach a yield of 1.1 tritium atoms per fusion neutron (breeding ratio 1.1).

(d) Deduce how large the loss fraction of tritium may be on a single trip through the tokamak.
Answer: The probability of a fusion reaction, per D or T atom, per 'single pass' through the reactor, is 1% $\Longrightarrow$ each reaction requires 100 turnarounds. Thus one produces 1 neutron which breeds 1.1 tritons. So we may lose 0.1 triton per triton, per 100 turnarounds. That is, the allowed loss fraction per single pass through the tokamak is 0.001.

(e) What do you conclude?
Answer: Our estimate is that a carbon divertor leads to a loss fraction of 1 where only 0.001 is tolerable. This is a big problem, and one of the reasons why carbon is not considered for a reactor.

Problems Chapter 14: Materials for fusion reactors

14.1 The blanket

The plasma in a fusion reactor is surrounded by a blanket.

(a) The blanket has three distinct functions, each of them essential to the success of the fusion reactor. Which three?
Answer: i) harvest the fusion power from the neutrons; ii) breed tritium using the reaction of the neutron with lithium; iii) shield everything behind the blanket, i.e. the superconducting coils and the vacuum vessel, from the neutron flux

(b) What is the typical thickness of the blanket?
Answer: typically 1 m; needed to stop a sufficiently large fraction of the neutrons

(c) Comment on the role of the blanket in determining the minimum size of a fusion reactor
Answer: Because there is always the 1 m of blanket, the performance of the reactor depends less sensitively on the size of the plasma. So, if someone tries to sell you a table top fusion reactor he/she must have forgotten to include the blanket in the design

(d) The blanket in ITER does not fulfil all three functions. Which function(s) does it have?
Answer: The blanket in ITER does have the shielding function. It is cooled, but the extracted heat is not used to produce electricity (why not?); ITER will use externally supplied tritium – the test blanket modules are only used to test various concepts of the tritium breeding technology

(e) Which blanket function will be tested by the 'test blanket modules' in ITER?
Answer: see above

(f) In a reactor, the blanket needs to be heavily cooled. Why?

Answer: Because almost all of the fusion power, plus the power generated in the tritium breeding process, must be removed from the blanket.

(g) As coolant various options are being considered, including water, helium and liquid lead-lithium. Comment on the pros and cons of each of these.

Answer: water is a good coolant for heat removal but does not allow use at the elevated temperatures that are needed for a good conversion efficiency, whereas also the materials require operation higher temperatures. Helium can be used at (much) higher temperature, but has the drawback that its cooling power is poor, so it must be used at very high pressure, with associated pumping power losses. Moreover, helium losses from the system are unavoidable, so using helium at such a large scale, and at high pressure, might be a problem because of losses: cost and availability! Liquid Lithium-Lead is great in many respects, but it is extremely corrosive, so the compatibility with other materials is a serious issue. Even more importantly, it is a good electrical conductor, which means that it will require very high pumping pressure (with associated power loss) when applied in an environment with strong magnetic fields – such as a fusion reactor.

(h) There is yet another possible coolant: FLiBe, a coolant also used in fission reactors. (Look up what it is). What is the advantage of FLiBe over lead-lithium? And what could be a drawback?

Answer: FLiBe is actually an interesting alternative to liquid Li-Pb, its advantage being the fact that it is an insulator – no issues with flowing it through a magnetic field. Compared to Li-Pb, its drawback is that its breeding properties are not sufficient for a self-sustaining reactor.

14.2 Materials under irradiation.

In a fusion reactor, most parts will be shielded from the neutron flux by the blanket, but the blanket itself, and especially the 'first wall' that separates the blanket from the plasma and vacuum, will be subjected to the neutron flux. Let's consider what happens to the material of the first wall.

(a) One of the elementary graphs that characterise a material is the engineering stress-strain curve. Sketch a typical stress-strain diagram and indicate in the graph: i) elastic modulus (YoungÕs modulus); ii) the yield stress; iii) the tensile strength; iii) the ductility.

