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Essential Neo-Riemannian Theory for Today's Musician

Laura Felicity Mason
lmason11@utk.edu

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I am submitting herewith a thesis written by Laura Felicity Mason entitled "Essential Neo-Riemannian Theory for Today's Musician." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Music, with a major in Music.

Brendan McConville, Major Professor

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Essential Neo-Riemannian Theory for Today's Musician

A Thesis Presented for the
Master of Music
Degree
The University of Tennessee, Knoxville

Laura Felicity Mason
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To my parents

Mark and Terry Mason

and my siblings

Samuel, Alyssa, and Beverly

for all of their love and support

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ABSTRACT

This thesis will build upon the foundation set by Engebretsen and Broman in *Transformational Theory in the Undergraduate Curriculum: A Case for Teaching the Neo-Riemannian Approach* (*Journal of Music Theory Pedagogy 21*) and Roig-Francoli's *Harmony in Context* (McGraw-Hill) by justifying the inclusion of neo-Riemannian Theory (NRT) in the undergraduate music theory curricula. This thesis also serves as a text for use by undergraduates to supplement a typical theory curriculum. While Engebretsen and Broman introduce the notion of NRT inclusion, and Roig-Francolí dedicates several pages in *Harmony* to its discussion, NRT remains uncommon in an undergraduate curriculum. NRT, an emerging and relevant analytical system, lends itself to bridging the transition from the chromatic harmony of the nineteenth century to the varied techniques of the twentieth century. NRT's flexibility assists comprehension of passages from various genres of music, old and new. In an effort to communicate the concepts of NRT to as many undergraduate perspectives as possible, examples and assignments feature musical works of the Common Practice Period, such as those of Beethoven and Liszt, as well as those drawn from the Rock-Pop Era from artists such as Ozzy Osbourne and The Beatles. This thesis addresses the application of NRT through various written, aural, and keyboard assignments that can be easily utilized in most undergraduate curricula. Through the use of written assignments, including examples for analysis as well as composition-exercises, students will achieve an understanding of NRT at the Analytical and Synthesis levels of Bloom's Taxonomy.

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INTRODUCTION

In 1987, David Lewin published his book *Generalized Musical Intervals and Transformations*, introducing Hugo Riemann's transformations without defining triads by their relationship to a key area, establishing transformational theory. From that time forward, others have added to this analytical theory. One specialized segment of this theory is neo-Riemannian theory (NRT), which deals only with major and minor triads and consists of three contextual inversions.[1]

NRT, an emerging analytical system, lends itself to bridging the transition from the chromatic harmony of the nineteenth century to the varied techniques of the twentieth century, while also showing the effective use of set theory on traditional tonal music. NRT's flexibility assists comprehension of passages from various genres of music, old and new, such as those of Beethoven and Liszt, as well as those drawn from the Rock-Pop Era from artists such as Ozzy Osbourne and The Beatles.

In the article "Transformational Theory in the Undergraduate Curriculum: A Case for Teaching the Neo-Riemannian Approach" (*Journal of Music Theory Pedagogy 21*), Engebretsen and Broman present the need for the inclusion of neo-Riemannian Theory (NRT) in the undergraduate music theory curricula. While Engebretsen and Broman introduce the notion of NRT inclusion, they do not provide curriculum to follow, though they offer several practical classroom applications and an order of inclusion. The only

[1] Richard Cohn, "Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective." *Journal of Music Theory*, 42, no. 2, Neo-Riemannian Theory (Autumn, 1998), 171.

texts that have attempted to address this need are Roig-Francolí's *Harmony in Context, 2nd ed.* and Joseph Straus's *Introduction to Post-Tonal Theory, 3rd ed.* Roig-Francolí dedicates several pages in *Harmony* to NRT discussion, but does not seek to include any material that links NRT to set theory. Straus, on the other hand, includes NRT in a section on transformational theory that is short, succinct, and not always accessible to the average undergraduate student.

Engebretsen and Broman's article inspired the writing of this thesis, which serves two purposes: a master's thesis *and* an introductory, supplemental text for an undergraduate curriculum. This textbook supplement is specifically relational to programs utilizing one of the following texts, though the text could enhance any twentieth century text: Miguel Roig-Francolí's *Understanding Post-Tonal Music,* Stefan Kostka's *Materials and Techniques of Post-Tonal Music, 4th ed.,* and Joseph Straus's *Introduction to Post-Tonal Theory, 3rd ed.* Because this thesis serves two purposes, master's thesis and introductory text, the reader will find more scholarly quotes in this thesis than would be found in the traditional undergraduate text.

My goal as a teacher and an author was to introduce the student to NRT in an accessible manner, while introducing more advanced topics of the theory, such as cycles, and including information regarding the use of set theory in conjunction with NRT. Because I wanted to maintain undergraduate accessibility, I have decided not to include information on pedagogically difficult works, such as Henry Klumpenhouwer's "Dualist Tonal Space and Transformation in Nineteenth-Century Musical Thought" and

"Some Remarks on the Use of Riemann Transformations," which have had great impact on NRT.

I have organized this thesis in a manner similar to that of a textbook, offering instructional text, followed by "Terms to Know," "Further Reading," and "Exercise Sections." The text addresses the application of NRT through various written, aural, and keyboard assignments that can be easily utilized in most undergraduate curricula, many of which can be modified for additional exercises of the same variety. Through the use of written assignments, including examples for analysis as well as composition-exercises, students will achieve an understanding of NRT at the Analytical and Synthesis levels of Bloom's Taxonomy.

PREFACE TO THE TEACHER

Music is constantly evolving. As music changes, music theory must evolve to define and catalog each adaptation with accuracy. To produce responsible theorists and musicians in general, the pedagogy and curriculum must be constantly updated to reflect the changes made in music.

While the traditional written tonal music theory curriculum, including aspects of harmony, counterpoint, part-writing, form, etc., is indispensable, the well-rounded musician cannot limit himself/herself to these practices alone. With this in mind, many music theory curricula include twentieth century analytical techniques in the last semester of undergraduate study. This semester generally covers topics such as pitch class analysis, twelve tone methodology, and aleatoric music. Generally, textbooks covering twentieth century analysis do not dedicate substantial time to new theories of tonal analysis, such as neo-Riemannian theory (NRT).[2]

The introduction of neo-Riemannian theory to an undergraduate curriculum following a basic study of pitch class analysis will offer reinforcement to concepts learned and will introduce another explanatory system into each musician's supply of analytical tools. Therefore, the purpose of this text is two-fold. First, this supplement (to a standard theory sequence text) will introduce basic NRT (also referred to as transformational theory, of which it is a faction) to the undergraduate curriculum,

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[2] Two exceptions are Joseph Straus's introduction of neo-Riemannian Theory under "Triadic Post-Tonality" in his third edition of *Introduction to Post-Tonal Theory* (2005) and Miguel Roig-Francolí's introduction in *Harmony in Context*, 2nd ed.

reinforcing concepts already learned in a twentieth century theoretical techniques course. Second, this text is meant to be a useful analytical tool for undergraduate students studying music of the Neo-Romantic, Minimalist, and Pop-Rock genres, which have largely diatonic, yet not classically-functioning, progressions.[3] Theorist Richard Cohn states, "Neo-Riemannian theory strips these concepts [triadic transformations, common-tone maximization, voice-leading parsimony, dual inversion, enharmonic equivalence, and the 'Table of Tonal Relations'] of their tonally centric and dualistic residues, integrates them, and binds them within a framework already erected for the study of the atonal repertories of our own century."[4] Therefore, introducing NRT in undergraduate courses provides a much-needed update to today's curriculum.

The material of this text should be introduced after a basic study of pitch class analysis. For instance, the author recommends introducing this material after completing Chapter 4 of Miguel Roig-Francolí's *Understanding Post-Tonal Music,* Chapter 5 of Stefan Kostka's *Materials and Techniques of Post-Tonal Music, 4th ed.,* or as a supplement to Chapter 4 of Joseph Straus's *Introduction to Post-Tonal Theory, 3rd ed.* Introducing this material in those sequences where indicated will provide the smoothest transition for the student.

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[3] Nora Engebretsen, and Per F. Broman, "Transformational Theory in the Undergraduate Curriculum: A Case for Teaching the Neo-Riemannian Approach." *Journal of Music Theory Pedagogy* 21 (2007), 40.

[4] Cohn, "Introduction," 169.

PREFACE TO THE STUDENT

What is Neo-Riemannian Theory?

Neo-Riemannian theory (NRT) is a segment of Transformational Theory. Transformational Theory addresses triadic chord successions through mathematical group theory. “Group Theory extracts the essential characteristics of diverse situations in which some type of *symmetry* or *transformation* appears. Given a non-empty set, a binary operation is defined on it such that certain axioms hold, that is, it possesses a structure (the group structure).”[5] More generally, transformational theory continues the tradition of Milton Babbitt and Allen Forte by *using mathematics to show the relationship between and among intervals*. By performing group transformations or, in our case, NRT, the musician can see meaningful relationships within these *triadic* progressions, just as prime forms of sets show us meaningful information about various pitch sets.

Because NRT transformations can be defined mathematically and the use of enharmonics becomes tedious, this text utilizes standard pitch class terminology and definitions. (See Tables 1 and 2 at the end of the Preface.) For more information about set theory, consult the Further Reading section at the end of Chapter One.

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[5] Flor Aceff-Sánchez, et al., “An Introduction to Group Theory with Applications in Mathematical Music Theory,” Publicaciones Electrònicas Sociedad Matemática Mexicana 15 (2012), 7.

Neo-Riemannian theory consists of three basic transformations: **parallel (P), relative (R),** and **leading-tone exchange (L, also called *Leittonwechsel*).** (See Figure 1 below.) A parallel transformation converts a major triad to the minor and vice versa by moving the third by half-step. In other words C Major is transformed to C Minor just as C Minor is transformed to C Major. A relative transformation converts a major triad to a minor triad by moving the fifth a whole-step up and vice versa, moves a minor triad to a major triad by moving the root down by whole-step (i.e. C Major to A Minor or A Minor to C Major). The leading-tone exchange transformation converts a major triad to a minor triad by moving the root down by half-step, and vice versa, from minor to major by moving the fifth up a half-step to become the root of the resulting triad (i.e. C Major to E Minor and E Minor to C Major).

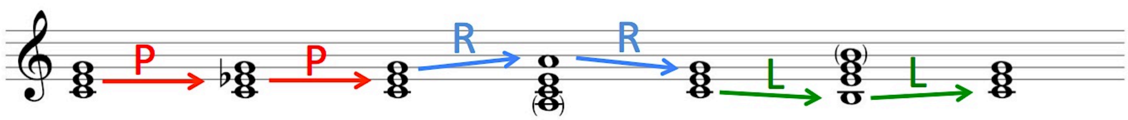

Figure 1: NRT transformations performed on a C major triad.

Why Study Neo-Riemannian Theory?

The mid to late nineteenth century gave rise to chromatic progressions heralded by passing motion and triadic progressions that divided the octave by thirds. Chromatic progressions ". . . were frequently expressed by means of bold modulations, innovative chord progressions, dissonance and resolutions and, in general, much less preparation for abrupt changes. These radical transformations gave rise, in music, to postromanticism and, finally, to atonality."[6] While this music is aurally aesthetic, it is non-functional according to traditional tonal harmony. The best way to describe these non-functional chords using traditional harmony is to say that they are "coloristic" chords or that they are modally borrowed. We must consider an alternative analytical system to accommodate these non-functional triads that are found in late nineteenth century music. NRT, seeks to address these progressions.

Preparing the Student for NRT

The student is advised to thoroughly review Table 1 and Table 2 below. The tables will ensure the student is well equipped to follow this text.

[6] Flor Aceff-Sánchez, et al., "Introduction to Group Theory," 12.

Table 1: Terms for Review

Term	Definition
enharmonic	In modern tuning, notes are considered enharmonic if they are the same sounding pitch although notated with different note names (e.g. A sharp and B flat or E and F
Forte class	The classification given to a pitch class (pc) set by Allen Forte (for instance 3-11). The first number indicates the cardinality and the second the ordinal position in Forte's list. See Further Reading for locations of Allen Forte's set class listing.
hexachord	A general collection of six pitches.
hexatonic	A scalar arrangement of six pitches within the same octave. A common example is the whole tone scale.
normal form	The unordered pitch class set, notated by [ ] in this text.
pitch class (pc) Pl: pitch classes (pcs)	Any group of pitches with the same name, which may occur in any octave, which are generally indicated by an integer (as seen in Figure X).
pitch class set (pc set) Pl: pitch class sets (pc sets)	*Any unordered amalgamation of pitch classes.* For instance, in a tonal work we may find the pitches C (0), E (4), and G (7) together. This creates the common triadic pc set [047].

Table 1 Continued: Terms for Review

prime form	The numerical representation of a set class, notated with ( ) in this text.
octave equivalency	Two pitches of the same name are considered one pc regardless of the octave in which the pitch occurs, reducing the number of pitches to twelve (0-11, notated 0-B).
modulo 12 (mod-12) arithmetic	Any integer larger than 11 is reduced to its equivalent within 0-11.[7]
Set class	The collection of equivalent normal order forms, through T_n/T_nI
T_n	Transposition, where n represents the number of ascending half steps
T_nI	T represents transposition by half steps n, and I represents Inversion, or a pc set's mod-12 complement.

[7] Ibid., 70.

Table 2: The Standard PC Integer Notation System Utilized in This Text	
Pitch Name	Pitch Class
C	0
C#/Db	1
D	2
D#/Eb	3
E/Fb	4
F	5
F#/Gb	6
G	7
G#/Ab	8
A	9
A#/Bb	A (or T for Ten)
B	B (or E for Eleven)

CHAPTER ONE: HISTORY AND INTRODUCTION

General Background

Hugo Riemann (1849-1919), music theorist and musicologist, is considered the founder of modern music theory thought.[8] His treatise, *Handbuch der Harmonielehre,* outlines harmonic function defined by root-interval structures rather than the scalar-based structures of the Common Practice Period to which we have become accustomed. In 1987, Lewin introduced Riemann's diatonic functions in a way in which they functioned outside of a diatonic key structure.[9] Lewin's interpretation of Riemann's transformations provided a technique to interpret music that was triadic, yet not necessarily functional according to traditional scalar practices. Lewin's work has been summarized: "In David Lewin's transformational approach to triadic relations, . . . he explores classes of contextual transformations that, following Riemann and Hauptmann, act upon consonant triads whose constituent pitches are arranged in a line of alternating minor and major thirds (transformations that largely comprise incremental shifts along this line that change one triad into another)."[10] Essentially, Lewin's application of (mathematical) group theory to music sought to show triadic relationships through intervallic relationships rather than traditionally defined diatonic ones.

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[8] Alexander Rehding, *Hugo Riemann and the Birth of Modern Musical Thought.* Cambridge University Press: Cambridge, 2003.

[9] David Lewin, *Generalized Musical Intervals and Transformations*. Yale University Press: New Haven, CT, 1987; reprint Oxford University Press, 2007.

[10] Douthett, Jack M., et al., "Introduction." In *Music Theory and Mathematics: Chords, Collections, and Transformations*. (Rochester, NY: University of Rochester Press, 2008), 4.

In 1989, Brian Hyer published his dissertation *Tonal Intuitions in "Tristan und Isolde"* in which he substantiated the use of three of Lewin's contextual inversions (Parallel, Leading-tone Exchange, and Relative) and one transposition (Dominant), which he represented solely by first letter (*P*,*L*, or *R*) instead of Lewin's shortened titles (such as MED). Hyer's chosen contextual inversions would become the basis of NRT transformations as would the use of the geometrical Tonnetz, which Hyer also revived from nineteenth century theory.[11]

One contextual inversion that that Hyer synthesized was the NRT Relative transformation from Lewin's MED and MED' transformations. Lewin's MED transformation performed on a triad results in that triad becoming the mediant of the resulting triad, i.e. (C,+) MED = (A,-).[12] The MED' transformation is the opposite of the MED transformation, i.e. (A,-) MED' = (C,+). Hyer combined the MED and MED' transformations into one transformation, which we use in NRT: the Relative (R) transformation. (See Figure 2.) The R transformation transforms a triad to its relative major or minor triad.

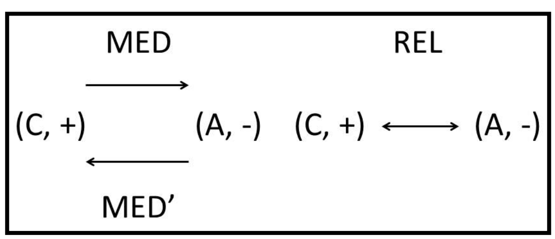

Figure 2: MED and MED' transformations and the NRT REL transformation

[11] Cohn, "Introduction," 171.
[12] Ibid., 176.

From the works of David Lewin, Brian Hyers, Henry Klumpenhouwer, Richard Cohn, and others, NRT has evolved to the theory that it is today.

Terminology

In NRT, triads are considered ordered pc sets and are referred to as **Klangs**. Lewin states, "Each Klang is an ordered pair (p, sign), where p is a pitch class and sign takes on the values + and – for major and minor respectively. The Klang models a harmonic object with p as root or tonic, an object whose modality is determined by the sign."[13] In NRT, chords are not arranged and weighted in the traditional tonal hierarchy. Cohn states, "In assuming the *a priori* status of consonant triads, neo-Riemannian theory leaves unaddressed the reasons that late nineteenth-century composers continued to favour triads as harmonic objects. Indeed, the adoption of a group-theoretic approach to relations between triads suggests that the internal structure of the individual triads might also be viewed group-theoretically, as a complex of equal weighted pitch-classes and intervals."[14] Per Broman states the concept this way: "The abandonment of diatonic context allows neo-Riemannian theory to model non-diatonic relationships among triads: any consonant triad can be connected to any other through

[13] Lewin, *Generalized Musical,* 175-76.

