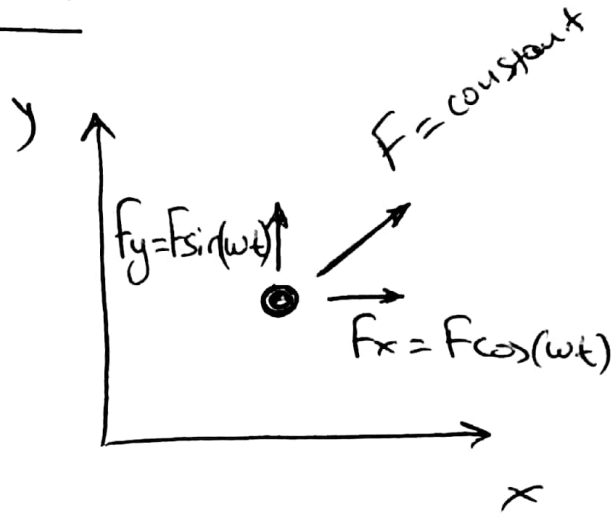
Θέμα 3ο



ω = constant

For $t=0 \rightsquigarrow U_0 = 0$

$U(t) = ?$

$S = ?$ for

$U = \phi$ for the first time.

$$F_x = m\frac{\partial U_x}{\partial t} \Rightarrow$$

$$\Rightarrow F\cos(\omega t) = m\frac{\partial U_x}{\partial t} \Rightarrow \partial U_x = \frac{F}{m}\cos(\omega t)\,\partial t \Rightarrow$$

$$\Rightarrow \int_0^{U_x} \partial U_x = \frac{F}{m}\int_0^t \cos(\omega t)\,\partial t \Rightarrow U_x = \frac{F}{m}\frac{1}{\omega}\left(\sin(\omega t)\right)\Big|_0^t \Rightarrow$$

$$\Rightarrow \boxed{U_x = \frac{F}{m\omega}\sin(\omega t)}$$

$$F_y = m\frac{\partial U_y}{\partial t} \Rightarrow F\sin(\omega t) = m\frac{\partial U_y}{\partial t} \Rightarrow$$

$$\Rightarrow \frac{F}{m}\sin(\omega t) = \frac{\partial U_y}{\partial t} \Rightarrow \frac{F}{m}\int_0^t \sin(\omega t)\,\partial t = \int_0^{U_y} \partial U_y \Rightarrow$$

$$\Rightarrow \frac{F}{m}\left(-\frac{1}{\omega}\right)\cos(\omega t)\Big|_0^t = U_y \Rightarrow U_y = -\frac{F}{m\omega}\left(\cos\omega t - 1\right) \Rightarrow$$

$$\Rightarrow \boxed{v_y = \frac{F}{m\omega} - \frac{F}{m\omega}\cos(\omega t)}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{F}{m\omega}\sin\omega t\right)^2 + \left(\frac{F}{m\omega} - \frac{F}{m\omega}\cos\omega t\right)^2} \Rightarrow$$

$$\Rightarrow |\vec{v}| = \sqrt{\frac{F^2}{m^2\omega^2}\sin^2(\omega t) + \left(\frac{F}{m\omega}\right)^2 - 2\frac{F^2}{m^2\omega^2}\cos\omega t + \frac{F^2}{m^2\omega^2}\cos^2\omega t}$$

$$\Rightarrow |\vec{v}| = \sqrt{\frac{F^2}{m^2\omega^2}\left(\sin^2(\omega t) + \cos^2(\omega t)\right) + \frac{F^2}{m^2\omega^2}\left(1 - 2\cos\omega t\right)} \Rightarrow$$

$$\Rightarrow |\vec{v}| = \sqrt{\frac{F^2}{m^2\omega^2}\left(1 + 1 - 2\cos\omega t\right)} \Rightarrow$$

$$\Rightarrow |\vec{v}| = \sqrt{\frac{F^2}{m^2\omega^2}\left(2 - 2\cos\omega t\right)} \Rightarrow$$

$$\Rightarrow |\vec{v}| = \frac{F}{m\omega}\sqrt{4\,\frac{1-\cos\omega t}{2}} \Rightarrow$$

$$\Rightarrow |\vec{v}| = \frac{2F}{m\omega}\sqrt{\sin^2\left(\frac{\omega t}{2}\right)} \Rightarrow$$

$$\Rightarrow \boxed{|\vec{v}| = \frac{2F}{m\omega}\sin\left(\frac{\omega t}{2}\right)}$$

second part of the question: $s = ?$ When $v = 0$ for 1st time

$$|\vec{v}| = 0 \Rightarrow \frac{2F}{m\omega}\sin\left(\frac{\omega t}{2}\right) = 0 \Rightarrow$$

$$\Rightarrow \sin\left(\frac{\omega t}{2}\right) = 0 \Rightarrow \frac{\omega t}{2} = \pi \Rightarrow \boxed{t_1 = \frac{2\pi}{\omega}}$$

$$\frac{\partial s}{\partial t} = \frac{2F}{m\omega}\sin\left(\frac{\omega t}{2}\right) \Rightarrow$$

$$\Rightarrow \int_0^s \partial s = \frac{2F}{m\omega}\int_0^{t_1} \sin\left(\frac{\omega t}{2}\right)$$

$$S = \frac{2F}{m\omega}\left(\frac{2}{\omega}\right)\int_0^{t_1}\left[\cos\left(\frac{\omega t}{2}\right)\right]' \partial t \Rightarrow$$

$$\Rightarrow S = -\frac{4F}{m\omega^2}\left[\cos\left(\frac{\omega t}{2}\right)\right]_0^{\frac{2\pi}{\omega}} \Rightarrow$$

$$\Rightarrow S = -\frac{4F}{m\omega^2}\left(\cos(\pi) - 1\right) \Rightarrow$$

$$\Rightarrow S = -\frac{4F}{m\omega^2}\left(-1-1\right) \Rightarrow S = 2\,\frac{4F}{m\omega^2} \Rightarrow$$

$$\Rightarrow \boxed{S = \frac{8F}{m\omega^2}}$$