Answer: see slides or wikipedia

(b) Now assume that the material is hardened by neutron irradiation. Draw in one stress-strain plot the curve for the same material before and after radiation hardening.

Answer: same initial slope, reaching higher tensile strength but breaking at much lower elongation (strain).

(c) Materials can best be used in a well-defined temperature window. Why is application of the material a problem at temperatures below this window? And why at temperatures above this window?

Answer: below: material becomes brittle (ductile-to-brittle transition temperature); above: thermal creep, recrystallisation.

(d) In the Sharpy test, the energy E is measured that is needed to break the test specimen. Sketch the device used in the Sharpy test, indicate the parts, and explain the measurement procedure (you may want to google this).

Answer: see slides

(e) Sketch the typical curves of E as a function of the temperature of a steel test specimen before and after neutron irradiation. Identify the differences and comment on their importance for the application of materials in a fusion reactor.

Answer: Two important differences. 1. the ductile to brittle transition moves to higher temperature, i.e. the applicability temperature window becomes smaller from the lower side. This is important, because it means that the temperature excursions – e.g. those due to ELMs – that can be allowed are smaller. It may also mean that a higher coolant temperature is required. 2. The energy needed to break the specimen in the ductile phase becomes smaller, too. So the material becomes more brittle also when it is applied in the right temperature window.

14.3 Swelling

Metals swell under neutron irradiation.

(a) Why is swelling a problem?

Answer: Several issues: swelling will lead to stress in the material, the swollen components change shape, may not fit anymore, swelling may lead to components coming loose, ...

(b) There are - at least - two fundamentally different causes of swelling. Which?

Answer: 1. defects in the lattice, formation of vacancies that under diffusion will cluster and form voids. 2. gas production in transmutation reactions. The gas gathers in voids and on grain boundaries and produces bubbles. If close to the surface, this also leads to flaking

(c) The neutron energy spectrum from a fission reactor is different from that from a fusion reactor. What is the most prominent difference?

Answer: The strong peak at 14 MeV in the fusion neutron spectrum, due to the d-t reaction

(d) How does that affect our ability to test materials for fusion reactors using fission reactors. Which is most different: the lattice damage (dpa) or the swelling? Why?

Answer: The lattice damage, i.e. the dpa, can be calculated and corrected for the different spectra. The cross-section of the transmutation reactions, however, is strongly dependent on the energy of the neutrons. So the fusion neutron spectrum produces much more helium relative to the dpa load than the fission spectrum. Hence, the irradiation in fission reactors is not representative for the swelling that can be expected in a fusion reactor.

(e) It has been found that different lattice structures of otherwise similar materials show very different swelling. Which is more swelling-resistant: bcc or fcc?

Answer: The bcc structure shows much less swelling than the fcc latice. This is important, as it means that the alloys that are concocted for fusion reactors must have the bcc structure. This consideration limits the allowable fraction of chromium in EUROFER to some 9%.

14.4 Tritium breeding

Although a fusion reactor uses the deuterium-tritium fusion reaction, the fuel that is brought to the reactor is not tritium, which does not exist in nature, but lithium. So we need to turn lithium into tritium as part of the fuel cycle.

(a) What is the half-life of tritium? If you have a stock, which fraction do you lose per year?

Answer: tritium half-life is 12.3 year. You lose approximately 6% of your stock every year

(b) Give the reactions of a neutron with ^{6}Li and ^{7}Li, respectively. Which of the two is exothermic? Which has the highest cross-section?

Answer: Both reactions produce a tritium and a ^{4}He, the reaction with ^{7}Li produces a neutron in addition. The reaction with ^{6}Li is exothermic (+4.8 MeV), hence occurs with thermal neutrons, and has by far the highest cross-section.

(c) Pure lithium is not a very practical material to work with. Why not?