[14] Richard Cohn, "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions." *Music Analysis* 15, no. 1 (1996), 12.

some combination of the P, L, and R transformations."[15] The term Klang and its notation system were adopted to remove the stigmatism of the diatonic key. For the purpose of this text, we will refer to Klangs simply as "triads," although we will adopt the Klang notation system (pc, sign).

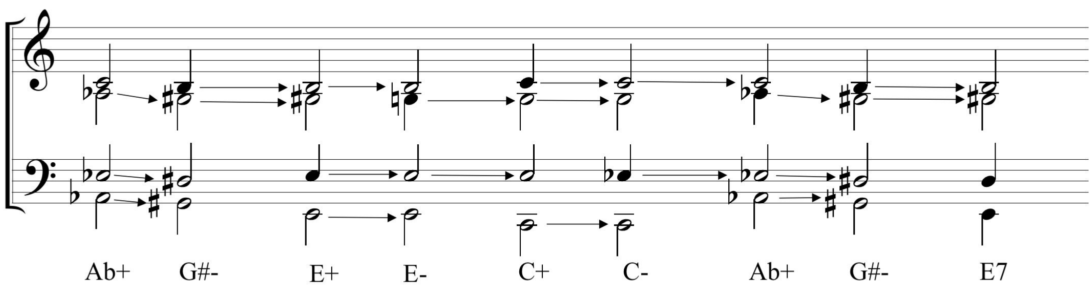

Figure 3: Richard Cohn's reduction of Brahms' Concerto for Violin and 'Cello, first movement, mm.268-79.[18]

In 1997, Cohn published the article, "Neo-Riemannian Operations, Parsimonious Trichords, and Their 'Tonnetz'," in which he introduced the term **parsimony.** Two chords are parsimonious if they share two pitches.[16] Skinner states, "Cohn's definition of generalized parsimonious voice-leading between triads requires two conditions: (1) two voices are retained as common tones, and (2) the third voice proceeds by an interval no greater than twice the smallest available interval in a given tuning."[17] In the

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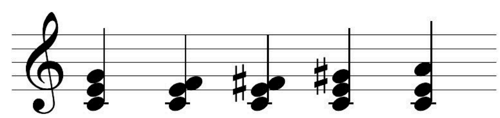
[15] Per F. Broman, "Reger and Riemann: Some Analytical and Pedagogical Prospects." *Svensk Tidskrift för Musikforskning* (2002), 16.

[16] Douthett, Jack M., et al. "Introduction," 5.

[17] Myles Leigh Skinner, "Toward a Quarter-Tone Syntax: Analyses of Selected Works by Blackwood, Haba, Ives, and Wyschnegradsky." PhD diss., (State University of New York at Buffalo, 2006), 198.

case equal temperament, the smallest interval available is the half-step/semitone. Notice in Figure 3 above, that Brahms maintains two pitches between triads as indicated by the open noteheads.[18] The maintained pitches may not be spelled the same way; they may be enharmonics. Parsimonious voice leading is considered directly relational to NRT as it creates the progressions which analyzed with NRT. Figure 4 gives examples of parsimonious voice leading among pc sets, where a "step" can be that of a half or a whole. [19]



pc set: [047] [045] [046] [048] [049]

Forte class: (3-11) (3-4) (3-8) (3-12) (3-11)

Figure 4: Possible Stepwise Transformations in a Triad[19]

Basic NRT Transformations

[18] Richard Cohn, "Neo-Riemannian Operations, Parsimonious Trichords, and Their 'Tonnetz' Representations." *Journal of Music Theory*, 41, no. 1 (Spring 1997), 35.

[19] Richard Plotkin, "Transforming Transformational Analysis: Applications of Filtered Point-Symmetry." (PhD diss., The University of Chicago, 2010), 53.

Lewin's MED transformation is not considered a neo-Riemannian transformation because it is not an **involution**. Involutions in mathematics are defined as an operation, which, when applied to its result, returns the original number.[20] In music, a transformation applied a second time returns the transformed chord to the original chord, which is the first common characteristic of the basic transformations.[21] For instance, G Major transformed by the Relative transformation becomes E minor. When the Relative transformation is performed on the resulting E minor, G Major results.

Neo-Riemannian theory consists of three basic parsimonious transformations: **parallel (P), relative (R),** and **leading-tone exchange (L, also called *Leittonwechsel*).** (See Figure 5 below.) A *parallel* transformation converts a major triad to the minor and vice versa by moving the third by half-step. These chords are parsimonious because they share the root and the fifth. A *relative* transformation converts a major triad to a minor triad by moving the fifth a whole-step up and vice versa, moves a minor triad to a major triad by moving the root down by whole-step. The *leading-tone exchange* transformation converts a major triad to a minor triad by moving the root down by half-step, and, from minor to major, by moving the fifth up a half-step to become the root of the resulting triad.

If the transformations were described according to the rules of traditional harmony (with key contexts), a parallel (P) transformation converts a major triad to its

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[20] Todd A. Ell and Stephen J. Sangwine, "Quaterion Involutions and anti-involutions." In *Computers and Mathematics with Applications,* 53, no. 1, (2007), 137-138.
[21] Cohn, "Neo-Riemannian Operations," 1.

parallel minor and vice versa. A relative (R) transformation converts a triad to its relative major or minor. The leading-tone exchange (L) transformation converts a triad to its iii (if the original is major) or VI (if the original chord is minor).

Cohn gives the characteristics of each transformation: “The three transformations share several properties: each is an involution; each is mode-altering (it maps a triad to its pitch-class inversion); and each transformation preserves two common tones, replacing the third with a pc a semitone away (in the case of P and L) or a whole tone away (in the case of R). This last property is particularly significant for the analysis of late Romantic music, where common-tone preservation and smooth voice-leading are characteristic stylistic features.”[22] **Mode-altering** can be thought of as the inversion of the triad; thus, [047] becomes [037] or vice versa. To visualize common-tone preservation, view Figure 5 below. Also, notice that they are involutions as performing the transformation to the resulting triad returns it to the original.

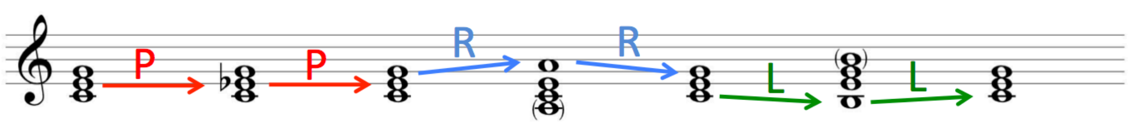

Figure 5: NRT transformations performed on a C major triad. Note each transformation is an involution.

The Tonnetz

[22] Cohn, "Maximally Smooth Cycles," 12.

Joe Argentino states, “The *Tonnetz* or ‘tone-network,’ is a two-dimensional graph in which the axes represent the three intervals of a trichord (traditionally major and minor chords).”[23] Because the transformations are involutions, the **Tonnetz** features only major and minor triads, while augmented and diminished triads have not been included in the basic transformations.[24] (See Figures 6 and 7.) The Tonnetz links neo-Riemannian theory transformations easier to perceive.

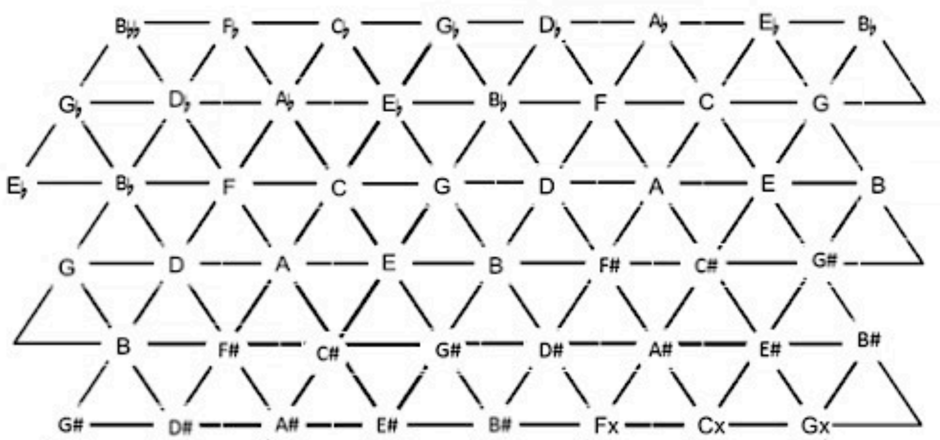

Figure 6: The Pitch Name Tonnetz

Under equal temperament, each pitch should appear only once on the Tonnetz, and the Tonnetz should wrap around on itself.[25] Due to the use of two dimensions (on

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[23] Joe R. Argentino, “Transformations and Hexatonic *Tonnetz* Spaces in Late Works of Schoenberg,” PhD diss., The University of Western Ontario (2010), 1.

[24] Cohn, "Maximally Smooth Cycles," 13.

[25] Engebretsen and Broman, "Transformational Theory," 45.

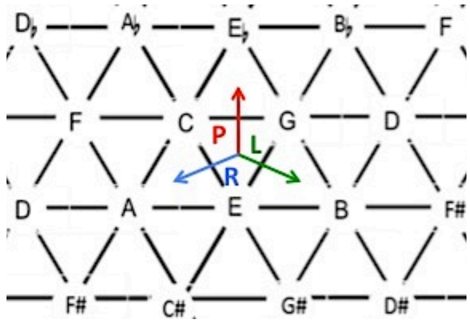

Figure 7: The basic transformations on the Tonnetz

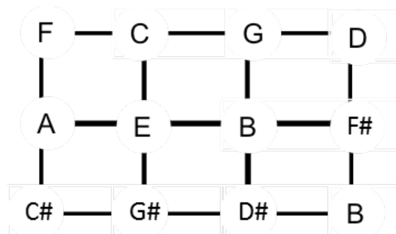

Figure 8: Euler's Tonnetz[26]

paper), this text's Tonnetz will feature repeated pitches in the order that they would appear if the Tonnetz wrapped around on itself (three-dimensional figure).[27]

The first Tonnetz was created in 1739 by Leonhard Euler.[28] (See Figure 8.) Ottakar Hostinský introduced the angled Tonnetz in 1879 and Riemann adapted it in 1880.[29] Riemmann's Tonnetz is shown in Figure 9.

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[26] Rachel W. Hall, "Playing Musical Tiles."http://people.sju.edu/~rhall/Bridges/london.pdf.

[27] A scrolling tonnetz may be accessed at http://en.wikipedia.org/wiki/File:TonnetzTorus.gif.

[28] Richard Cohn, *Audacious Euphony Chromaticism and the Consonant Triad's Second Nature* (New York: Oxford University Press, 2012), 28.

[29] Reinhold Behringer and John Elliott, "Linking Physical Space with the Riemann Tonnetz for Exploration of Western Tonality," in *Music Education*. NY: Nova Science Publishers, Inc., 1.

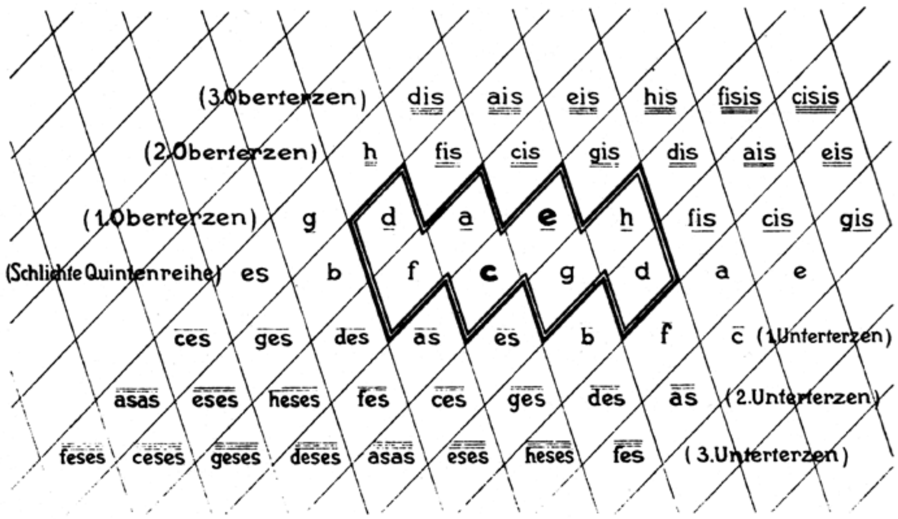

Figure 9: Riemann's Tonnetz[30]

Modern theorists Lewin, Hyer, and Cohn have updated and modified the Tonnetz to fit their needs.[31] One such update was the use of pitch classes.[32] As discussed earlier, the use of pcs eliminates the need to use enharmonics.[33] Dmitri Tymockzo states, "Ultimately, pitch classes are important because they provide a language for making generalizations about pitches."[34] Because NRT translations do not function within a key, reducing triads to pc sets makes their translation much easier.

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[30] Justin London, "Some Non-Isomorphisms Between Pitch and Time," (April 2001.) http://www.people.carleton.edu/~jlondon/some_non-isomorphisms.htm.

[31] Dmitri Tymoczko, "Discrete Voice-Leading Lattices," in *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 412.

[32] Alissa S. Crans, Thomas M. Fiore, and Ramon Satyendra, "Musical Actions of Dihedral Groups," *The Mathematical Association of America, Monthly 116* (June–July 2009), 488.

[33] Notes are considered enharmonic if they are the same in pitch (in modern tuning) though bearing different names (e.g. F sharp and G flat or B and C flat).

[34] Dmitri Tymockzo, "Harmony and Voice-Leading" in *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 31.

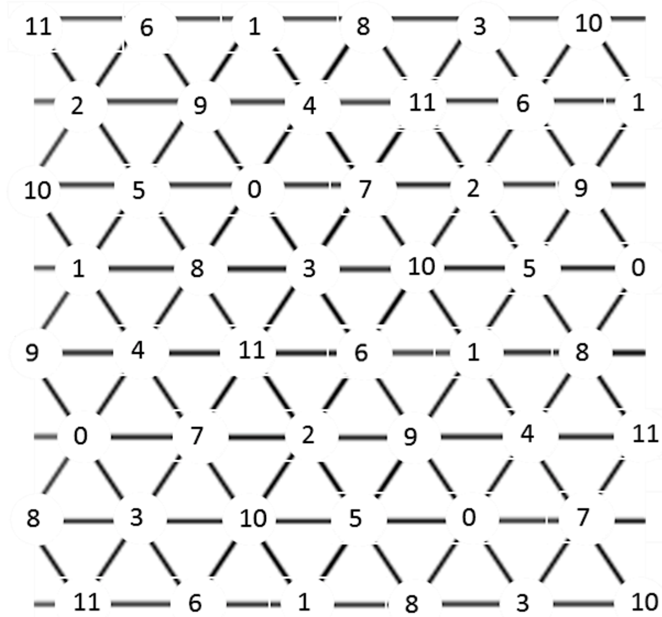

Figure 10: The Pitch Class Set Tonnetz

Another such modification was Jack Douthett and Peter Steinbach's chickenwire Tonnetz.[35] (See Figure 11.)[36] The chickenwire Tonnetz shows the relationship of triads through the use of root-named chords rather than showing the individual connectivity of pitches. Uppercase letters indicate major chords while lowercase letters trace minor chords. As shown by the key, solid lines represent the parallel transformation of chords, dashed lines represent the relative transformation of chords, and dotted lines represent leading-tone exchange transformations. For instance, (Eb,-) is transformed to (B,+) through the leading-tone exchange (as indicated by the dotted line).

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[35] Dmitri Tymockzo. "The Generalized Tonnetz," 2. http://dmitri.mycpanel.princeton.edu/tonnetzes.pdf.

[36] Milan Kidd, *An Introduction to the Practical Use of Music-Mathematics*, 5. http://www.math.uchicago.edu/%7Emay/VIGRE/VIGRE2006/PAPERS/Kidd.pdf.