Answer: it reacts with water, air, oxygen, and is extremely corrosive

(d) Therefore, several other forms of lithium are being considered for application in the blanket. Give two examples.

Answer: Possibility 1: liquid Li-Pb alloy. Possibility 2. Lithium-containing ceramics (several different forms are being considered). The two leading concepts for the blanket use these two concepts.

(e) Clearly not all neutrons that are produced in a fusion reaction end up in the blanket. How is it possible to still have a net tritium breeding ratio larger than unity? Which materials can be used to achieve this?

Answer: You make use of 'neutron multipliers', for which especially Beryllium and Lead are considered. A threshold energy is needed for this process, so the achievable multiplication factor is limited.

(f) Give two reasons why the net tritium breeding ratio of the blanket has to be significantly larger than 1. What is - approximately - the value that is aimed for in blanket design?

Answer: The net breeding ratio of the blanket must be sufficient to 1. compensate for the losses (decay! and simple loss from the system), and 2. breed a stock pile to start up the next reactor. Normally the values 1.1-1.15 are quoted as the target for the breeding blanket development program.

(g) Explain how the burn-up fraction of the (burning) fusion reactor is related to the required breeding ratio. Would you rather have a high or low burn-up fraction, from the perspective of tritium management? And from the perspective of achieving burn?

Answer: from the perspective of achieving burn, you want to have the lowest possible He content in the plasma (because it reduces the reaction rate), with a very low burn-up fraction as a consequence. For tritium management, this means that the tritium has to cycled through the reactor, the gas separation plant, the cryo-plant, the fuel injection system dozens of times. Each cycle may take as much as a day (so we are looking at losses due to decay) and it will be a tough challenge to keep the losses – due to tritium retention in the reactor itself, or to losses through permeation of the vessel or the gas handling system – in that entire process at an acceptable level.

14.5 Eurofer and ODS steel

Over the past decades a steel has been developed and tested that could be used for the first wall in fusion reactors. It is called EUROFER (as it was developed in Europe).

(a) What is the main element in the EUROFER alloy, after Iron? Which percentage, approximately?

Answer: Eurofer contains approximately 9% of chromium.

(b) Which element that is abundant in normal stainless steel had to be removed from the alloy for EUROFER for application in fusion? And why?

Answer: Normal steel contains some 20% of nickel, which is unacceptable in a neutron environment due its activation properties. Nickel had to be take out of the mix.

(c) EUROFER has good radiation properties. Yet it has a severe limitation. What?

Answer: the main limitation of EUROFER is the upper boundary of its temperature window, which at 550 C is too low for application with elevated wall and coolant temperatures

(d) An alteration of EUROFER has been proposed (and successfully tested) that remedies the above limitation: ODS. What is that?

Answer: in Oxide Dispersion Strengthened steel, nano-particles (oxides) are dispersed in the steel. This makes the steel much stronger (impediment to the movement of defects) and increases the temperature at which it becomes weak by some 100 C, to 650 C.

(e) ODS steel turned out to have an additional benefit: it reduces swelling. How?

Answer: the nano-particles create spaces in the lattice that can store a large quantity of gas

(f) So ODS steel has great promise. But there is no such thing as a free lunch. Mention two problems – of entirely different nature – of ODS steels.

Answer: First, ODS is very expensive. Second, when you process it, in particular when welding, the welds will not have the same properties as the original ODS steel. In general, there will be a gradient of the oxides away from the weld. Thus the welds become the weak – less neutron-hard – spot of your structure.

14.6 IFMIF

There is a strong need to realise a test facility that could be used to test and validate materials for fusion reactors. This facility goes under the name 'International Fusion Materials Irradiation Facility' (IFMIF). IFMIF is in the conceptual phase. Some of the technology is being developed, but construction of the project hasn't started. In fact, it has not even been decided where it will be built.

(a) Why is it not sufficient to use fission reactors to test materials for fusion?

Answer: The difference in neutron spectrum, in particular the lack of the strong 14 MeV peak in the fission spectrum

(b) Why can we not test the materials in large tokamaks like JET, when running them in d-t?

Answer: we can, but the whole point of doing ITER is that its neutron fluences will be many orders of magnitude larger than those in JET (which is presently the only reactor in the world that can use tritium). So to test if a material will survive 5 years in ITER would take decades or centuries in JET.

(c) Or yet another smart idea: use the fusor as a source of fusion neutrons. Cheap, simple, steady state, produces the correct fusion neutron spectrum ... so what's the snag?

Answer: same problem, the neutron flux density is not high enough. Yet, in a small device one can place the sample very close to the source, so that helps

(d) Explain the working of IFMIF: how does it produce the neutrons?

Answer: IFMIF uses the beam-on-target principle of which we had already seen that it can be a very efficient neutron source yet not produce net energy. IFMIF accelerates deuterium to produce a 40 MeV beam, which is directe towards the target: a jet of liquid lithium. The typical reactions of the deuteron with lithium produce ^{7}Be or ^{4}He and a neutron. The neutron spectrum from these reactions is a similar to that from the d-t fusion reaction.

(e) How large is the test volume of IFMIF in which the highest neutron flux is achieved?

Answer: only 0.5 liter!

(f) IFMIF can only test small materials samples. To qualify materials or components for use in a fusion plant, a 'components test facility' would be needed. Can you propose a good candidate technology?

Answer: This is a question of serious debate in the fusion community. For instance, the compact (Spherical) tokamak has been proposed for this purpose. It may not be suitable as a reactor concept (due to the high heat loads and the neutron protection of the central solenoid) but as a compact high intensity neutron source it might work fine. Just note that a component test facility is a machine that looks a lot like a fusion reactor, but it does not need to have steady-state capability, or does it need to be self-heating - - a driven system is fine for this purpose.

Problems Chapter 15:The Roadmap

15.1 ITER

ITER is the Mondial Joint experiment in fusion, the essential next step towards a fusion power plant. A few things you ought to know about ITER.

(a) Which are the 7 parties that have signed the ITER agreement?

Answer: Europe(represented by the European Commission); Japan; USA; Russia; China; S-Korea; India

(b) What is the goal of ITER in terms of the 'power multiplication factor' Q and pulse duration? (give two distinct goals)

Answer: Q=10 during a pulse of 500 s; Q=5 in steady state

(c) When should these goals be achieved according to the present planning.

Answer: ah, this is a tricky one. Until very recently, the official planning had 2020 for first plasma and 2027 for the start of d-t operations. So the actual goals could be achieved a few year after that, say 2030. But it is a public secret that this planning is unrealistic by a wide margin. First plasma could come as late as 2026. But the new planning isn't official yet.

(d) What is the estimated cost of the experiment (the investment needed for the construction, not the running cost).

Answer: Mostly the cost is quoted at some 15 Billion Euro. Because of the in-kind contributions of the parties, no exact number is known.

(e) ITER wil start by operating in hydrogen and/or helium, then plans to have campaign in which deuterium is used, to finally go to the deuterium-tritium mixture. Why this sequence?

Answer: With hydrogen or helium as working gas, there is no nuclear contamination or activation of the machine, so you can still go in and modify or repair things. With these gases, a lot can be tested. But the operation is sensitive to the gas in several aspects. In particular, it is much more difficult to achieve H-mode – and therefore high confinement – when working in hydrogen compared to deuterium or tritium. So the exploration of the plasma physics and confinement regimes that are relevant to high performance may not be possible without contaminating and activating the machine. Deuterium is somewhere halfway: it produces neutrons but much less than the d-t mixture, is doesn't contaminate the machine, but also in confinement properties it is not quite as good as tritium.

(f) the mission of ITER is partly to confirm the predictions for confinement and performance. But in other areas ITER will enter entirely new territory. Give two areas where ITER explores uncharted waters and indicate why ITER is ground-breaking in these fields.