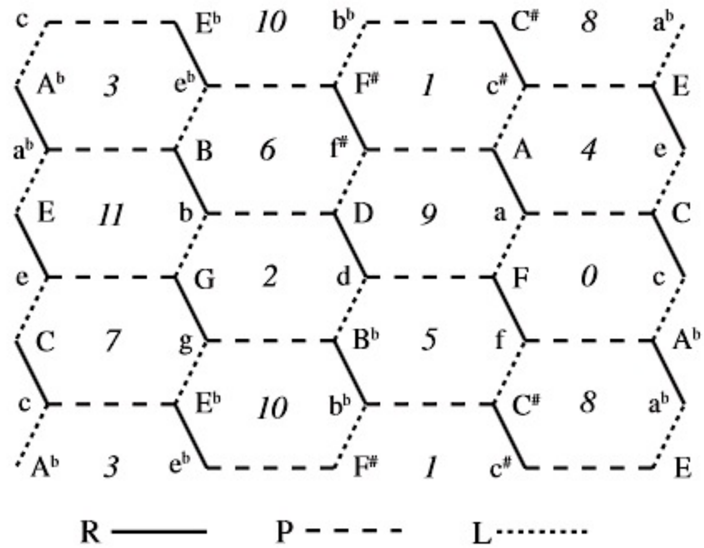

Figure 11: Douthett and Steinbach's chickenwire Tonnetz[36]

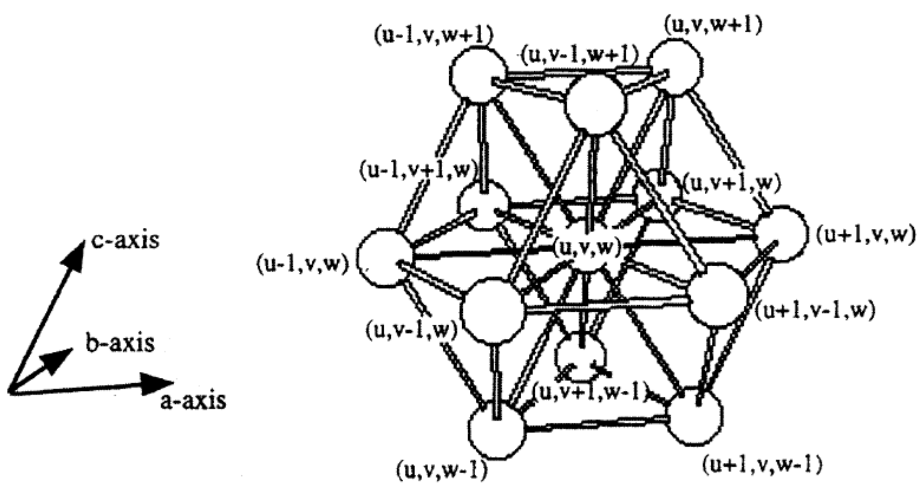

Figure 12: A region of Edward Gollin's 3D Tonnetz.[37]

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[37] Edward Gollin, "Some Aspects of Three-Dimensional 'Tonnetze'," *Journal of Music Theory*, 42, no. 2, Neo-Riemannian Theory (Autumn, 1998), 202.

In 1998, Edward Gollin introduced the 3D tonnetz, pictured above (Figure 12). While further expansions, like Gollin's, have been made to the Tonnetz and others have created Tonnetze of their own,[38] we will use the Riemann/Hyers Tonnetz (both with pitch names and pcs – Figures 6 and 10).

Navigating the Tonnetz

To navigate the Tonnetz, first realize that chords are transformed by changing one pitch at a time. When (G,+) is transformed to (G,-) via the parallel transformation, think of the edge of the triangle resting on *B* being flipped over onto *Bb* (Figure 13a). When (G,+) is transformed to (E,-) via the relative transformation, think of the edge of the triangle resting on *D* being flipped over onto *E* (Figure 13b). When (G,+) is transformed to (B,-) via the leading-tone exchange transformation, think of the edge of the triangle resting on *G* being flipped over onto *F#* (Figure 13c).

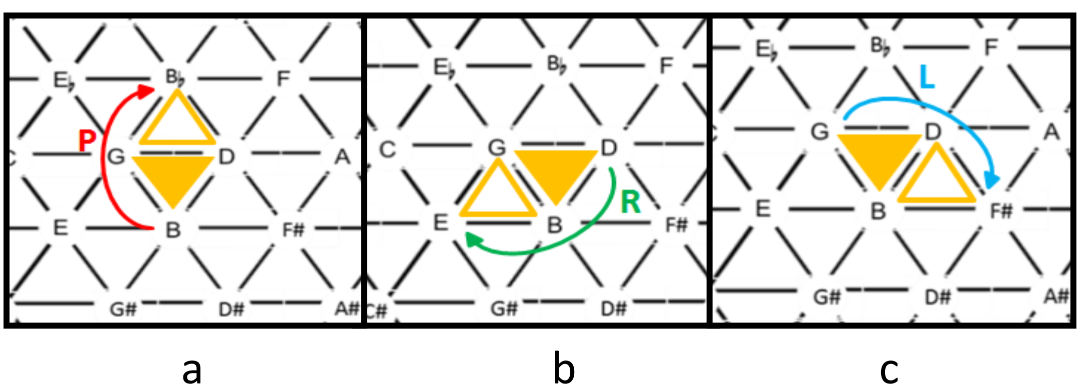


Figure 13: The basic transformations

[38] Cohn, *Audacious,* 28.

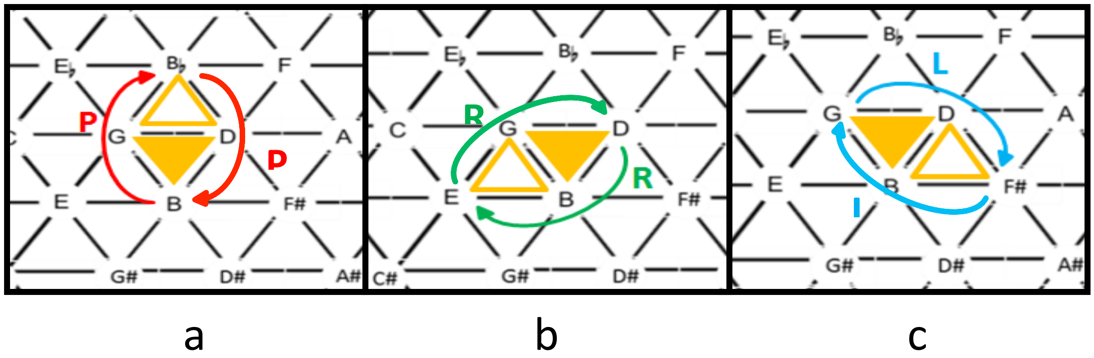


Figure 14: Transformations of triads and their involutions.

Because all of the transformations are involutions, repeating the transformation will return the triad to the original triad. Therefore, the transformations can be undone by performing the same motions in reverse. (See Figure 14.)

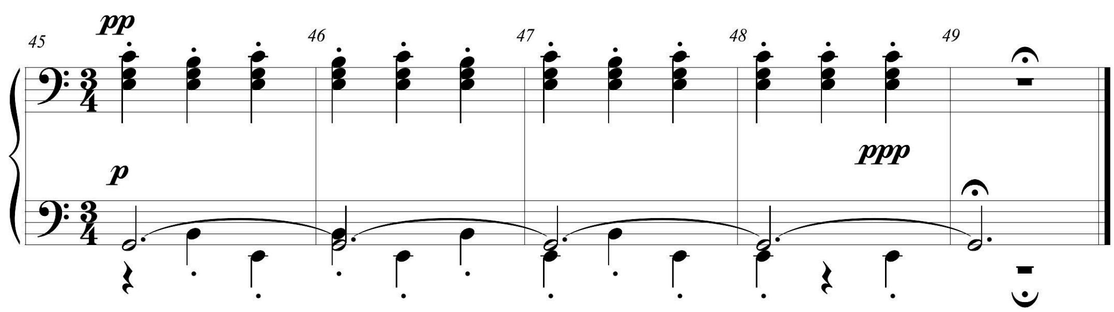


Figure 15: Prokofiev, *Fugitive Visions, Op. 22, No. 4,* mm. 45-49. Public Domain.

To chart the opening of any progression:

1. Identify the triads and their progression.
2. Mark the first triangle in an identifying manner. (We will use a solid triangle in this text.)

3. Use arrows to show the progression on the Tonnetz.
4. Identify which pitches change between the chords.
5. After identifying the changing pitches, chart transformations on the Tonnetz, indicating the transformation by the appropriate indicators (P, L, or R).

Take for example, mm. 45-49 of Prokofiev's *Fugitive Visions, Op. 22, No. 4* (Figure 15). Following the steps given above, the process would be:

1. Identify the triads and their progression:

   (C,+) (E,-) (C,=) (E,-) and so on. . .

2. Mark the first triangle in an identifying manner.

   For this exercise, a solid triangle will be used. (See Figure 16 below.)

3. Use arrows to show the progression on the Tonnetz.

   Done in green below.

4. Identify which pitches change between the chords. *In this example, B to C and vice versa.*

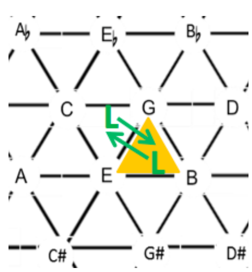


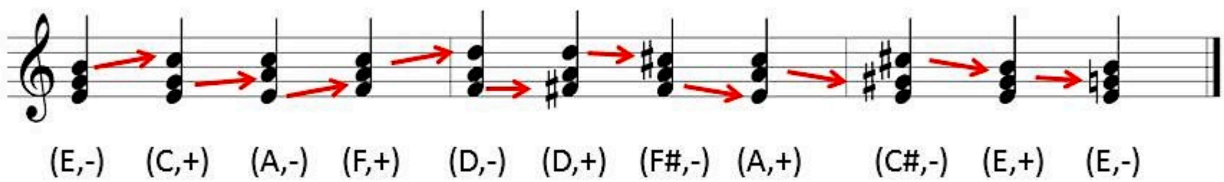
Figure 16: The Tonnetz representation of Prokofiev, *Fugitive Visions, Op. 22, No. 4,* mm. 45-49

5. Identify the transformations with the appropriate indicators (*P*, *L*, or *R*).

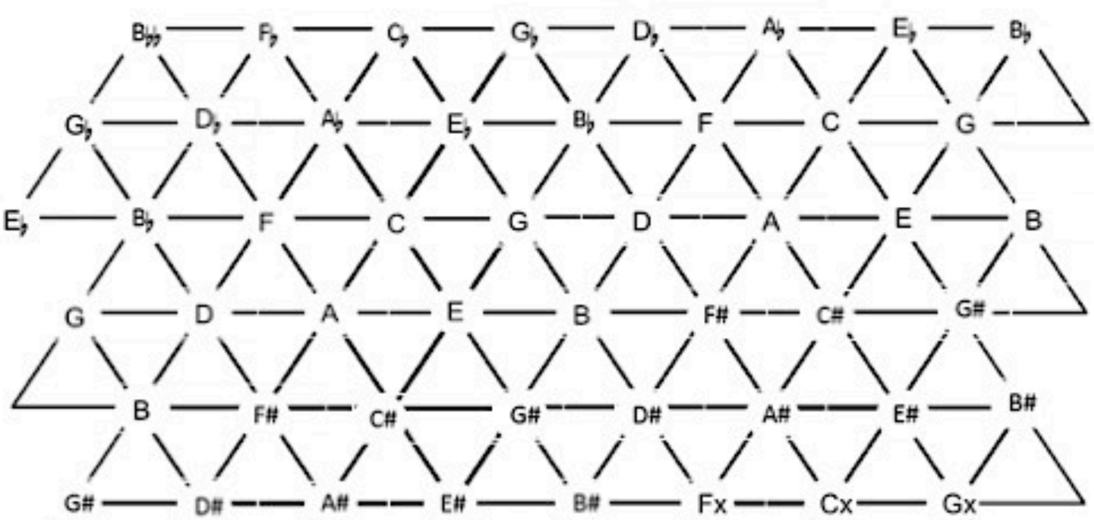
(C,+) to (E,-) is a leading-tone exchange. Shown in green above.

∇ *Learning Activity 1: Navigating the Tonnetz*

→ Repeat the steps given above for the progression below on the Tonnetz below.



→Which transformations result? Is there anything unique about the results?



Your resulting navigation should look like that in Figure 17 below. Notice that the progression LRLRP is performed twice to return to the original triad.

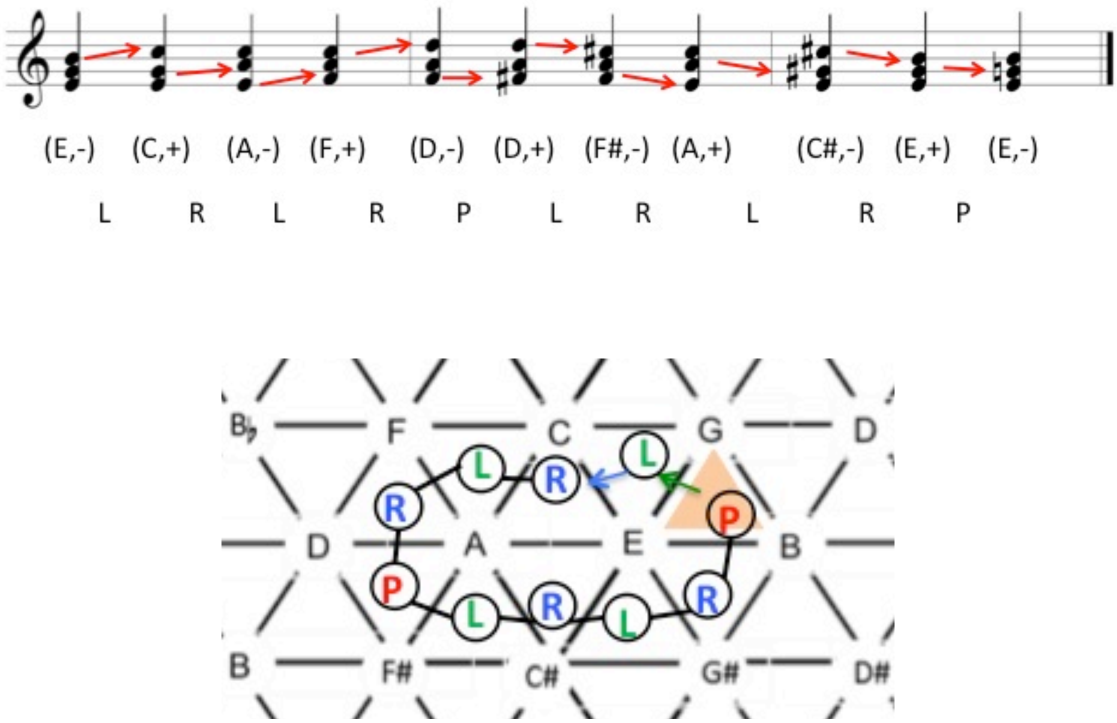


Figure 17: The sample progression charted

In recent years, many other music theorists have added to this theory and begun to build upon the foundations laid for them. A bibliography offering readings on the history and fundamentals of NRT is given in the *Further Reading* portions at the end of this chapter.

Terms to Know

Circle of Fifths	Mode-altering
Enharmonic	Octave equivalency
Involution	Parsimony
Klang	Pitch Class
Leittonwechsel	Pitch Class Set
Mod-12	Tonnetz(e)

Further Reading on Neo-Riemannian Theory

Cohn, Richard. “Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective.” *Journal of Music Theory* 42, no. 2, Neo-Riemannian Theory (Autumn, 1998): 167-180.

Cohn, Richard. "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions." *Music Analysis* 15, no. 1 (1996): 9-40. Read Section I.

Cohn, Richard “Neo-Riemannian Operations, Parsimonious Trichords, and Their ‘Tonnetz’ Representations.” *Journal of Music Theory*, 41, no. 1 (Spring 1997): 1-66.

Gollin, Edward, and Alexander Rehding. *The Oxford Handbook of Neo-Riemannian Music Theories*. New York: Oxford University Press, 2011.

Kidd, Milan. *An Introduction to the Practical Use of Music-Mathematics*. http://www.math.uchicago.edu/%7Emay/VIGRE/VIGRE2006/PAPERS/Kidd.pdf Read all.

Klumpenhouwer, Henry. "Some Remarks on the Use of Riemann Transformations." *Music Theory Online* 0, 9 (July 1994).

Lewin, David. *Generalized Musical Intervals and Transformations*. Yale University Press: New Haven, CT, 1987; reprint Oxford University Press, 2007.

Skinner, Myles Leigh. "Toward a Quarter-Tone Syntax: Analyses of Selected Works by Blackwood, Haba, Ives, and Wyschnegradsky." PhD diss., State University of New York at Buffalo, 2006. Read the section "Conventional Neo-Riemannian Transformations," 196-199.

Further Reading on Pitch Classes and Pitch Class Sets

Roig-Francolí, Miguel A. "Introduction to Pitch-Class Set Theory." in *Understanding Post-Tonal Music*. Boston: McGraw-Hill, 2008. Read pp. 69-104.

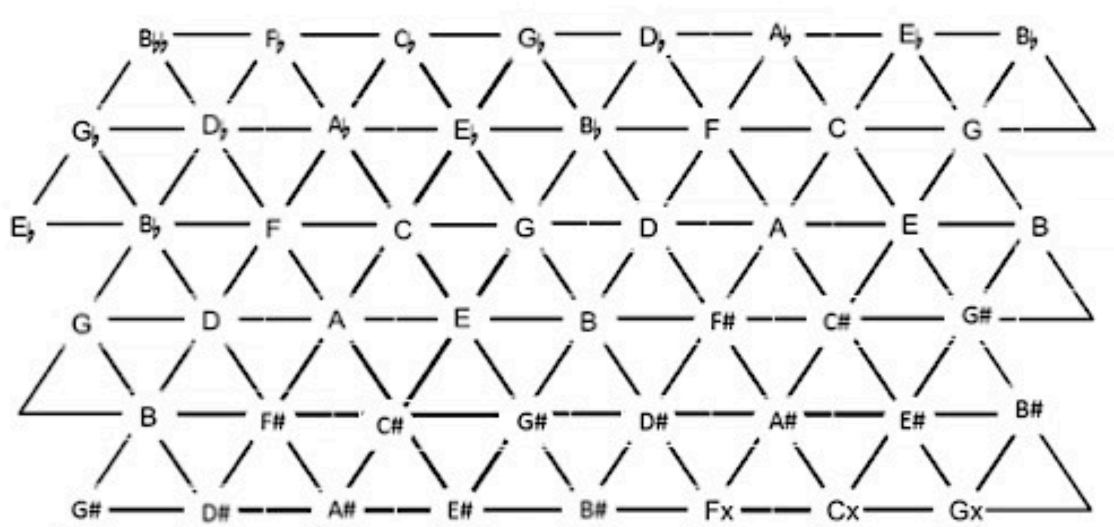
Straus, Joseph. "Centricity, Referential Collections, and Triadic Post-Tonality." in *Introduction to Post-Tonal Theory, 3rd ed*. Upper Saddle River, NJ: Prentice Hall, 2004. Read pp. 130- 181.

Whittall, Arnold. *The Cambridge Introduction to Serialism.* UK: Cambridge University Press, 2008. Read pp. 1-3.

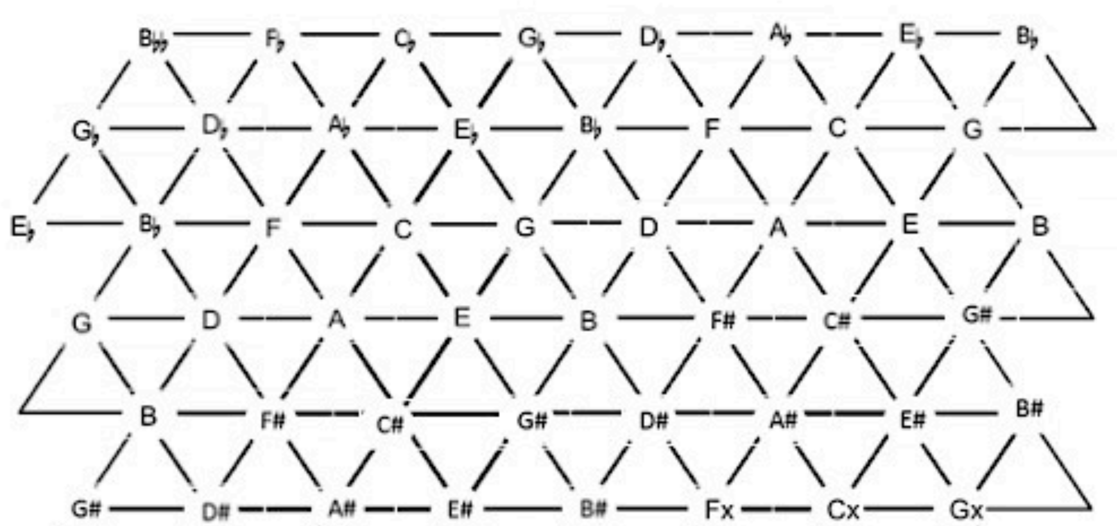
Suggestion for the Teacher: Exercises may be completed during the class period as part of the lesson or given as homework.

Δ Written Exercises

1. Chart each chord on the Tonnetz and chart a P transformation, followed by an L transformation, followed by an R transformation.
    a. Start on (E,-).
    b. Start on (D,+).
    c. Start on (Db,+).

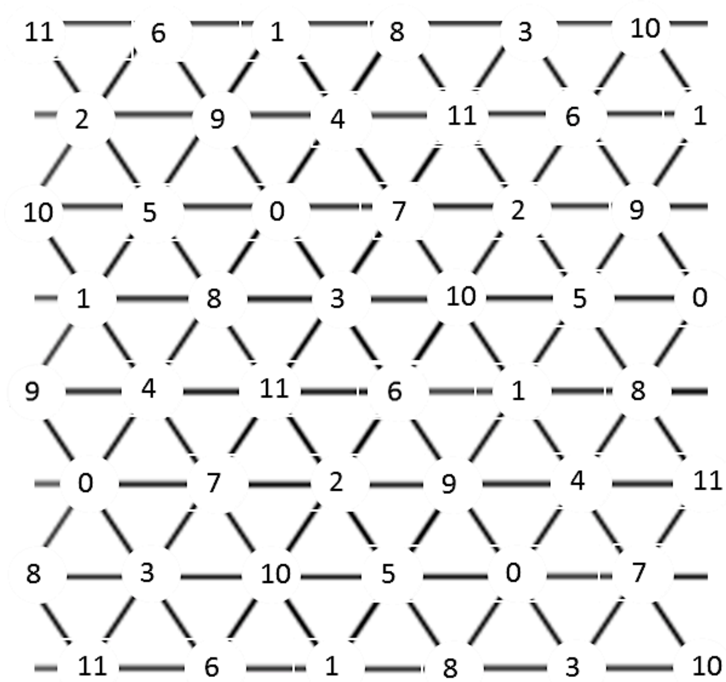


2. On the Tonnetz, chart the transformations of the following chord progression: (C,+) ⇨ (A,-) ⇨ (F,+) ⇨ (D,-) ⇨ (Bb,+) ⇨ (G,-) ⇨ (Eb,+). (In this text, arrows [⇨] will be used to indicate the movement from triad to triad within a progression.) Does a pattern emerge from these transformations?



Suggestion for the Teacher:
Students may also complete the same task using any set of your choosing.

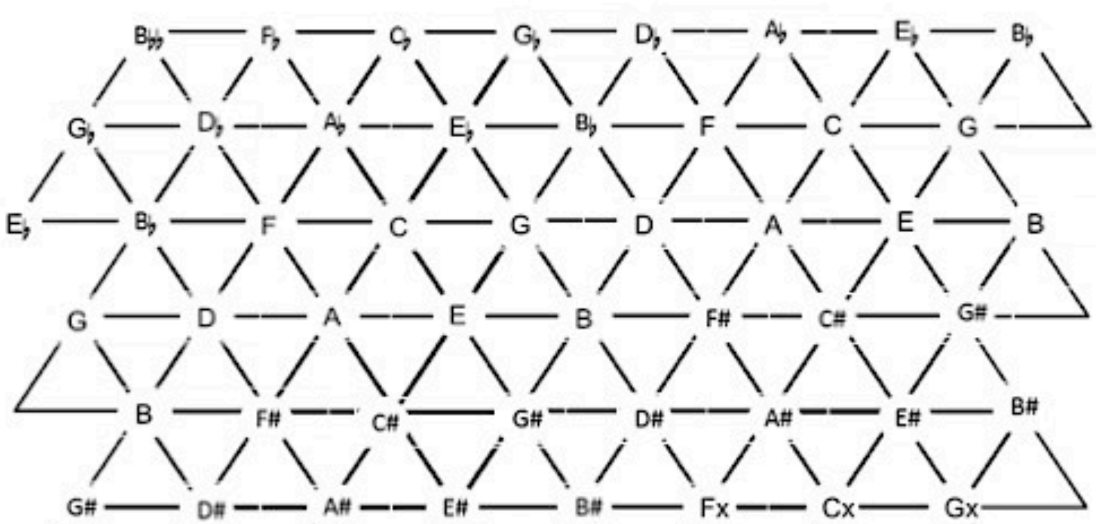
3. Perform each NRT transformation on PC set [047]. Which PC sets result? What is the relationship of the resulting sets?



4. On the Tonnetz, identify the transformations that must be made to progress from (D,+) to (Gb,+) and the triads in the progression. To progress to (Gb,+), you will have to make multiple transformations.

(D,+) ______ ______ ______ (Gb,-)

P ____ ____ *L*



6. Analyze mm. 1-8 of Christian Sinding's *Rustle of Spring, Op. 32, No. 3* by tracing NRT transformations.

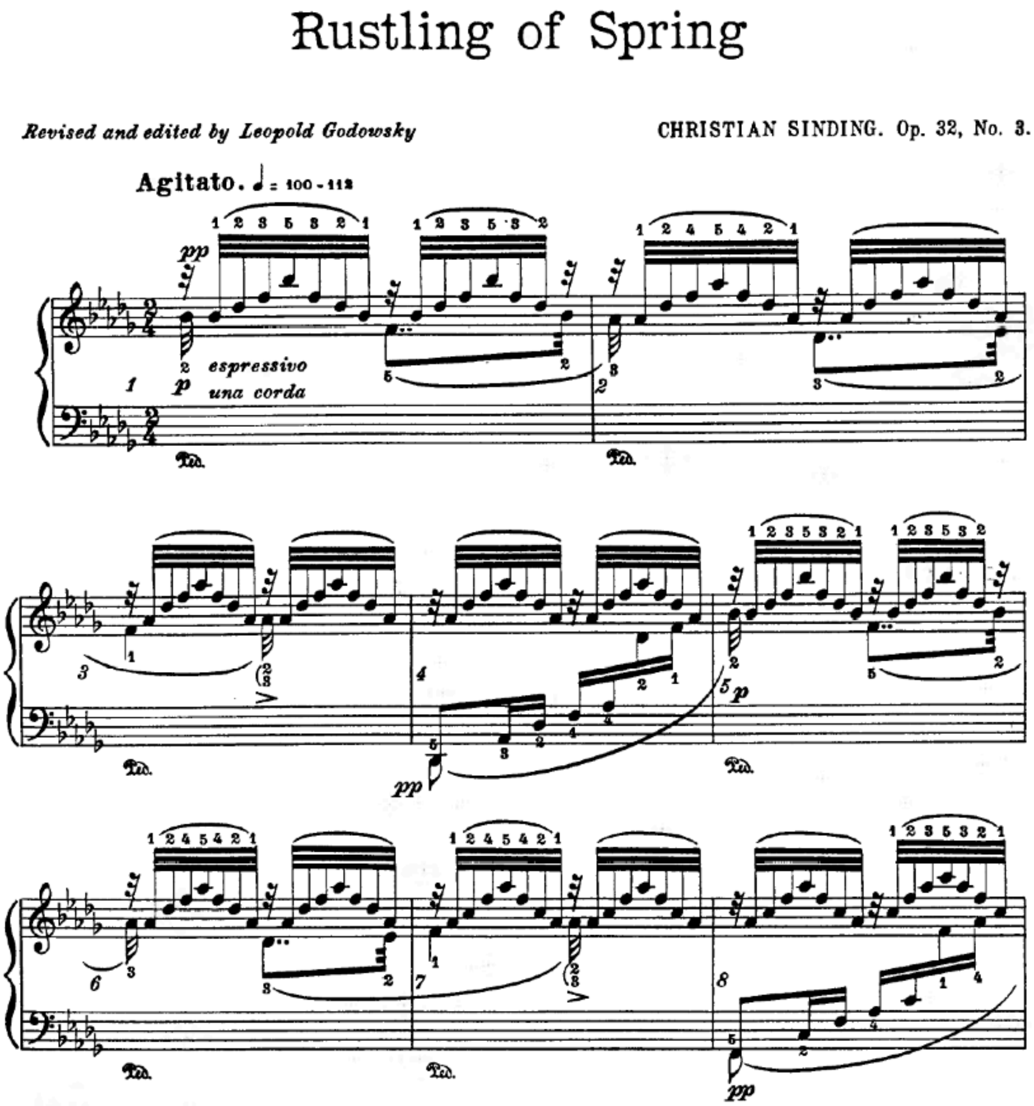
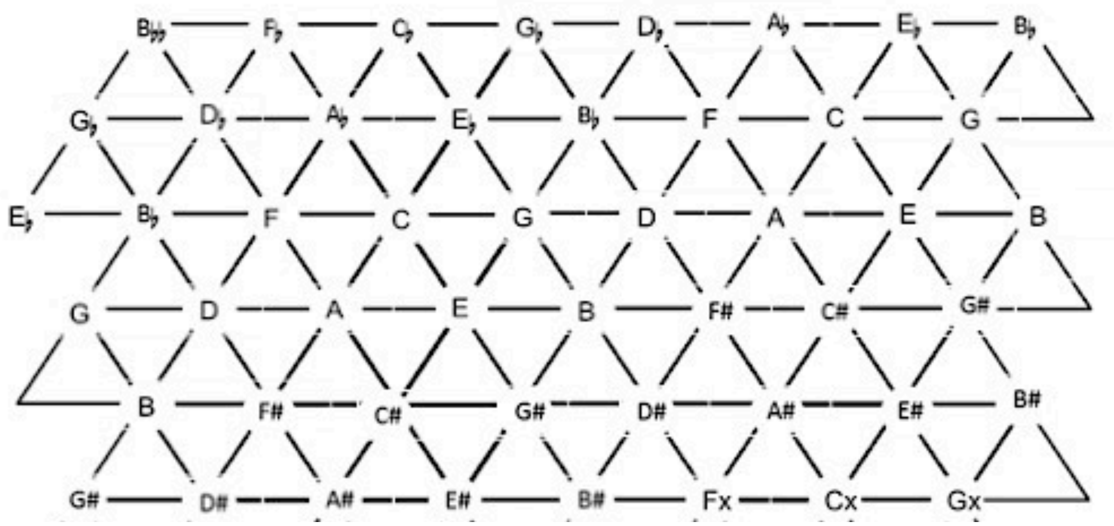


Figure 18: mm. 1-8 of Christian Sinding's *Rustle of Spring, Op. 32, No. 3*. Public Domain.



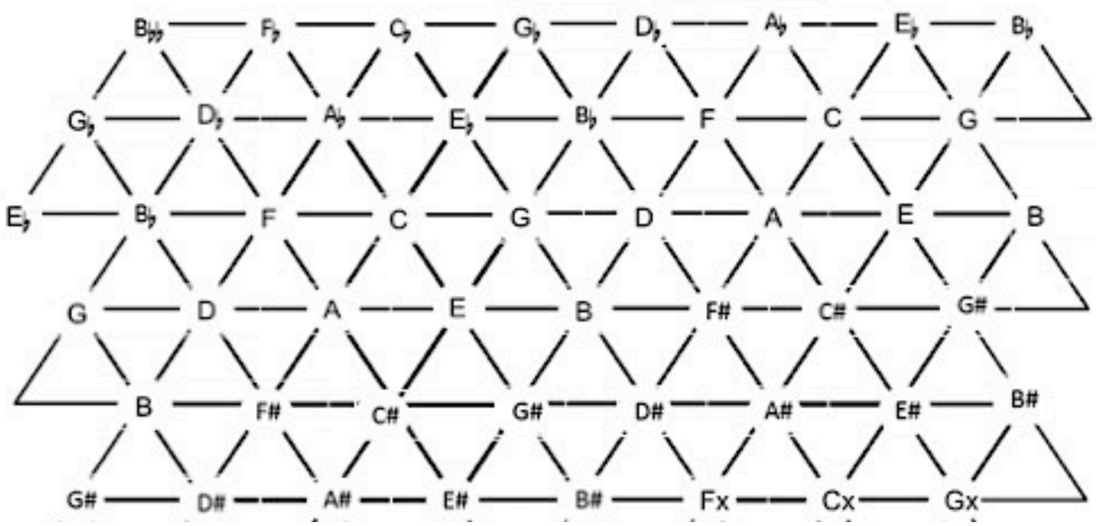
7. Analyze mm. 4-8 of Erik Satie's *Le Piccadilly* by tracing NRT transformations.

Le Piccadilly

Marche

Erik Satie
(1904)

Figure 19: mm. 4-8 of Erik Satie's *Le Piccadilly*. Public Domain.


B♭♭ F♭ C♭ G♭ D♭ A♭ E♭ B♭
G♭ D♭ A♭ E♭ B♭ F C G
E♭ B♭ F C G D A E B
G D A E B F# C# G#
B F# C# G# D# A# E# B#
G# D# A# E# B# Fx Cx Gx

Δ Keyboard Exercise

1. Play the progression from Written Exercise #4 using SAB parsimonious voice leading.

Δ Composition Exercises

1. Write an 8 bar progression using *P*, *L*, *R* transformations. After you have decided on a progression, add rhythmic values and create a melodic line, including non-chord tones.
2. Write a short composition with ternary (ABA') form where A modulates to B through 3 transformations and where B modulates to A' through 3 transformations.

Δ Aural Skills Exercises (Instructions for the Teacher)

1. Break the class into 3 parts: SAB. Have each section move one voice at a time to create PLR transformations. Ask students to identify the transformations.
2. Break the class into 3 parts: SAB. Ask students to complete transformations without identifying which voice should move, i.e. say "parallel." The voice singing $\hat{3}$ would move to b$\hat{3}$.
3. Play the progression (C,+) ⇨ (A,-) ⇨ (F,+) ⇨ (D,-) ⇨ (Bb,+)⇨(G,-) ⇨(G,+) ⇨ (C,+). Have students identify the changing voice and chart the transformations on the Tonnetz. (Other listening exercises may be conducted in the same manner.)
4. Listen to the first 12 measures of Richard Strauss's *Also Sprach Zarathustra* and ask students to identify the two transformations that occur.

CHAPTER TWO: CYCLES AND COMPOUND TRANSFORMATIONS

The Cyclic Nature of the NRT Transformations

Often times an NRT analysis includes a cycle, which is a series of repeated NRT transformations. In 1996, Lewin wrote the article "Cohn Functions" detailing Cohn's work that uses mod-12 when considering the construction of **non-trivial cycles** of pitch class sets.[39] Non-trivial cycles are series of triads that are at least 3 pc sets in length.[40]

In the same year, Cohn wrote "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," detailing his theory of **maximally smooth cycles***.* Maximally smooth cycles are constructed such that any pc set in the cycle becomes another pc set in the cycle by moving a single note by a half-step. The most famous of these cycles is the **circle of fifths,**[41] which is considered maximally smooth due to the change of one pitch of the scale to create another scale. For instance, C Major becomes G Major when F is raised to become F#.)[42]

In Figure 20, Cohn has reduced mm. 268-79 of Brahms' *Concerto for Violin and 'Cello*, first movement to show the maximally smooth cycle that Brahms wrote. Notice that the changing notes have filled-in noteheads. Also, note that the open noteheads show the two maintained pitches from one chord to the next. The maintained pitches are also indicated by

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[39] Douthett, et al. "Introduction," 4.
[40] Douthett, et al., "Filtered Point-Symmetry and Dynamical Voice-Leading." In *Music Theory and Mathematics: Chords, Collections, and Transformations*. (Rochester, NY: University of Rochester Press, 2008), 76.
[41] Douthett, et al. "Introduction," 4.
[42] Myles Leigh Skinner**,** "Toward a Quarter-Tone Syntax,"158.

arrow. Realize that this progression is maximally smooth because the pitches that move do so by half-step.