Answer: First of all: ITER will be the first experiment in which a plasma will be confined that is dominated by self-heating. This is entirely new, it means that several highly nonlinear feedback loops in the plasma become important that simply aren't active without the self-heating. New ground altogether. Secondly, ITER will be a groundbreaking experiment concerning: plasma-wall interaction (intensity of fluxes, fluencies of particles orders of magnitude higher than ever before); neutron loading of the walls, activation – ITER is a nuclear installation; Thirdly: using those neutrons, the test blanket modules will explore the breeding of tritium; ...

(g) the ITER parties contribute 'in-kind' to the project. Why did they not agree to simply creating a joint fund to build the reactor?

Answer: The parts of ITER have been assigned a 'value', expressed in 'ITER units of account'. The negotiations, and the splitting up of the procurement packages, were done on the basis of that 'value'. Countries could agree to a package based on their own estimation of the cost, in their own country, with their own industry. The cost of a part in any currency ($ or Euro) could be very different in different countries.

(h) during the ITER negotiations, many 'procurement packages' were split between several, in some cases all, partners. E.g. 2 segments of the vacuum vessel will be manufactured by S-Korea, the remaining 5 by Europe. Why, do you think, did the parties want to split such tasks?

Answer: politically speaking, countries participate in ITER because they want to be in on the development of fusion power, and therefore they want their industry to become involved in all of the key technologies. That is why the countries tried to get a piece of every ITER component.

15.2 Conceptual Power Plant Study

The Power Plant Conceptual Study considers 4 models (A-D)of increasing advancedness. Let us investigate what these advances are. You want to use Table 1 from the PPCS report at hand as a reference.

(a) Which is the largest model in terms of major radius: A or D?

Answer: A is the largest, D is the smallest.

(b) What is the trend in terms of net electric output power, going from model A to D

Answer: All models were designed for approximately the same net electric output power.

(c) What are the most striking aspects that lead to improvements in plasma performance (confinement) going from model A to D?

Answer: Mostly, the higher models are assumed to have a higher density and β-limit. Further improvements to the performance by higher elongation and triangularity. And, the bootstrap fraction is increased. This does not improve confinement directly, but reduces the amount of current drive that is needed to achieve steady state.

(d) What are the most striking improvements in the technology that lie at the basis of the improved plant performance?

Answer: Most importantly: higher models use hot walls, i.e. high temperature coolants. This allows much better thermodynamic efficiency in the electricity generation, while at the same time it is better (essential) for the first wall and limiter materials. This is a major technological step. Besides that, a higher efficiency of the current drive system is assumed. But notice a. that there is no indication of the technological advances underlying this improvement, and b. that the real improvement here is the much higher bootstrap fraction, up to 76% in model D, so that much less current drive power is needed in the first place. (so the increase of 60 to 70% H&CD efficiency is not that significant.)

(e) Comment on the relative amounts of power that are generated in the divertor and the blanket.

Answer: Of the fusion power, 80% goes to the neutrons, hence the blanket, to start with. The 20% that goes to the α-particles reaches the edge by diffusion, but then a large fraction is radiated away before it reaches the divertor. So in the end, only a small fraction of the fusion power ends up in the divertor. (Still enough to be a challenge in terms of heat load!). Further, the reactor breeds its own tritium, a proces that, depending on the reactions that are used, can add some 25% to the fusion power. So, it is very important to get the conversion of heat to electricity from the blanket right, whereas in the divertor the driving argument for the choice of cooling system is the concern about the plasma-facing materials. (In the PPCS report you'll find the 'Blanket Gain' in table 1. It is 1.17 or 1.18 for the models A, C and D, and an amazing 1.39 for model B.)

(f) Comparing the net electric output to the fusion power, factoring in efficiency of the electricity generation (Carnot sets an upper limit here) and the power used by the current drive system, it appears that the numbers don't add up. Which source of energy is implicit in these numbers?