Figure 20: Cohn's reduction of Brahms' Concerto for Violin and 'Cello, first movement, mm.268-79.[43]

The LP/PL Cycle

One type of cycle that is frequently found in literature is the LP/PL cycle. Brahms' Concerto charted on the Tonnetz below serves as a nice example of this cycle type. (See Figure 21.[43]) Note that the Tonnetz should be three-dimensional and would continuing rolling on itself; in which case, the progression would start exactly where it began.

While speaking of the development of NRT, Joe Argentino states, "The characteristic trait of hexatonic passages is the disappearance of tonally driven triadic progressions (directed fifth motions), replaced by, as an example, symmetrical divisions of the octave (major third motions), where motion from one chord to the next is frequently characterized by efficient,

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[43] Cohn,"Maximally Smooth Cycles," 15.

smooth voice-leading and an abundance of common tones."[44] These passages are related by thirds. Brahms's progression is one such progression: it is an LP cycle.

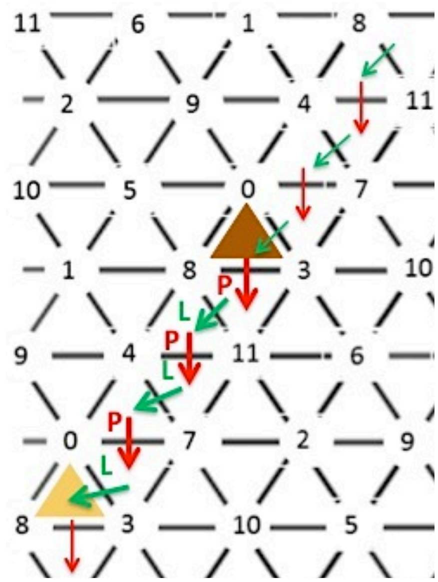


Figure 21: The charted Tonnetz for Brahms' *Concerto for Violin and 'Cello*, first movement, mm. 268-79.

LP and PL cycles are the same cycle only in reverse. For example:

LP: (C,+) *L* = (E,-) *P* = (E,+) *L* = (G#,-) *P* = (Ab,+) *L* = (C,-) *P* = (C,+)

PL: (C,+) *P* = (C,-) *L* = (Ab,+) *P* = (G#,-) *L* = (E,+) *P* = (E,-) *L* = (C,+)

There are four hexachords that result from the aggregate pcs involved in four distinct triadic starting points for LP/PL cycles: (C,+), (Db,+), (D,+), and (Eb,+). (See Figure 22.) Notice that there are no shared triads between hexachords, because each of the four hexachords is a

[44] Argentino, "Transformation," 1.

closed cycle. If the pcs are taken from each of the resulting triads and combined, the repeated pitches are cancelled, and then the hexachord results.

(C,+) (E,-) (E,+) (G#,-) (Ab,+) (C,-) (C,+)

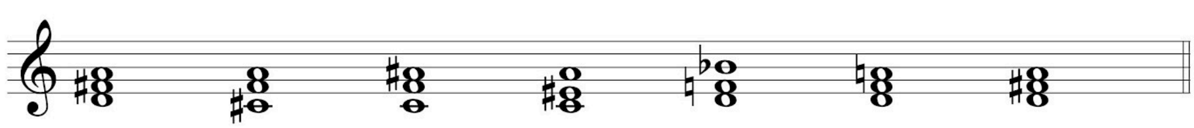
[047] [47B] [48B] [8B3] [803] [037] [047] = [03478B]

Performing either PL or LP three times will result in the original triad (as seen above). Engebretsen and Broman state, "These cycles are often referred to as a hexatonic cycles [sic] because each engages all and only those pitch-classes forming a hexatonic collection."[45] Skinner states, "The conventional PL cycle . . . is a significant compositional resource for late Romantic composers such as Wagner, Franck, Liszt, Mahler, and Richard Strauss."[46]

	P		L		P		L		P		L		
C,+		C,-		Ab,+		Ab,-		E,+		E,-		C,+	[3478B0]
Db,+		Cb,-		A,+		A,-		F,+		F,-		Db,+	[014589]
D,+		D,-		Bb,+		Bb,-		Gb,+		Gb,-		D,+	[12569A]
Eb,+		Eb, -		B,+		B,-		G,+		G,-		Eb,+	[2367AB]

Figure 22:The PL cycles

∇ *Learning Activity 1: Charting a Cycle on the Tonnetz*



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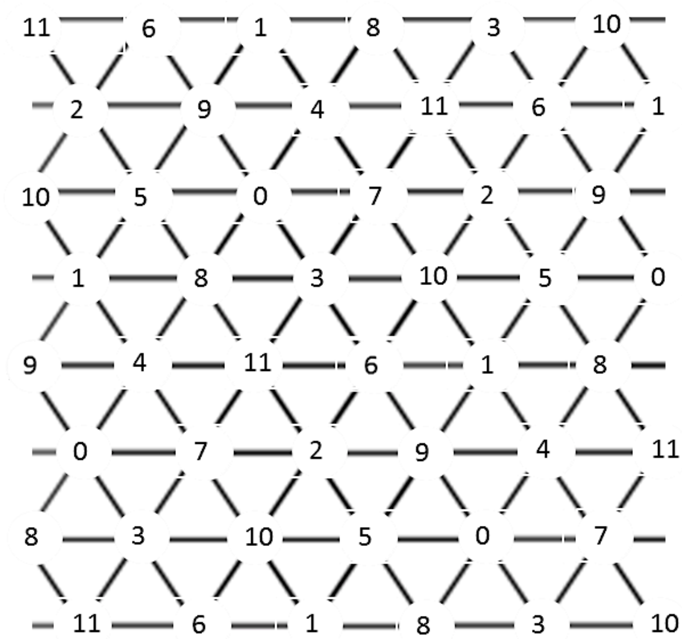
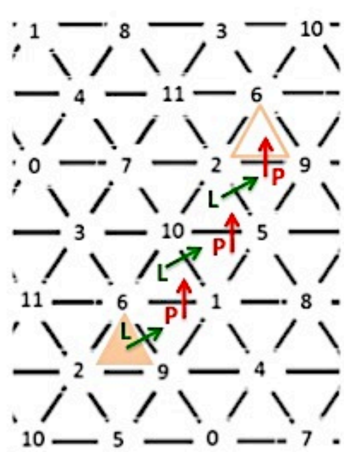
[45] Engebretsen and Broman, "Transformational Theory," 48.
[46] Skinner, "Quarter-Tone Syntax," 223.

→ Following the instructions from Chapter One, chart the cycle above[47] on the Tonnetz just as you would a non-cyclic progression.

→ (Hint: First, spell the triads using integers.)



Notice that the cycle ends on the same triad on which it begins. After completing the exercise above, your answer should look like:



[47] Milan Kidd, *An Introduction,* 4.

Notice that the LP hexachords divide up the twenty-four major and minor chords into four distinct tonal regions, and all are all members of the 3-11 set class. Richard Cohn states, ". . . What is unique about set-class 3-11, together with its nine-note complement, is the capacity of its member sets to form an ordered set of maximally smooth successions that is long enough to be perceived as a cycle . . . yet short enough that it does not exhaust all the members of its set-class."[48] In other words, each of the hexatonic cycles come from set class 3-11 and form short cycles, which each utilize only six of the 24 triads within the set-class.

"'Hexatonic' is a nickname for set-class 6-20, whose prime form is (0, 1, 4, 5, 8, 9). As one of the all-combinatorial hexachords, 6-20 maps itself under three distinct transpositions and three distinct inversions. Consequently, there are only four distinct pc sets, each of which

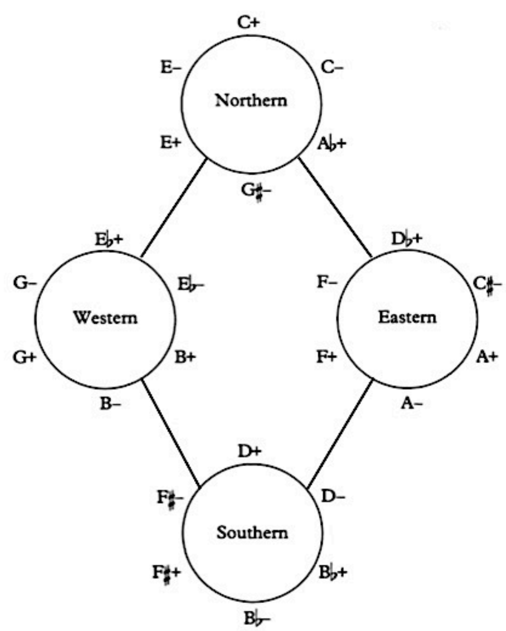


Figure 23: Richard Cohn's Illustration of The Hyper-hexatonic System[49]

[48] Cohn, "Maximally Smooth Cycles," 17.
[49] Cohn, "Introduction," 175.

supplies the fund of pcs from which the triads in a single hexatonic system are drawn." Each of the hexachords in Figure 23 has a regional name given by Cohn. The cycles are related to each other through the R transformation and form Cohn's Hyper-hexatonic system (see Figure 23). The (Ab,+) of the Northern cycle is transformed by R to (F,-) of the Eastern cycle. Likewise the (F,+) of the Eastern cycle is transformed by R to (D,-) of the Southern cycle and so forth. Figure 24 shows the adjacent cycles as they would appear on the Tonnetz. When read from left to right, Figure 24 shows portions of the LR cycle.

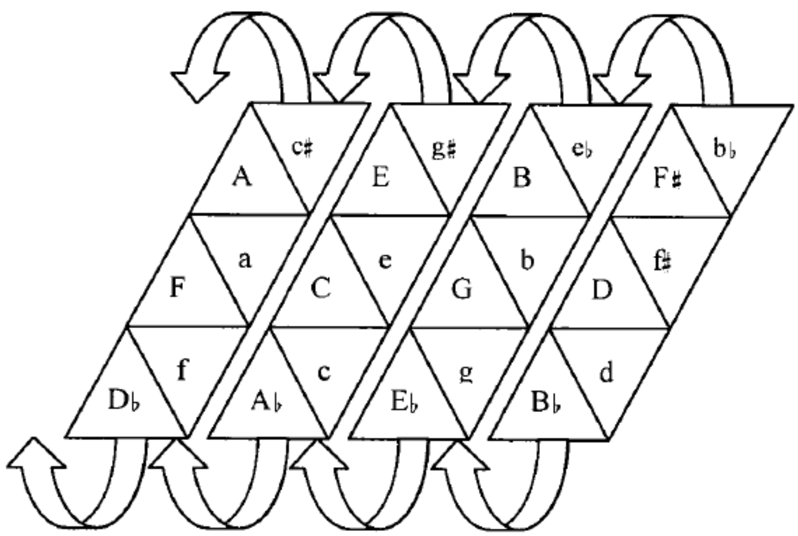


Figure 24: Adjacent LP transformations of Forte class 3-11 on the Tonnetz[50]

The PR/RP Cycle

Unlike the hexatonic LP/PL cycle, the PR/RP cycle creates a series of triads whose pcs break down into an octatonic collection.[51] Figure 25 shows a PR cycle. In the case of the top

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[50] Argentino, "Transformations," 7.

row of the chart, we can see that when the pcs are taken from each of the resulting triads and combined, the repeated pitches are cancelled, and then the octatonic collection results. Notice the PR/RP cycle yields only three different set classes.

(C,+) (C,-) (Eb,+) (Eb,-) (Gb,+) (Gb,-) (A,+) (A,-)

[047] [037] [37A] [36A] [6A1] [691] [914] [904] = [0134679A]

	P	R	P	R	P	R	P	R	
C,+	C,-	Eb,+	Eb,-	Gb,+	Gb,-	A,+	A,-	(C,+)	[0134679A]
D,+	D,-	F,+	F,-	Ab,+	Ab,-	B,+	B,-	(D,+)	[235689B0]
Db,+	Db,-	E,+	E,-	G,+	G,-	Bb,+	Bb,-	(Db,+)	[124578AB]

Figure 25: The PR cycles

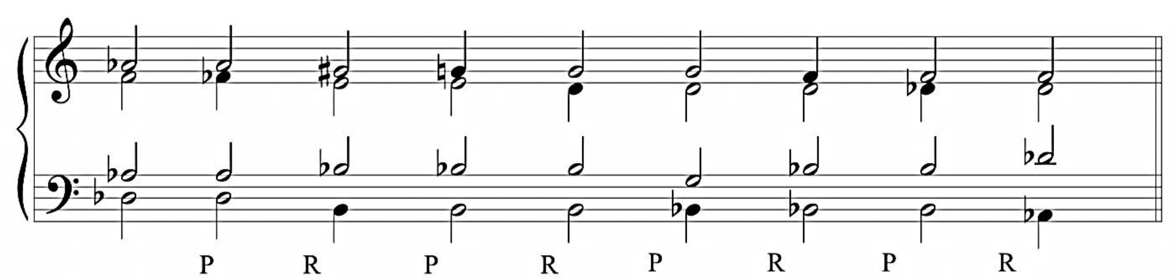

Figure 26: Sample RP cycle[52]

[51] Engebretsen and Broman, "Transformational Theory," 49.
[52] Ibid., 50.

The LR/RL Cycle

The complete LR cycle starting on (C,+) may be seen in Figure 27. Notice that the LR cycle goes through the entire series of consonant triads. Engebretsen and Broman state, “The LR cycle, which moves along the horizontals of the *Tonnetz*, differs from the LP and PR cycles in that it progresses through all twenty-four consonant cycles, rather than partitioning them into shorter cycles.”[53] Richard Cohn states, “The complete <LR> cycle is too long to sustain compositional interest under normal circumstances.”[54]

(C,+)	*R*	=	(A,-)	(A,-)	*L*	-	(F,+)
(F,+)	*R*	=	(D,-)	(D,-)	*L*	=	(Bb,+)
(Bb,+)	*R*	=	(G,-)	(G,-)	*L*	=	(Eb,+)
(Eb,+)	*R*	=	(C,-)	(C,-)	*L*	=	(Ab,+)
(Ab,+)	*R*	=	(F,-)	(F,-)	*L*	=	(Db,+)
(Db,+)	*R*	=	(Bb,-)	(Bb,-)	*L*	=	(Gb,+)
(Gb,+)	*R*	=	(Eb,-)	(Eb,-)	*L*	=	(B,+)
(B,+)	*R*	=	(G#,-)	(G#,-)	*L*	=	(E,+)
(E,+)	*R*	=	(C#,-)	(C#,-)	*L*	=	(A,+)
(A,+)	*R*	=	(F#,-)	(F#,-)	*L*	=	(D,+)
(D,+)	*R*	=	(B,-)	(B,-)	*L*	=	(G,+)
(G,+)	*R*	=	(E,-)	(E,-)	*L*	=	(C,+)

Figure 27: The RL cycle

[53] Engebretsen and Broman, “Transformational Theory," 49.
[54] Cohn, “Neo-Riemannian Operations,” 36.

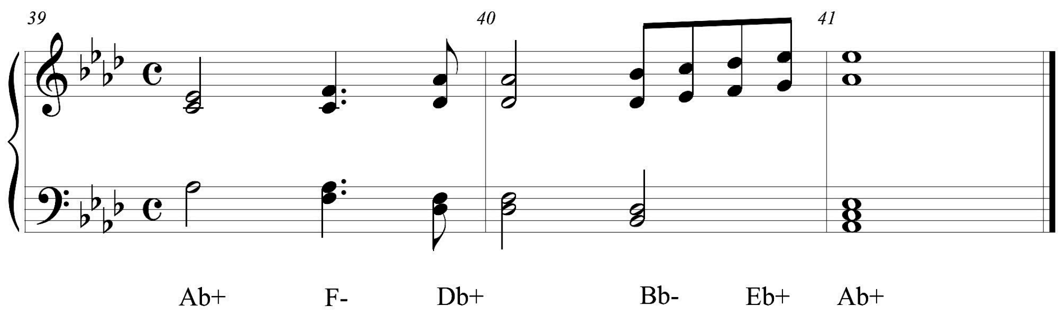

Figure 28: Wagner's *Parsifal*, Act I, mm. 39-41. Public Domain.

While the complete RL/LR cycle is too long to use in composition, it is possibly the most common translation chain found in literature. RL cycles can be found in diatonic and non-diatonic settings. A diatonic setting can be seen below in mm. 39-41 of Wagner's *Parsifal*, Act I, giving us another way of looking at this common progression (I-vi-IV-ii-V-I). (See Figure 28.)

Another famous example of an RL cycle is Ludwig van Beethoven's *Ninth Symphony.*[55] (See Figure 29.) The reduction of Beethoven's progression is shown in Figure 30 below. In Figure 31 below, the progression is charted on the Tonnetz. In Figure 32, Roig-Francolí gives a more detailed reduction of the second half of the chain. Notice that *a* shows the progression with the temporary tonicizations of each triad, while *b* eliminates the dominant triads of the dominant ⇨ tonic progressions to show the RL cycle that unifies the progression.

[55] Cohn, "Maximally Smooth Cycles," 33.

Figure 29: Beethoven's *Ninth Symphony*[56]

[56] Miguel A. Roig-Francolí, *Harmony in Context 2nd Ed.,* (NY: McGraw-Hill Companies, Inc., 2011), 735.

Figure 30: Roig-Francolí's reduction of Beethoven's *Ninth Symphony*[57]

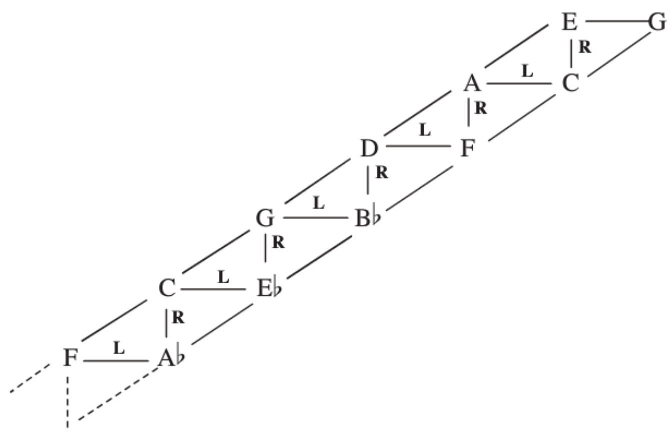

Figure 31: Roig-Francolí's diagram of Beethoven's *Ninth Symphony*[58]

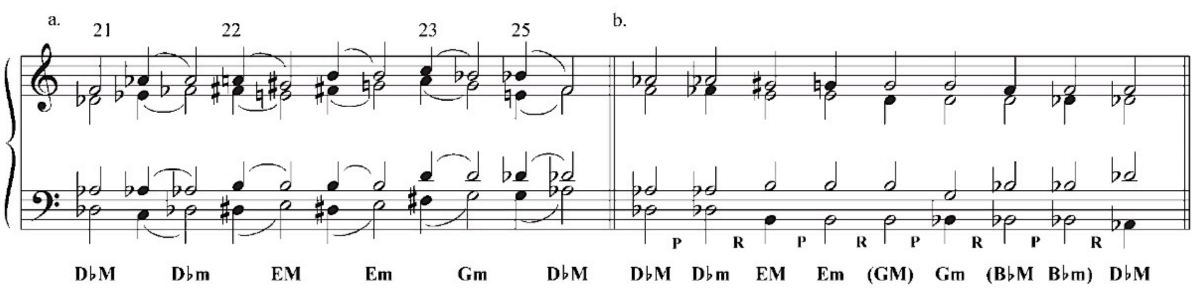

Figure 32: Roig-Francolí's reduction of Beethoven's *Ninth Symphony*[59]

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[57] Ibid., 735.
[58] Ibid., 735.
[59] Ibid., 735.

Another example of the RL cycle is that of Schubert's *Sonata in B-flat Major*, fourth movement, mm. 119-130. (See Figure 33.) Notice that Schubert interrupts the cycle with P in m. 123 but immediately returns to utilizing an RL cycle.

Figure 33: Capuzzo Analysis of Schubert, *Sonata in B-flat Major* (1828), IV, mm. 119-130[60]

Compound Transformations

Performing Compound Transformations

The NRT transformations P, L, and R were introduced in chapter one. Each of those transformations performed individually results in a new triad. **Compound transformations** are a combination of two or more individual transformations performed in succession to produce the new triad. While cycles have their place in literature, compound transformations can be found in many genres, including Pop-Rock music. In Figure 34, Lennon and McCartney have

[60] Capuzzo, Guy. "Neo-Riemannian Theory and the Analysis of Pop-Rock Music." *Music Theory Spectrum* 26, no. 2 (2004): 182.

started an RL cycle in *All My Loving*, which they dismiss with the compound transformation RP, (D,+) ⇨ (B,+). (See Figure 35.)

To perform compound transformations, take the nearest movement in the direction of the resulting triad on the Tonnetz and follow in logical route, using the shortest route possible. In the case of the RP transformation in *All My Loving*, (D,+) *R* = (B,-) *P* = (B,+).

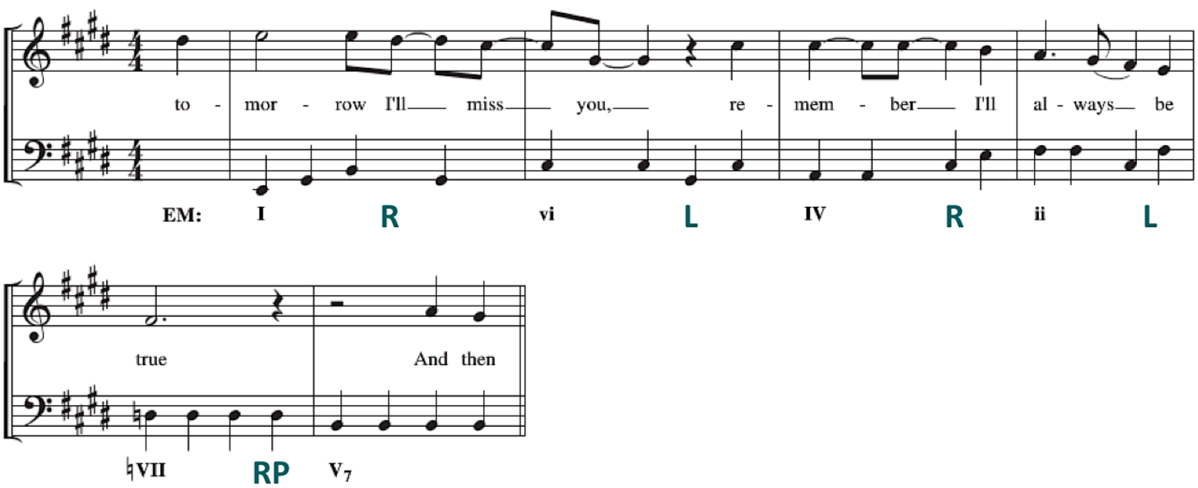


Figure 34: John Lennon and Paul McCartney's *All My Loving* from the album *With the Beatles*. Used with Permission.

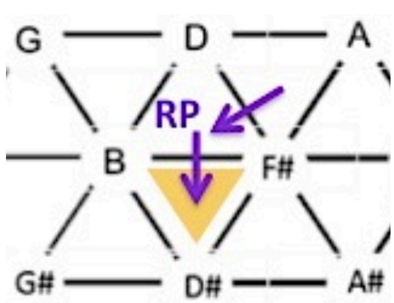


Figure 35: The compound transformation in John Lennon and Paul McCartney's *All My Loving* from the album *With the Beatles*

∇ Learning Activity #2: Compound Transformations

Start on (C#,-) and perform *PR*. Perform *PR* on the resulting triad. What is the final result?

(C#,-) *PR* = _____ *PR* = _____

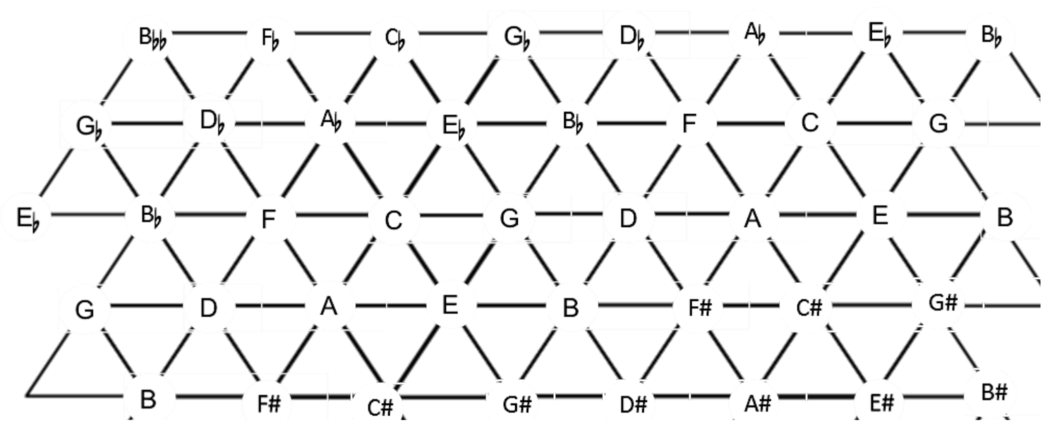
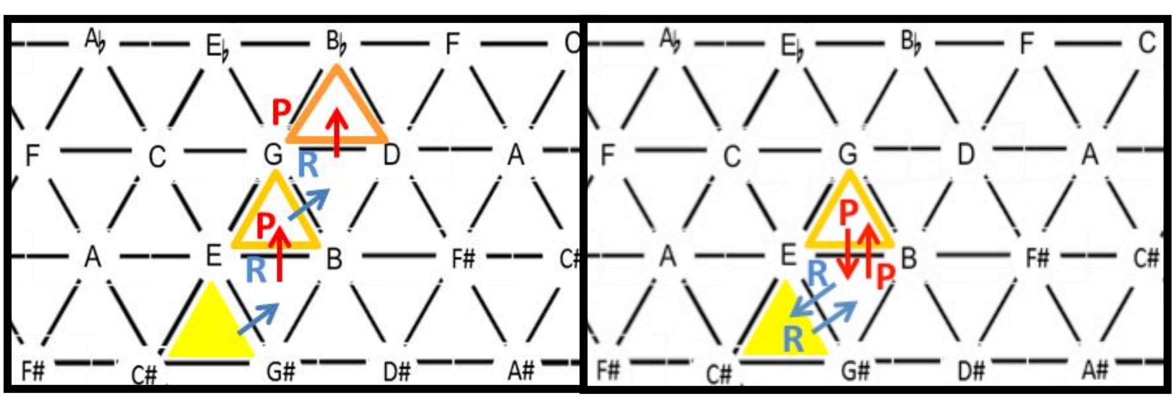


Figure 36 shows the progression (C#,-) ⇨ (E,-) ⇨ (G,-), which is the sollution to the activity above. (See Figure 36a.) Notice that unlike the individual transformations, compound



a b

Figure 36: The RP transformation is not an involution

transformations are not involutions. To return to the original triad, one must perform the opposite compound transformation. (C#,-) *RP* = (E,-) while (E,-) *PR* = (C#,-). (See Figure 36b.)

In the article "Neo-Riemannian Theory and the Analysis of Pop-Rock Music," Capuzzo successfully demonstrates the usefulness of NRT in describing the non-traditional progressions that are often found in popular music written within the last 60 years. One such piece that Capuzzo uses to demonstrate NRT's effectiveness is Ozzy Osbourne's "Flying High Again."[61] In Figure 37, Capuzzo has written out the guitar line in both standard notation and tablature, demonstrating the voice-leading parsimony.[62]

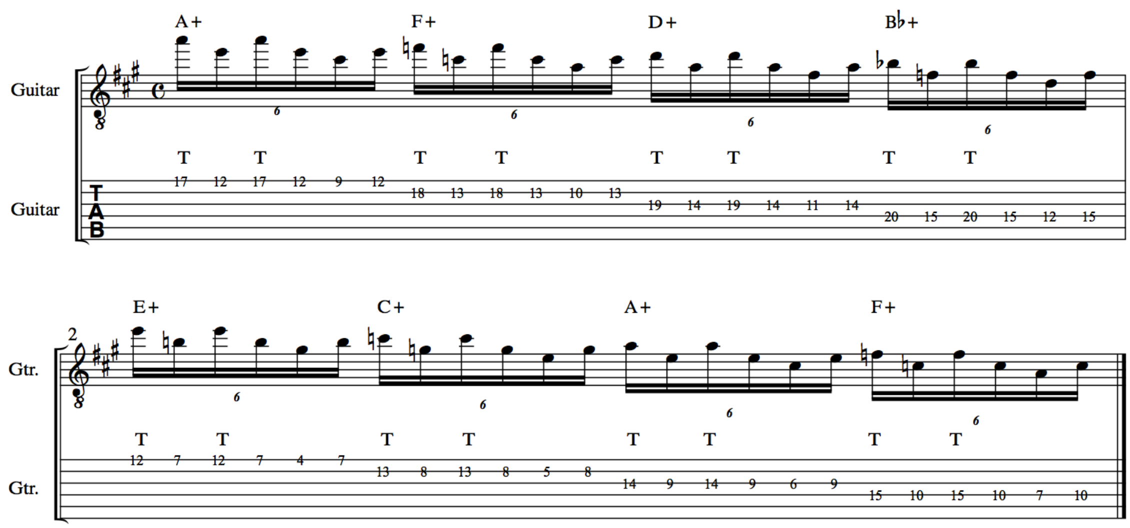


Figure 37: Capuzzo's analysis and reduction of Ozzy Osbourne's "Flying High Again"[61]

In Figure 38, Capuzzo reduces the progression to triad names and shows first the compound transformations performed above the chords and then demonstrates the progression (PL ⇨ RP ⇨ PL) is transposed (i.e. T_7) the second time. Or if related by

[61] Ibid., 184.
[62] Ibid., 184.

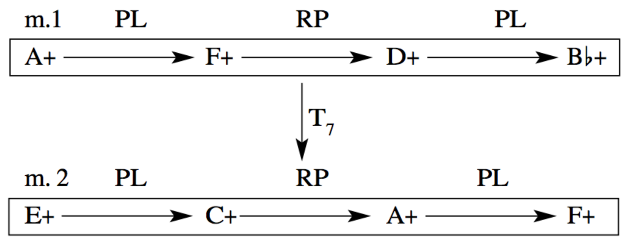


Figure 38: Capuzzo's transformational network of Ozzy Osbourne's "Flying High Again"[61]

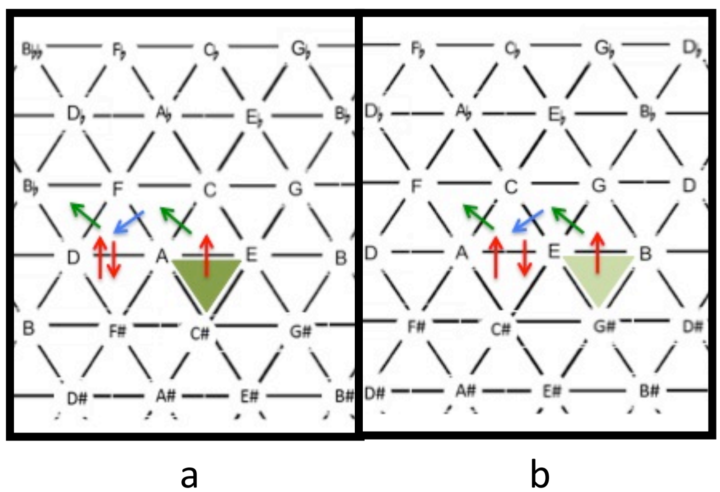


Figure 39: Ozzy Osbourne's "Flying High Again" charted on the Tonnetz. M.1 is shown in *a*, while m. 2 is shown in *b*.

transformation, the chords are the result of an RL transformation. [i.e. (A,+) *RL* (E,+) and so on.] Therefore, the entire progression can be shifted on the Tonnetz by RL, which shifts their placement over two triangles to the right.

Another rock song that utilizes compound transformations is Bob Dylan's "Lay, Lady, Lay." Figure 40 shows both occurrences of the progression: first as the original and second by the transposition T_n of seven. Figure 41 shows both instances charted on the Tonnetz. Like

Osbourne's "Flying High Again," the chords while transposed by seven are the result of an RL transformation [i.e. (A,-) *RL* (E,-)].

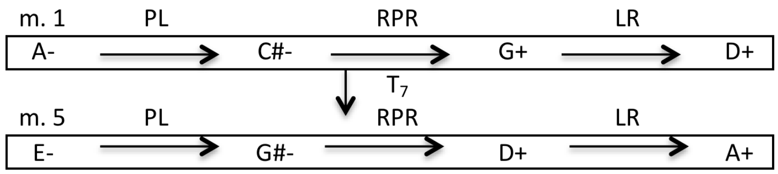


Figure 40: The progression from Bob Dylan's "Lay, Lady, Lay"[63]

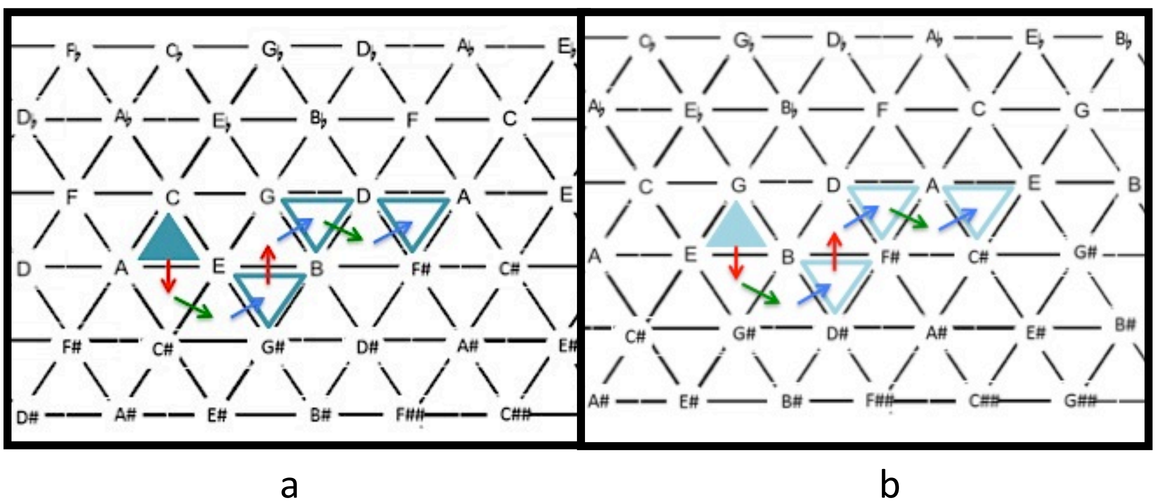


Figure 41: The charted progression from Bob Dylan's "Lay, Lady, Lay"

An Example Compound: PRL

An example of a compound transformation occurs when (C,+) becomes (G,-), and its NRT progression could be mapped as follows: (C,+) *P* = (C,-) *R* = (Eb, +) *L* = (G,-). Thus, the compound transformation (C,+) *PRL* = (G,-). (See Figure 42.)