Answer: Take model B for instance. It produces 1.33 GW of net electric output power. It consumes – only counting the H&CD power – 270 MW generated with 60% efficiency, hence 450 MW. So it must generate at least 1.8 GW of electric power. The fusion power is 3.6 GW, which if converted to electricity with as much as 40% efficiency, clearly produces less then 1.8 GW. The solution to the riddle is, of course, the additional power generated in the blanket, in the tritium breeding process. This blanket power is not counted as 'fusion power', but it is very significant. Realise that this means that besides lithium, the neutron multiplier beryllium must also be considered as a fuel of the reactor. It is consumed and must be replenished.

(g) Why do these reactors need 'current drive'? This is important, since the current drive system is a major power consumer.

Answer: The tokamak needs a toroidal plasma current, and inductive current drive can only supply that current for a limited pulse duration (why?). For steady-state operation it is therefore mandatory to have non-inductive current drive

(h) For model A (near term technology) the power efficiency of the current drive system is supposed to be 60%. In present day experiments, the best current drive systems are the Neutral Beam injectors. What is the power efficiency of today's NBI systems?

Answer: This may need some googling. The answer is: less than 30%, typically closer to 25%. The 60% efficiency quoted is banking on advances in the CD efficiency that haven't been proven yet.

(i) The power needed for the current drive system makes up a very large part of the 'recirculating power', i.e. the power needed to run the reactor itself. Why is this power so much lower in model D than model A?

Answer: Coming back to the same issues from a different angle. Most important factors: a) the bootstrap current, which is assumed to go up to 76% in model D; and b) due to the much better plasma performance and thermodynamic efficiency of the power generation, model D is much smaller than model A and therefore requires much less plasma current (14 MA in model D, compared to 31 MA in model A). Further, the efficiency of the H&CD systems is assumed to see a further increase, from 60% to 70%. A high bootstrap fraction can be achieved by placing the pressure gradient close to the edge of the plasma, where (because of the ϵ-dependence) the highest fraction of trapped particles is found. This does presume excellent profile control, as well as the ability to run the discharge in an equilibrium which has most of the current – namely the bootstrap current – on the outside of the plasma column. Hence, a relatively flat or inverted q-profile. Such modes of operation are called 'advanced scenarios', they are being explored in present day experiments and also ITER will have the possibility to test these scenarios.

(j) Safety is an important point in the PPCS study. Of particular concern in fission reactors is the 'residual decay heat'. Why is this a concern (under what circumstances) and what is the situation in a fusion reactor.

Answer: The activated parts of a reactor will continue to produce heat, due to radioactive decay, after the reactor has been shut down. This heat must be removed, or else parts of the reactor can melt and bad things may happen. In present day fission reactors, this means that there must be emergency cooling systems that will work even if something went wrong with the reactor, its control systems etc. This was what caused the problems in Fukushima. The issue is relevant in fusion reactors, too. but is less stringent. The fusion reactors in PPCS have been designed to not need any form of active cooling for the decay heat.

(k) In the formula for the Cost of Electricity, no performance parameter has a particularly strong influence (the power 0.4 being the strongest dependency). Even more striking, the influence of the obvious parameters Availability and Thermodynamic Efficiency (of the conversion cycle) also is around the square root dependence. How is that possible?

Answer: The generic answer is that a large fraction of the cost of the reactor is not associated with the tokamak or its performance, but rather with the 'balance of plant', i.e. all the other stuff, and the cost of financing. So even if the cost of running the tokamak to produce a kWh is vanishingly small, the kWh has a bottom price below which it cannot go. The dependence on the 'tokamak' factors should be strong (power 1, typically) when these are the dominant ones, i.e. if the total price is much greater than the bottom price. And this dependence becomes weak if the bottom price is approached. Clearly, the PPCS calculations put the cost of electricity somewhere in between those two asymptotic situations.