[63] Ibid.,188.

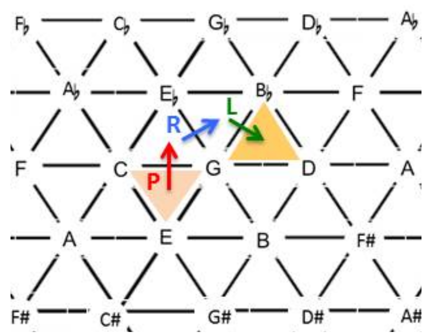


Figure 42: The compound transformation of (C,+) to (G,-).

If we reverse the progression, we arrive at the same triad. (C,+) *L* = (E, -) *R* = (G,+) *P* = (G,-). The compound transformation would be (C,+) *LRP* = (G,-). (See Figure 43.)

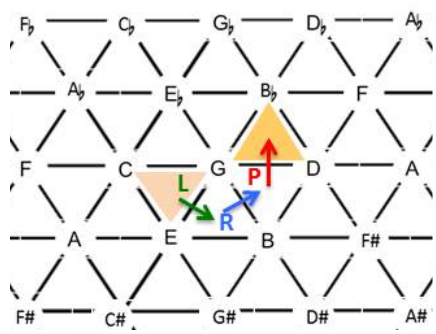


Figure 43: The LRP transformation of (C,+) to (G,-)

∇ Learning Activity #3: Compound Transformations

Complete the first compound transformation:

(C,+) *PRL* = (G,-), which performed in steps looks like

(C,+) *P* = (C,-) *R* = (Eb, +) *L* = (G,-)

and perform it a second time, starting with (G,-)

(G,-) *PRL* = ______

(G,-) *P* = ____ *R* = ____ *L* = ____

Notice also that the compound transformation PRL is an involution just as a single transformation is an involution even though other compound transformations are not involutions. (G,-) *P* = (G,+) *R* (E,-) *L* = (C, +). Figure 44 illustrates the involution. Likewise, the compound transformation LRP is an involution.

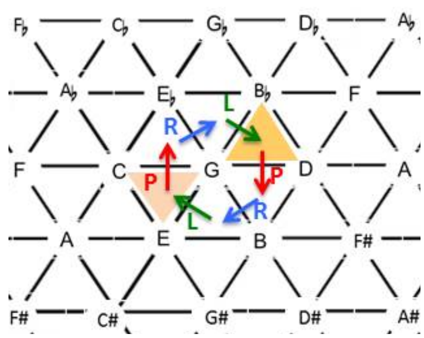


Figure 44: The Involution of the PRL compound transformation

Lewin's DOM Transformation

One of Lewin's single transformations, DOM, is not considered a single NRT transformation since it is not an involution. The DOM transformation moves a triad directly two triangles to the left. In other words, a triad moves to itself subdominant. For example, (D,+) *DOM* = (G,+). (See Figure 45a.) DOM is also not an NRT transformation because the DOM transformation is a compound transformation (*RL*). Likewise, DOM', which is the opposite of the DOM transformation, is the compound transformation (*LR*). (See Figure 45b.) An example of the DOM/compound *RL* transformation is mm. 37-43 of Rachmaninoff's *Prelude Op.32, No.*

4. (See Figure 46.) In Figure 46, on can see that Rachmaninoff begins an *LR* cycle, but dismisses it by performing a double transformation (*RL*) to move from (G,-) to (D,-) and then performs the double transformation again to move from (D,-) to (A,-). After passing between the triads for two measures, Rachmaninoff performs a *P* transformation.

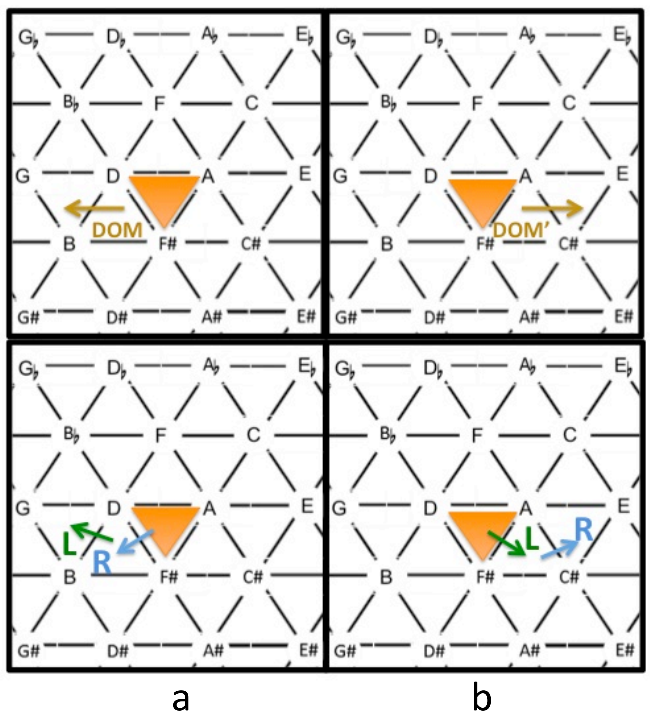


Figure 45: The DOM and DOM' transformations shown as compound transformations.

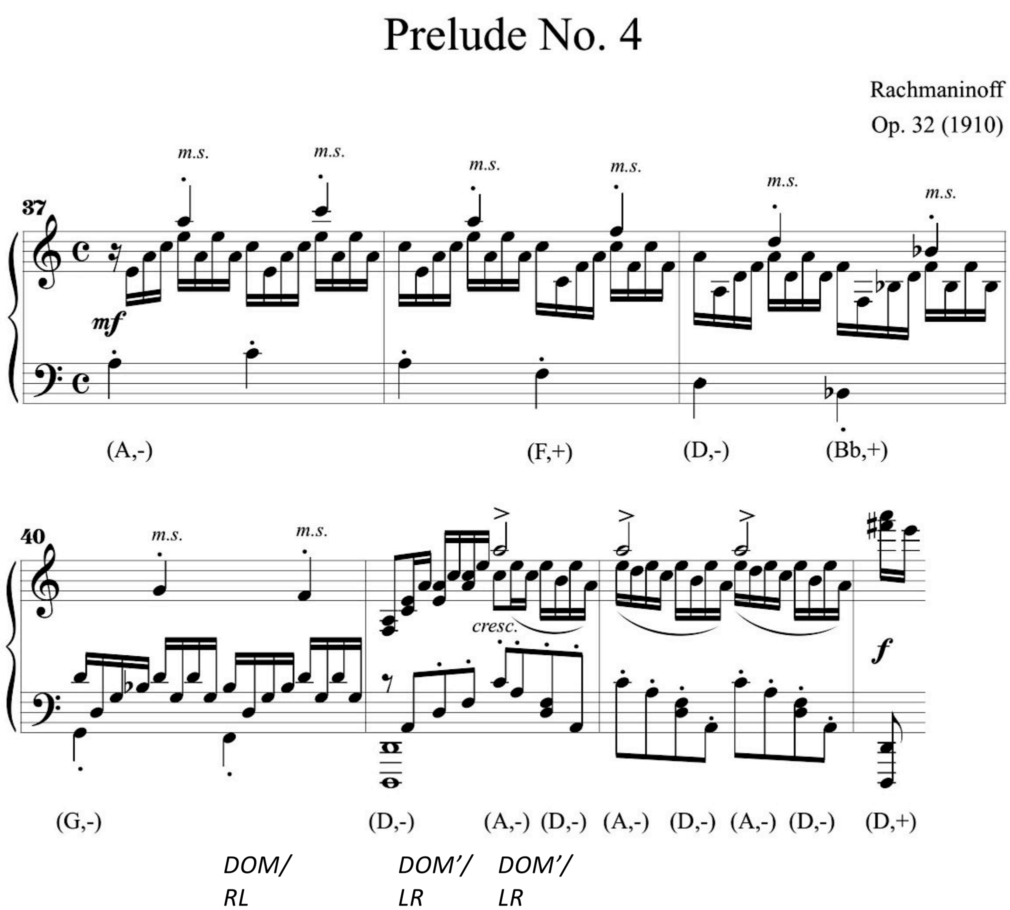


Figure 46: Sergei Rachmaninoff's *Prelude Op. 32, No. 4,* mm. 37-43. Public Domain.

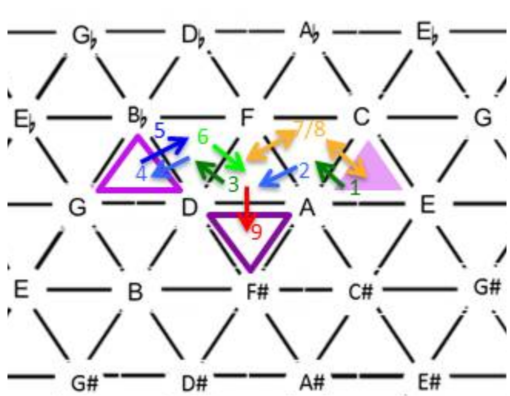


Figure 47: Rachmaninoff's *Prelude Op. 32, No. 4,* mm. 37-43 shown on the Tonnetz

Terms to Know

Circle of Fifths

Compound transformation

Maximally Smooth Cycle

Mod-12

Non-Trivial Cycles

Further Reading on NRT Cycles

Clugh, John, Nora Engebretsen, and Jonathan Kochavi. "Scales, Sets, and Interval Cycles: A Taxonomy." *Music Theory Spectrum* 21, no. 1, (Spring 1999): 74-104.

Cohn, Richard Lawrence. *Audacious Euphony: Chromaticism and the Consonant Triad's Second Nature*. New York: Oxford University Press, 2012. Read Chapter 2.

Cohn, Richard. "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions." *Music Analysis* 15, no. 1 (1996): 9-40. Read Section I.

Douthett, Jack M., Martha M. Hyde, Charles J. Smith, and John Clough. "Filtered Point-Symmetry and Dynamical Voice-Leading." In *Music Theory and Mathematics: Chords, Collections, and Transformations*. (Rochester, NY: University of Rochester Press, 2008), 72-106. Read pp. 72-77.

Further Reading on Compound Transformations

Capuzzo, Guy. "Neo-Riemannian Theory and the Analysis of Pop-Rock Music." *Music Theory Spectrum* 26, no. 2 (2004): 177-200. Read All.

Cohn, Richard. "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions." *Music Analysis* 15, no. 1 (1996): 9-40. Read Section I.

Kidd, Milan. *An Introduction to the Practical Use of Music-Mathematics*. http://www.math.uchicago.edu/%7Emay/VIGRE/VIGRE2006/PAPERS/Kidd.pdf. Read all.

Δ Written Exercises

1. Describe the relationship of the chords in the treble clef of the piano accompaniment of mm. 1-3 of *Charlie Rutlage* by Charles Ives by using NRT transformations. Does a cycle occur?

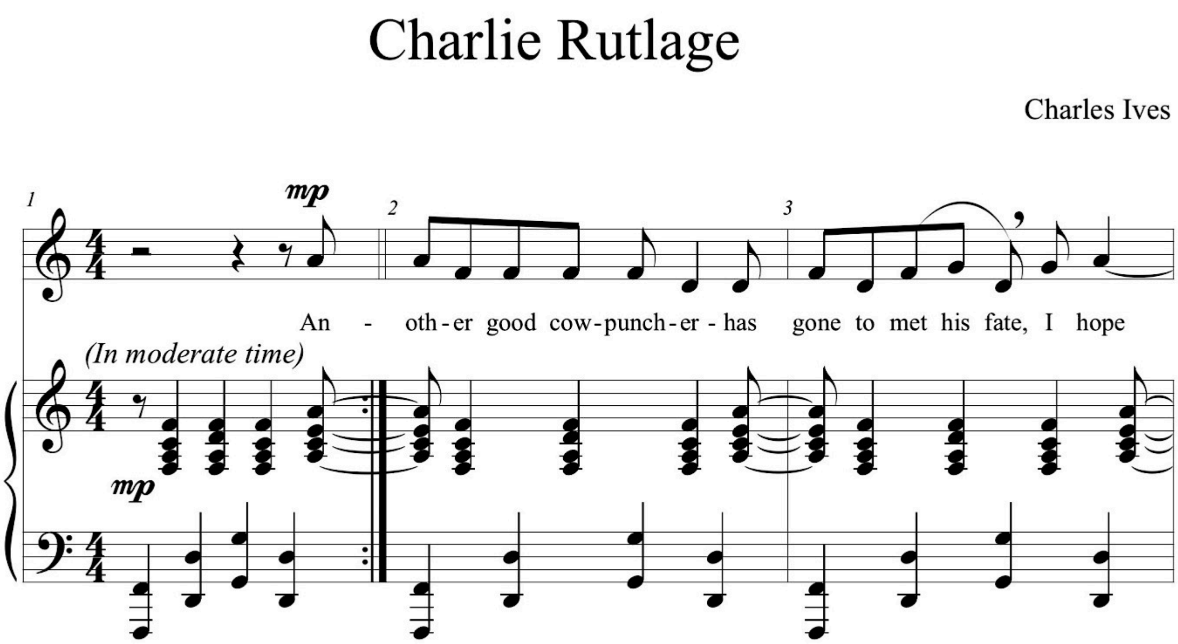
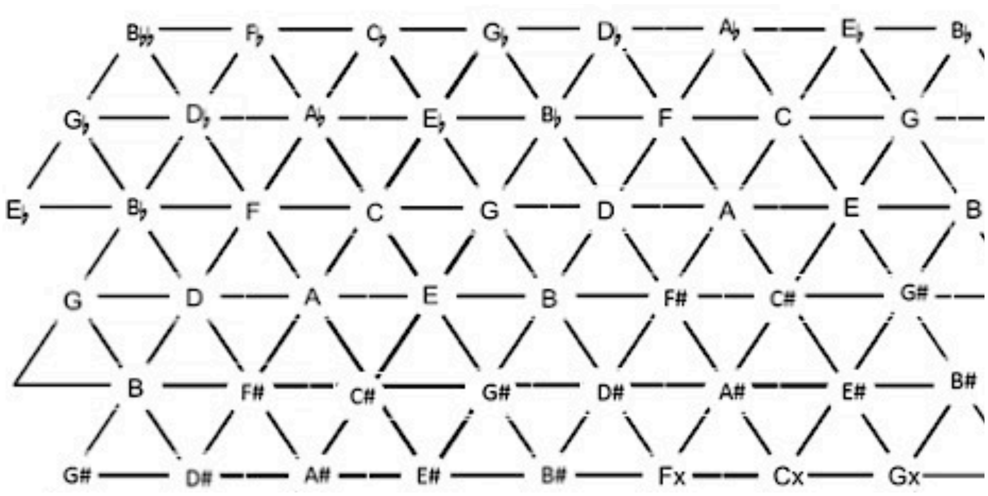
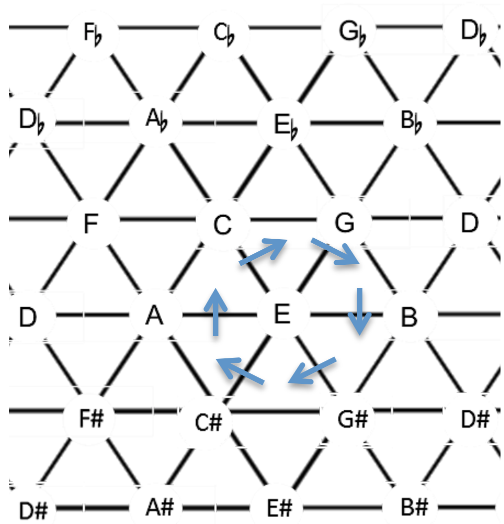


Figure 48: mm. 1-3 of *Charlie Rutlage* by Charles Ives. Public Domain.



2. Identify the cycle, beginning on (E,-).



3. Chart the progression on the Tonnetz and identify the cycle of Schubert's Overture to *Die Zauberharfe*, opening *Andante* (as reduced by Richard Cohn).[64]

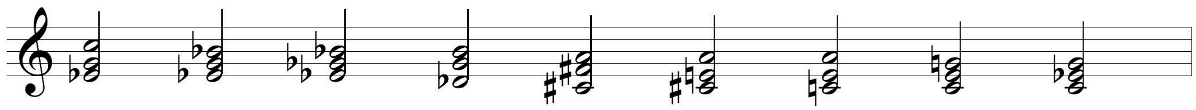
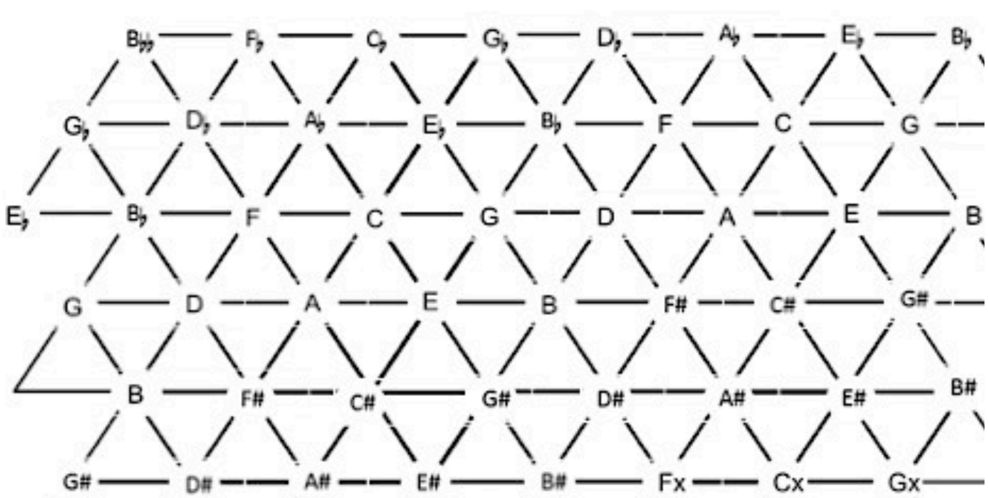


Figure 49: Schubert's Overture to *Die Zauberharfe*, opening *Andante* (as reduced by Richard Cohn).[62]



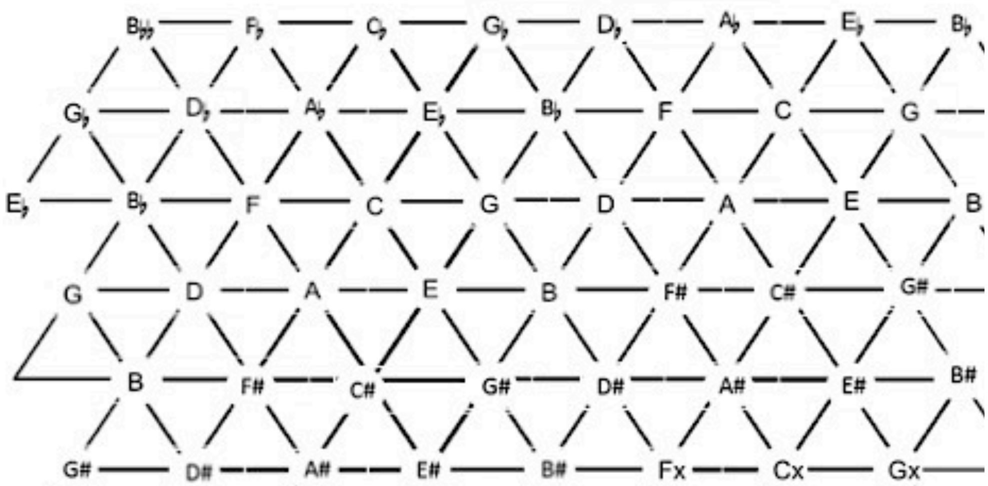
4. Navigate the following two cycles on the Tonnetz.

Starting on (B,-): P ⇨ R ⇨ L ⇨ P ⇨ R ⇨ L

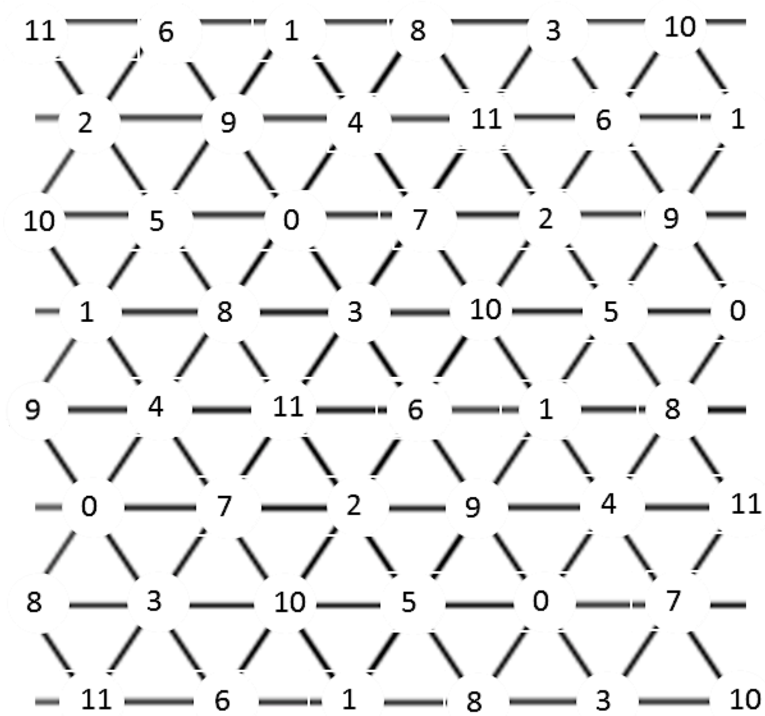
[64] Cohn, "Neo-Riemannian Operations," 35.

Starting on (F#,+): L ⇨ P ⇨ R ⇨ L ⇨P ⇨ R

Which pitches are shared by the two cycles? Which Triads are shared by the two cycles?

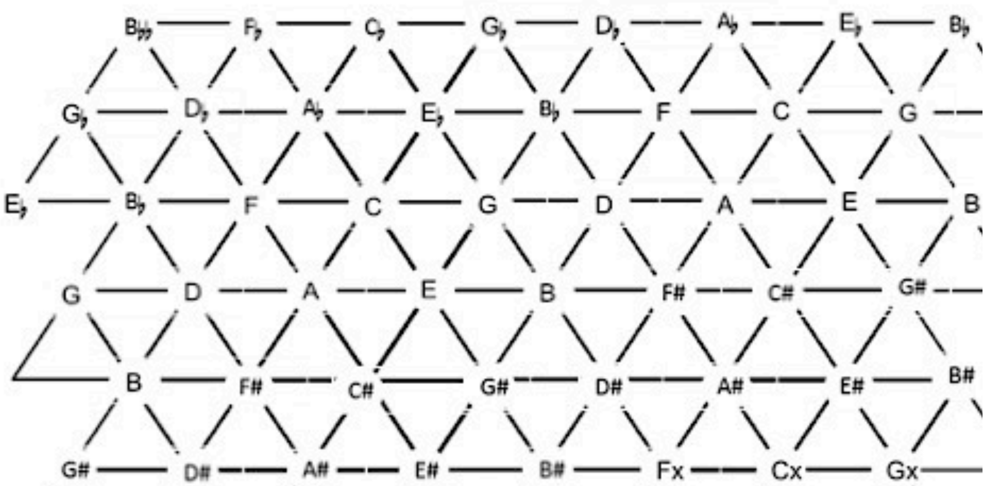


5. Complete a full PLR cycle on pc set [269]. Give the Tn/TnI relationships for each pc set to the original.



6. Perform the following compound transformations, identifying and marking each resulting Triad on the Tonnetz, starting with (Ab,+).

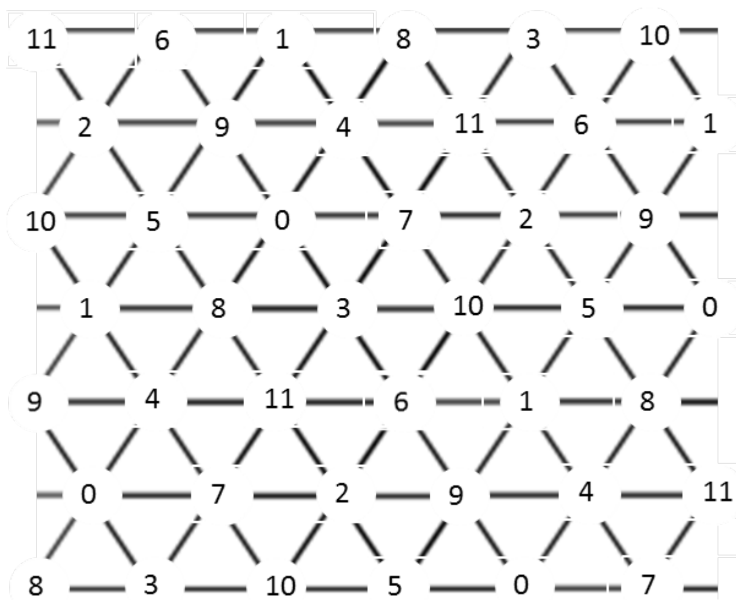
   RP ⇨ *RL* ⇨ *RL* ⇨ *PR* ⇨ *LR*



7. Perform the following compound transformations, identifying and marking each resulting Triad on the Tonnetz, starting with [058].

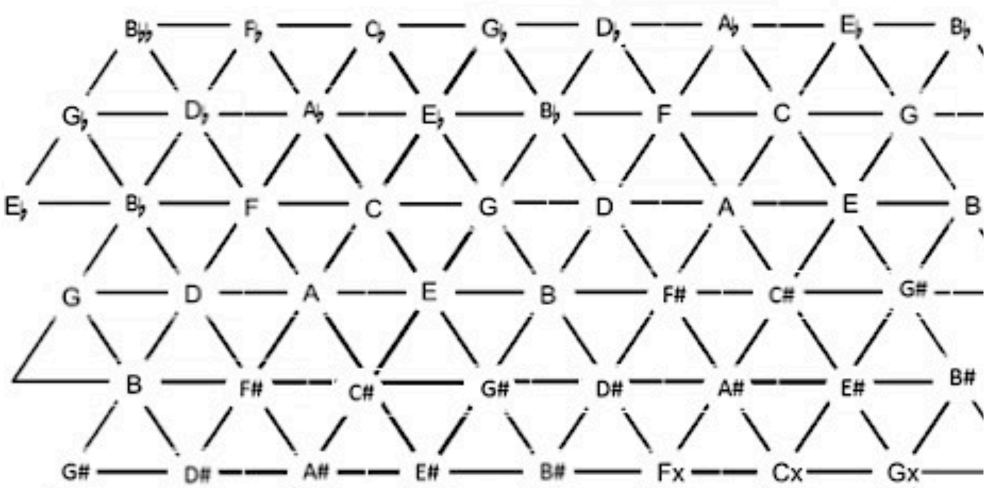
   RP ⇨ *RL* ⇨ *RL* ⇨ *PR* ⇨ *LR*

   Which pcs are shared the most shared? What pc set do they create?



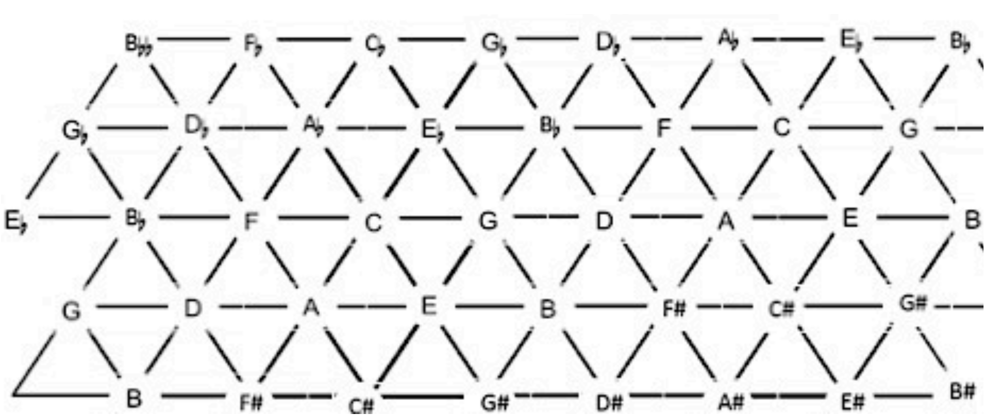
8. Analyze mm. 34-38 of Ernest Bloch's *Poems of the* Sea, II. Chanty using NRT transformations.

Figure 50: mm. 34-38 of Ernest Bloch's *Poems of the* Sea, II. Chanty. NY: G. Schirmer, 1923. Used with Permission.



9. Analyze the selection from Erik Satie's *Gnossienne No. 4* using NRT transformations.

Figure 51: Erik Satie's *Gnossienne No. 4.* Public Domain.



10. Analyze the selection from Dmitri Kabalevsky's *Sonatina in C Major, Op.13, No.1* using NRT transformations.

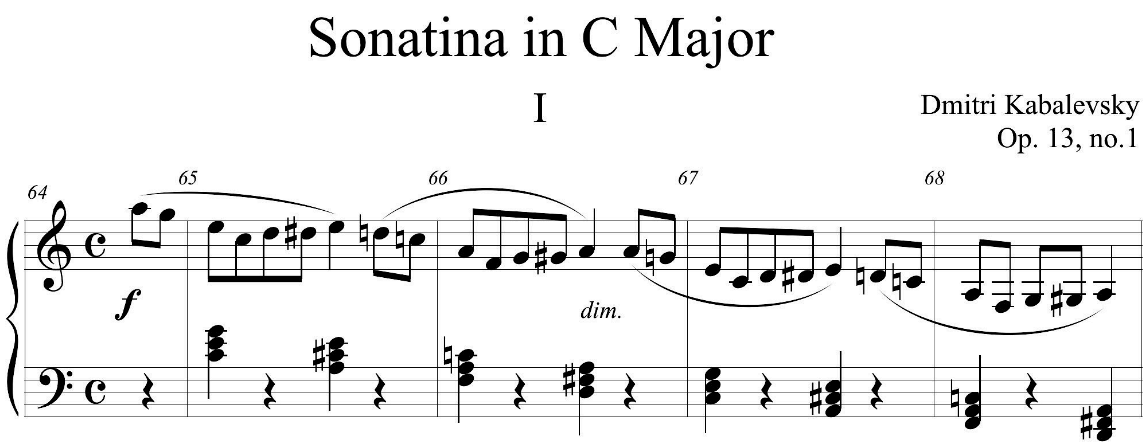
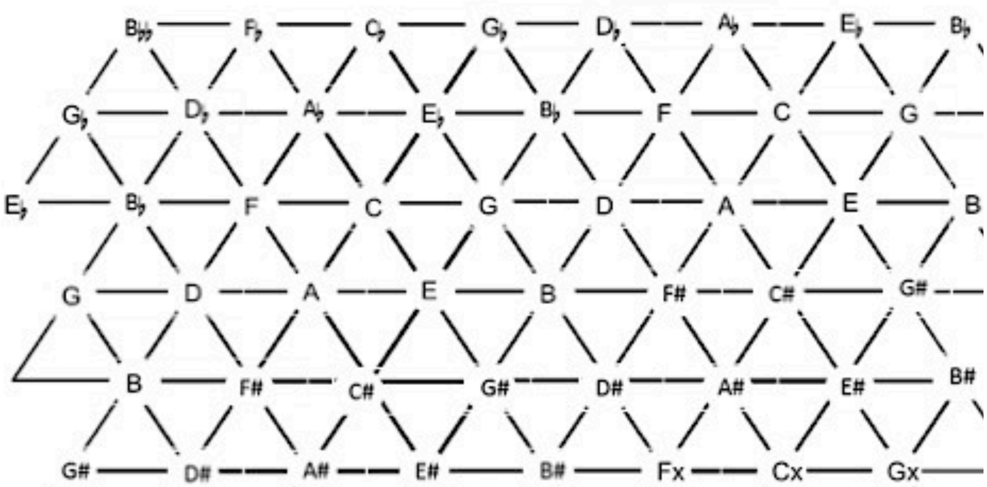


Figure 52: mm. 64-68 of Dmitri Kabalevsky's *Sonatina in C Major, Op.13, No.1.* Public Domain.



Δ Composition Exercise

1. Complete an 8-bar composition, using only the pitches from the cycle in Written Exercise #1.
2. Complete a composition of at least 8 bars containing at least 2 compound transformations.

Δ Keyboard Exercise

1. Play the progression (Db,+) ⇨ (Bb,-) ⇨ (G,-) ⇨ (E,-) with parsimonious voice-leading.

IN CONCLUSION: TRANSFORMATIONAL THEORY

Neo-Riemannian theory is a faction of a much larger theory, transformational theory. As an infant theory, transformational theory continues to develop. In recent years, many improvements have been made in several areas, including additional transformations, seventh chords, and pc sets that do not have major and minor sonorities or are larger than triads. (See Further Reading at the end of this section.) Transformational theory has added single transformations, some of which may not be involutions and transformations performed on augmented triads, a chord commonly found between parsimonious major and minor triads of late-Romantic progressions.[65] With each advance of NRT and transformational theory, changes are made to the Tonnetz and new Tonnetze are created.

The benefits of such an analytical system are numerous. Transformational theory as a whole allows for the analysis of many genres, including late Romantic works, Impressionism, aleatoric, Neo-Romantic, Minimalist, and Pop-Rock. Transformational theory allows the theorist to analyze music in concrete mathematical terms, defining relationships that are not seemingly present when analyzed with traditional Roman numeral analysis. When combined with digital technologies, Tonnetze and new representations of progressions give clarity and structure to passages that appeared to have none.

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[65] Popoff, Alexandre. "Towards A Categorical Approach of Transformational Music Theory." *ArXive-prints* (April 2012): 18.

While there are still musical areas to be hypothesized and explored within transformational theory, distinguishing works are abundant. I suggest those who are interested in further study familiarize themselves with the following pieces of literature.

Further Reading

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Cohn, Richard. “Square Dances with Cubes.” Journal of Music Theory 42, No. 2, Neo-Riemannian Theory (Autumn, 1998): 283-296.

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Taylor, Benedict. “Modal Four-Note Pitch Collections in the Music of Dvořák’s American Period.” *Music Theory Spectrum*, 32, no. 1 (Spring 2010): 44-59.

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Bass, Richard. "Enharmonic Position Finding and the Resolution of Seventh Chords in Chromatic Music." *Music Theory Spectrum* 29, no. 1 (2007): 73-100. http://www.jstor.org/stable/10.1525/mts.2007.29.1.73. Accessed August 24, 2012.

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VITA

Laura Felicity Mason received her Bachelor's degree in Music Education from Clearwater Christian College with a proficiency in piano. While at the University of Tennessee, Knoxville, she studied Music Theory. Laura holds a graduate teaching assistantship in the School of Music, where she teaches aural skills and music theory. She has special interests in Music Theory Pedagogy, Grundgestalt, and neo-Riemannian Theory. In addition to her responsibilities at the University, Laura enjoys working as a church pianist and teaching private piano and theory lessons.